



GLOBAL JOURNAL OF SCIENCE FRONTIER RESEARCH: A  
PHYSICS AND SPACE SCIENCE  
Volume 22 Issue 7 Version 1.0 Year 2022  
Type: Double Blind Peer Reviewed International Research Journal  
Publisher: Global Journals  
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

# Noncommutative Quantum Gravity and Symmetry of Klein-Gordon Equation

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*GJSFR-A Classification:* DDC Code: 523.1 LCC Code: QB981



*Strictly as per the compliance and regulations of:*



# Noncommutative Quantum Gravity and Symmetry of Klein-Gordon Equation

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**Abstract-** In the paper 'A New Approach to Quantum Gravity'[1], we suggest a new approach to quantum gravity. Using this theory, we can study the noncommutative gravitational field in momentum space. In this paper, we obtain the general form of the Klein-Gordon equation in noncommutative gravitational field. Then we find the symmetry associated with noncommutative gravity from the Klein-Gordon equation. We study black hole in momentum space and conclude that the event horizon of black holes is formed by the dipoles in momentum space with limit state.

## I. INTRODUCTION

In the paper 'A New Approach to Quantum Gravity'[1], we suggest a new theory of quantum gravity, give the propagator of the graviton, solve the difficulty of the Feynman integral divergence, and give evidence to prove that this theory is classical equivalent to the general theory of relativity. In this paper, we discuss the multi-graviton system and the self-interaction between gravitons. In momentum space, we obtain the general form of the Klein-Gordon equation in the gravitational field and find the symmetry associated with noncommutative gravity. We give the metric of the gravitational field of the multi-graviton system. There are singularities in this metric, which can produce black holes.

In section 2, we give a brief review of the quantum gravity theory suggested in the paper[1]. In section 3, we discuss the multiple-graviton system with self-interaction, giving the metric of the multiple-graviton system. In section 4, we calculate the Klein-Gordon equation in gravitational field. Due to the specificity of the metric of curved space caused by gravitational field in momentum space, from the Klein-Gordon equation in curved space, we obtain the general form of the Klein-Gordon equation in gravitational field, which is exactly the usual form of the Klein-Gordon equation in quantum field theory. Then we find the symmetry associated with noncommutative gravity from the Klein-Gordon equation. In section 5, by transforming the metric of gravitational field from coordinate space to momentum space, we get the isolated singularities, which is the event horizon of the black hole. This type of isolated singularity means that the horizon is formed by limit state dipoles.

## II. A BRIEF REVIEW OF QUANTUM GRAVITY

In this section, we briefly review the theory of quantum gravity suggested in the paper[1]. More details can be found in [1].

Since the introduction of the uncertainty principle into the general theory of relativity, we get a wave packet approximate to the Dirac  $\delta$ -function as follows

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$$\xi^i(x, r) = \begin{cases} \xi^r = r + C^r(x) \exp(-\frac{r}{l_P}) \\ \xi^\theta = \theta(x) \\ \xi^\phi = \phi(x) \\ \xi^t = t + C^t(x) \exp(-\frac{|t|}{t_P}) \end{cases} \quad (2.1)$$

It can be explained as a semiclassical graviton. The Lagrangian density can be written as

$$\mathcal{L} = -\frac{\eta^{\mu\nu}}{2} \frac{\partial \xi^i(x, r)}{\partial x^\mu} \frac{\partial \xi^j(x, r)}{\partial x^\nu} \eta_{ij} \quad (2.2)$$

The free field equation is

$$\partial^\mu \partial_\mu \xi^i = 0 \quad (2.3)$$

From the free field equation, we obtain Green's function

$$\tilde{G}^i(k) = \begin{cases} \tilde{G}^r(k) = -\frac{1}{(k^r)^2} \cdot \delta\left(k^r - \frac{i}{l_P}\right) \\ \tilde{G}^\theta(k) = -\frac{1}{(k^\theta)^2} \\ \tilde{G}^\phi(k) = -\frac{1}{(k^\phi)^2} \\ \tilde{G}^t(k) = -\frac{1}{\omega^2} \cdot \delta\left(\omega - \frac{i}{t_P}\right) \end{cases} \quad (2.4)$$

Compare Green's function (2.4) with the usual Feynman propagator, it can be seen that the generalized functions  $\delta\left(k^r - \frac{i}{l_P}\right)$  and  $\delta\left(\omega - \frac{i}{t_P}\right)$  give the regularization to the integral over  $k^r$  and  $\omega$  of the usual Feynman propagator. According to the properties of the Dirac  $\delta$ -function, we just need to give singularity on the integral paths without calculating specific integrals when calculating the Feynman diagrams. So that the difficulty of divergence of the Feynman integral over large virtual momenta of graviton has been solved.

The energy-momentum tensor of graviton is

$$\begin{aligned} T_{\mu\nu} &= \eta_{\mu\nu} \mathcal{L} - \frac{\partial \mathcal{L}}{\partial(\partial^\mu \xi^i)} \partial_\nu \xi^i \\ &= -\frac{\eta_{\mu\nu}}{2} \partial^\lambda \xi^i \partial_\lambda \xi^j \eta_{ij} + \partial_\mu \xi^i \partial_\nu \xi^j \eta_{ij} \end{aligned} \quad (2.5)$$

In the general theory of relativity, the energy-momentum tensor of gravitational field itself is

$$\begin{aligned}
 t_{\mu\nu} &= \frac{1}{8\pi G} \left( \frac{1}{2} \eta_{\mu\nu} R^{(1)} - R_{\mu\nu}^{(1)} \right) \\
 &= \frac{1}{8\pi G \cdot C} \left( \frac{1}{2} \eta_{\mu\nu} \frac{\partial \xi^i}{\partial x^\kappa} \frac{\partial \xi_i}{\partial x_\kappa} - \frac{\partial \xi^i}{\partial x^\mu} \frac{\partial \xi_i}{\partial x^\nu} \right)
 \end{aligned} \quad (2.6)$$

Up to a factor of a constant, Eq.(2.5) and Eq.(2.6) are equivalent. This shows that the quantum gravity theory established by a completely different method in the paper[1] is classical equivalent to the general theory of relativity.

### III. MULTI-GRAVITON SYSTEM WITH SELF-INTERACTION

According to the model of graviton suggested in the paper[1], the noncommutative space made up of  $\vee$  posets is flat. If we depend on traditional differential geometry to explain the spacetime limited by the uncertainty principle, the 4-dimensional space made up of mathematical points can be interpreted as curved, this is the gravitational field in the sense of the general theory of relativity. Let  $g_{\mu\nu}$  be the metric of the gravitational field. To describe a graviton, we used both coordinate systems  $x^\mu$  and  $X^\mu$ . If a graviton is excited at point  $x$ , the locally inertial coordinate system  $\xi^\alpha$  at point  $x$  can be written as

$$\xi^\alpha(x, X)|_x = X + C^\alpha(x) \cdot \exp\left(-\left|\frac{X}{L_P(x)}\right|\right)\Big|_{X=0} \quad (3.1)$$

where  $L_P(x) \equiv L_P^\mu(x)$ ,  $X \equiv X^\mu$ .

In the case of multiple-graviton, due to the ductility of gravitons, gravitons elsewhere in a multi-graviton system will act on a point  $x$  together. Therefore, we must also consider the self-interaction between gravitons caused by the ductility of gravitons. Specifically, if another graviton is excited at a distance of  $l \equiv l^\mu = (l^1, l^2, l^3, l^4)$  to point  $x$ , the locally inertial coordinate system  $\xi$  at point  $x$  caused by this graviton can be written as

$$\begin{aligned}
 \lambda(\xi^\alpha) &= \xi^\alpha((x+l), |l|) \\
 &= X + C^\alpha(x+l) \cdot \exp\left(-\left|\frac{l}{L_P(x+l)}\right|\right)
 \end{aligned} \quad (3.2)$$

Then in multi-graviton system the locally inertial coordinate system  $\xi^\alpha$  at point  $x$  have to written as

$$\begin{aligned}
 \lambda(\xi^\alpha) &= X + \int d^4l \xi^\alpha((x+l), |l|) \\
 &= X + \int d^4l \left( C^\alpha(x+l) \cdot \exp\left(-\left|\frac{l}{L_P(x+l)}\right|\right) \right)
 \end{aligned} \quad (3.3)$$

This expression shows that there is the self-interaction between gravitons.

For the gravitational field in vacuum, the field  $C^\alpha(x+l)$  in Eq.(3.3) satisfy the free field equation Eq.(2.3), the solution is

$$C^\alpha(x+l) = \int d^4k \left( C^\alpha(k) \exp(ik(x+l)) + (C^\alpha(k))^* \exp(-ik(x+l)) \right)$$

$$= \int d^4k \left( C^\alpha(k) \exp(ikx) \exp(ikl) + (C^\alpha(k))^* \exp(-ikx) \exp(-ikl) \right) \quad (3.4)$$

where  $k \equiv k_\mu$  is the energy-momentum conjugate to  $x^\mu$ .

From Eq.(3.3) and Eq.(3.4) we have

$$\begin{aligned} \frac{\partial \lambda(\xi^\alpha)}{\partial x^\mu} &= \frac{\partial \int d^4l \xi^\alpha((x+l), |l|)}{\partial x^\mu} = \frac{\partial \int d^4l \left( C^\alpha(x+l) \cdot \exp\left(-\left|\frac{l}{L_P(x+l)}\right|\right) \right)}{\partial x^\mu} \\ &= \int d^4l d^4k \left[ \left( \frac{\partial(C^\alpha(k) \exp(ik(x+l)))}{\partial x^\mu} \right. \right. \\ &\quad \left. \left. + \frac{\partial((C^\alpha(k))^* \exp(-ik(x+l)))}{\partial x^\mu} \right) \cdot \exp\left(-\left|\frac{l}{L_P(k)}\right|\right) \right] \\ &= \int d^4l d^4k \left[ \left( ik_\mu C^\alpha(k) \exp(ikx) \exp(ikl) \right. \right. \\ &\quad \left. \left. - ik_\mu (C^\alpha(k))^* \exp(-ikx) \exp(-ikl) \right) \cdot \exp\left(-\left|\frac{l}{L_P(k)}\right|\right) \right] \\ &= \int d^4l d^4k \left( ik_\mu C^\alpha(k) \exp(ikx) \exp\left(\frac{\pm ik L_P(k) - 1}{|L_P(k)|} \cdot |l| \right) \right. \\ &\quad \left. - ik_\mu (C^\alpha(k))^* \exp(-ikx) \exp\left(\frac{\mp ik L_P(k) - 1}{|L_P(k)|} \cdot |l| \right) \right) \\ &= \int d^4k \left( \frac{2|L_P|}{1 \mp ik L_P} ik_\mu C^\alpha(k) \exp(ikx) - \frac{2|L_P|}{1 \pm ik L_P} ik_\mu (C^\alpha(k))^* \exp(-ikx) \right) \end{aligned} \quad (3.5)$$

Note that in Eq.(3.5),  $L_P^\mu(x+l)$  in coordinate space has been transformed to  $L_P^\mu(k)$  in momentum space, and we denote  $L_P^\mu(k)$  as  $L_P(k)$  for short. The modulus of  $L_P(k)$  is  $(l_P, t_P)$ ,  $L_P(k)$  has only 3 degrees of freedom, the phase angle  $(\theta, \phi, \pm t_P)$ .

Then the metric in momentum space can be written as follows

$$\begin{aligned} g_{\mu\nu} &= \frac{\partial \lambda(\xi^\alpha)}{\partial x^\mu} \frac{\partial \lambda(\xi^\beta)}{\partial x^\nu} \eta_{\alpha\beta} \\ &= \frac{\partial \int d^4l \xi^\alpha((x+l), |l|)}{\partial x^\mu} \frac{\partial \int d^4l \xi^\beta((x+l), |l|)}{\partial x^\nu} \eta_{\alpha\beta} \\ &= \int d^4k \left( \frac{2|L_P|}{1 \mp ik L_P(k)} ik_\mu C^\alpha(k) \exp(ikx) - \frac{2|L_P|}{1 \pm ik L_P(k)} ik_\mu (C^\alpha(k))^* \exp(-ikx) \right) \end{aligned}$$

$$\begin{aligned}
 & \cdot \int d^4 k' \left( \frac{2|L_P|}{1 \mp i k' L_P(k')} i k'_\nu C_\alpha(k') \exp(i k' x) - \frac{2|L_P|}{1 \pm i k' L_P(k')} i k'_\nu (C_\alpha(k'))^* \exp(-i k' x) \right) \\
 & = \int d^4 k d^4 k' \quad 4|L_P|^2 \left( - \frac{k_\mu k'_\nu}{(1 \mp i k L_P)(1 \mp i k' L_P)} C^\alpha(k) C_\alpha(k') \exp[i(k+k')x] \right. \\
 & \quad - \frac{k_\mu k'_\nu}{(1 \pm i k L_P)(1 \pm i k' L_P)} (C^\alpha(k))^* (C_\alpha(k'))^* \exp[-i(k+k')x] \\
 & \quad \left. + \frac{k_\mu k'_\nu}{1 + k L_P \cdot k' L_P} [C^\alpha(k) (C_\alpha(k'))^* + (C^\alpha(k))^* C_\alpha(k')] \right)
 \end{aligned} \quad (3.6)$$

It can be written as follows

$$\begin{aligned}
 g_{\mu\nu} = \int d\vec{k} d\omega d\vec{k}' d\omega' & \left( 4|L_P|^2 \left( - \frac{C^\alpha(k) C_\alpha(k') \exp[i(k+k')x]}{(1 \mp i k L_P)(1 \mp i k' L_P)} \right. \right. \\
 & - \frac{(C^\alpha(k))^* (C_\alpha(k'))^* \exp[-i(k+k')x]}{(1 \pm i k L_P)(1 \pm i k' L_P)} \\
 & \left. \left. + \frac{C^\alpha(k) (C_\alpha(k'))^* + (C^\alpha(k))^* C_\alpha(k')}{1 + k L_P \cdot k' L_P} \right) \cdot k_\mu k'_\nu \right)
 \end{aligned} \quad (3.7)$$

where  $k \equiv k^\mu = (\vec{k}, \omega)$ .

#### IV. SYMMETRY OF KLEIN-GORDON EQUATION

Let's study the real scalar particles in the gravitational field. Complex scalar fields are completely similar.

In curved spacetime, the Lagrangian density of real scalar particle with spin 0 is

$$\mathcal{L} = g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + m^2 \Phi^2 \quad (4.1)$$

where  $m$  is the mass of scalar particle.

Then in the spacetime with the metric  $g_{\mu\nu}$ , the Klein-Gordon equation is

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left( \sqrt{-g} g^{\mu\nu} \frac{\partial \Phi}{\partial x^\nu} \right) - m^2 \Phi = 0 \quad (4.2)$$

where  $g = \text{Det } g_{\mu\nu}$  is the scalar density.

It can be written as

$$g^{\mu\nu} \frac{\partial^2 \Phi}{\partial x^\mu \partial x^\nu} + \frac{1}{2g} \frac{\partial g}{\partial x^\mu} g^{\mu\nu} \frac{\partial \Phi}{\partial x^\nu} + \frac{\partial g^{\mu\nu}}{\partial x^\mu} \frac{\partial \Phi}{\partial x^\nu} - m^2 \Phi = 0 \quad (4.3)$$

Eq.(4.2) can be written as follows

$$g^{\mu\nu} \frac{\partial^2 \Phi}{\partial x^\mu \partial x^\nu} + \frac{1}{2g} \frac{\partial g}{\partial x^\mu} g_{\mu\nu}^{-1} \frac{\partial \Phi}{\partial x^\nu} - g_{\mu\nu}^{-1} \frac{\partial g_{\mu\nu}}{\partial x^\mu} g_{\mu\nu}^{-1} \frac{\partial \Phi}{\partial x^\nu} - m^2 \Phi = 0 \quad (4.4)$$

The inverse of the metric  $g_{\mu\nu}$  is

$$g_{\mu\nu}^{-1} = \frac{1}{g} [g^*]^{\mu\nu} \quad (4.5)$$

where  $[g^*]^{\mu\nu}$  is the adjoint matrix of the metric  $g_{\mu\nu}$ .

Then Eq.(4.4) can be written as follows

$$g^{\mu\nu} \frac{\partial^2 \Phi}{\partial x^\mu \partial x^\nu} + \left( \frac{1}{2g} \frac{\partial g}{\partial x^\mu} - \frac{1}{g} [g^*]^{\mu\nu} \frac{\partial g_{\mu\nu}}{\partial x^\mu} \right) g_{\mu\nu}^{-1} \frac{\partial \Phi}{\partial x^\nu} - m^2 \Phi = 0 \quad (4.6)$$

For the metric (3.7), we have

$$\frac{1}{2g} \frac{\partial g}{\partial \vec{x} \partial t} = \frac{1}{g} [g^*]^{\mu\nu} \frac{\partial g_{\mu\nu}}{\partial \vec{x} \partial t} \quad (4.7)$$

where  $x^\mu = (\vec{x}, t)$ .

So that Eq.(4.6) can be written as

$$g^{\mu\nu} \frac{\partial^2 \Phi}{\partial x^\mu \partial x^\nu} - m^2 \Phi = 0 \quad (4.8)$$

Therefore in the gravitational field, the Klein-Gordon equation (4.2) is the usual form in quantum field theory, which can be written as follows

$$(\square^2 - m^2) \Phi = 0 \quad (98)$$

where  $\square^2$  is the usual D'Alembertian operator in curved spacetime as follows

$$\square^2 = g^{\mu\nu} \frac{\partial^2}{\partial x^\mu \partial x^\nu} \quad (4.10)$$

In general, in curved space, the Klein-Gordon equation is Eq.(4.2). Due to the specificity of the metric (3.7) of noncommutative gravitational field in momentum space, Eq.(4.8) has the following symmetry:

For the transformation from Minkowski spacetime to curved spacetime, the Klein-Gordon equation be invariant if the transformation of the local inertial system (or the cotangent frame of spacetime)  $\lambda(\hat{\xi})$  is as follows:

$$\lambda(\hat{\xi})|_x : r^i \rightarrow r^i + C^i(x) \exp\left(-\frac{r^i}{L_P^i}\right) \quad (4.11)$$

where the pole of the spherical polar coordinate system  $r^i$  is  $x$ .

Eq.(4.8) and Eq.(4.11) show the symmetry associated with noncommutative gravity. It can be interpreted as follows:

For the transformation from Minkowski spacetime to curved spacetime, if the Klein-Gordon equation is required to be invariant, we need to introduce the noncommutative gravitational field with ductility which in Eq.(4.11). And the noncommutative gravitational field must satisfy the field equation. In this way, the metric  $g_{\mu\nu}$  of spacetime is constrained, so that not all curved spacetime is physically possible. The ductility of graviton produces the self-interaction of gravitational field, which can produce black holes, and keep the symmetry of the Klein-Gordon equation associated with noncommutative gravity. The ductility of graviton can also be understood as that the graviton itself has the gravitational charge, thus affecting the geometry of spacetime around the graviton, and also produces the self-interaction of gravitational field. The most important feature of the ductile structure is: due to the characteristics of the ductile function, by quantizing the noncommutative gravitational field with ductility, we can obtain the Feynman rule without diverged integrals.

## V. BLACK HOLE

From the last term in the integrand of the metric (3.7), it can be seen that there are singularities as follows

$$k \cdot k' = -\frac{1}{L_P(k)} \frac{1}{L_P(k')} \quad (5.1)$$

$L_P(k)$  has 3 degrees of freedom, the phase angles  $(\theta, \phi, \pm t_P)$ . If the phase angles of the  $L_P(k)$  and  $L_P(k')$  in Eq.(5.1) are opposite, Eq.(5.1) be true, which also means

$$|k| = |k'| = \left| \frac{1}{L_P(k)} \right| = \left| \frac{1}{L_P(k')} \right| = \left( \frac{1}{l_P}, \frac{1}{t_P} \right) \quad (5.2)$$

Eq.(5.1) and Eq.(5.2) independent of the source flow of gravity. From Eq.(5.1) we can see that the phase angles of  $k$  and  $k'$  are the same as  $L_P(k)$  or  $L_P(k')$ , so that the singularity in momentum space must be in pairs at the same point in coordinate space, and the phase angles of the paired singularities are opposite.

If the field  $C^\alpha(x)$  given by the free field equation is an analytical function, then except for singularities shown in Eq.(5.1), the integrand in Eq.(3.6) is analytic, so these singularities are the isolated singularities. The metric be singular while the integral paths pass through these singularities. This can be interpreted that the event horizon in coordinate space is the set of isolated singularities in momentum space.

From the viewpoint of hydrodynamic, this type of isolated singularity is a drain or source of the flow in momentum space, where the complex potential is the integrand in Eq.(3.6). From Eq.(3.6), it can be seen that the isolated singularity in the integrand in Eq.(3.6) is the limiting case of a dipole in which the source and drain of the same strength are infinitely close and the strength increases infinitely.

In the paper[1], we get the energy-momentum tensor Eq.(2.5) of gravitational field itself. Recall Eq.(3.5), the expression clearly shows the self-interaction between gravitons. Therefore in the multi-graviton system, due to the self-interaction of the gravitational field, the energy-momentum tensor Eq.(2.5) should be written as follows



$$T_{\mu\nu} = -\frac{\eta_{\mu\nu}}{2} \partial^\lambda \lambda(\xi^i) \partial_\lambda \lambda(\xi^j) \eta_{ij} + \partial_\mu \lambda(\xi^i) \partial_\nu \lambda(\xi^j) \eta_{ij} \quad (5.3)$$

The eigenvalue  $\lambda(\xi)$  is given by Eq.(3.3). Using Eq.(3.5) we get the density of the energy-momentum tensor as follows

$$\begin{aligned} T_{\mu\nu}(x) = & -\frac{\eta_{\mu\nu}}{2} \int d^4k \left( \frac{2|L_P|}{1 \mp ikL_P} ik^\lambda C^\alpha(k) \exp(ikx) - \frac{2|L_P|}{1 \pm ikL_P} ik^\lambda (C^\alpha(k))^* \exp(-ikx) \right) \\ & \cdot \int d^4k' \left( \frac{2|L_P|}{1 \mp ik'L_P} ik'_\lambda C_\alpha(k') \exp(ik'x) - \frac{2|L_P|}{1 \pm ik'L_P} ik'_\lambda (C_\alpha(k'))^* \exp(-ik'x) \right) \\ & + \int d^4k \left( \frac{2|L_P|}{1 \mp ikL_P} ik_\mu C^\alpha(k) \exp(ikx) - \frac{2|L_P|}{1 \pm ikL_P} ik_\mu (C^\alpha(k))^* \exp(-ikx) \right) \\ & \cdot \int d^4k' \left( \frac{2|L_P|}{1 \mp ik'L_P} ik'_\nu C_\alpha(k') \exp(ik'x) - \frac{2|L_P|}{1 \pm ik'L_P} ik'_\nu (C_\alpha(k'))^* \exp(-ik'x) \right) \end{aligned} \quad (5.4)$$

Notice that there are the isolated singularities  $k \cdot k' = -\frac{1}{L_P(k)} \frac{1}{L_P(k')}$  in Eq.(5.4), and these singularities are unacceptable for energy-momentum. We can avoid this problem in this way: in the derivation of the energy-momentum of gravitational field itself Eq.(2.6) and Eq.(5.4), we have used the mass-shell condition, therefore it is considered that, the energy-momentum of a real graviton must satisfy  $|k| < \left( \frac{1}{l_P}, \frac{1}{t_P} \right)$ , the graviton with the energy-momentum  $|k| \geq \left( \frac{1}{l_P}, \frac{1}{t_P} \right)$  must be off-shell. So that the state of isolated singularity  $|k| = \left( \frac{1}{l_P}, \frac{1}{t_P} \right)$  is the limit state that cannot be reached by the on-shell graviton. Therefore, the event horizon is formed by dipoles composed of virtual gravitons that can reach the limit state, which cannot radiate like the gravitational wave composed of real gravitons.

## VI. CONCLUSION

In the paper 'A New Approach to Quantum Gravity'[1], we suggest a new theory of quantum gravity. Using this theory, we can study the noncommutative gravitational field in momentum space. The metric of this curved space caused by the gravitational field is special in momentum space, by this metric, we obtain the general form of the Klein-Gordon equation in the gravitational field. Then we find the symmetry associated with noncommutative gravity from the Klein-Gordon equation. The noncommutative gravitational field in momentum space has singularities, which means the event horizon of black holes. From this metric, we conclude that the event horizon of black holes is formed by the dipoles in momentum space with limit state.

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