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Product of Special Function and Polynomial Associated Via Pathway Fractional Integral Operator

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PRODUCTOFSPECIALFUNCTIONANDPOLYNOMIALASSOCIATEDVIAPATHWAYFRACTIONALINTEGRALOPERATOR

Product of Special Function and Polynomial Associated Via Pathway Fractional Integral Operator

Hemlata Saxena [α] & Danishwar Farooq [σ]

Abstract- In present paper we introduce four theorems using pathway fractional integral operator involving product of Srivastava polynomial and generalized Struve function. Our results are quite general in nature. We obtain our results in term of hypergeometric function. Certain special cases of the main results are also obtained here. Our results will help to extend some classical statistical distribution to wider classes of distribution, these are useful in practical applications.

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I. Introduction

Let $f(x)\,\epsilon\, L(a,b), \alpha\,\epsilon\,\mathbb{C}, R(\alpha) > 0$, then left sided Reimann – Liouville fractional integral operator is defined as [9]

$$\left(\mathrm{I}_{0^+}^{\alpha} f\right)(x) = \frac{1}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1} f(t)dt \text{ where } R(\alpha) > 0. \qquad \ldots\ldots\ldots(1.1)$$

Let $f(x)\,\epsilon\, L(a,b), \eta\,\epsilon\,\mathbb{C}, R(\eta) > 0, a > 0$ and a "Pathway parameter" $\alpha < 1$. Then the pathway fractional integration operator is defined by [6], also see [10]

$$\left(\mathrm{p}_{0^+}^{(\eta,\alpha)} f\right) = \mathrm{x}^{\eta}\int_0^{\frac{x}{a(1-\alpha)}} \left(1-\frac{a(1-\alpha)t}{x}\right)^{\frac{\eta}{1-\alpha}} f(t)dt \qquad \ldots\ldots\ldots\ldots(1.2)$$

when $\alpha = 0$, $a = 1$ and η is replaced by $\eta - 1$ in (1.2) it yields

$$\left(\mathrm{I}_{0^+}^{\eta} f\right)(x) = \frac{1}{\Gamma(\eta)}\int_0^x (x-t)^{\eta-1} f(t)dt \qquad \ldots(1.3)$$

Fractional integration operators play an important role in the solution of several problems of diversified fields of science and engineering. Many fractional integral operators like Riemann – Liouville, Weyl, Kober, Erdely – Koberand Saigo operator are studied by various workers due to their applications in the solutions of integral equation arising in several problems of many areas of physical, engineering and technological science. A detailed description of these operators can be found in the survey paper by Srivastava and Saxena [17].

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17. Srivastava H. M and Saxena R. K; Operator of Fractional integration and their applications. Appl. Math. Comput. 118, (2001), 1-52.

In this paper, we consider following functions defined as follows:
The Struve function of order p is given by

$$H_p(z) = \left(\frac{z}{2}\right)^{p+1} \sum_{K=0}^{\infty} \frac{(-1)^k}{\Gamma\left(k+\frac{3}{2}\right)\Gamma\left(k+p+\frac{1}{2}\right)} \left(\frac{z}{2}\right)^{2k} \quad \dots\dots\dots\dots\dots\dots(1.4)$$

The Struve function and its more generalization are found in many papers [1,2,4,12,7,13,14,15].The generalized Struve function studied by [13] as follows:

$$H_{l,\varepsilon}^{\lambda}(z)= \sum_{K=0}^{\infty} \frac{(-1)^k}{\Gamma\left(\lambda k+\frac{1}{\varepsilon}+\frac{3}{2}\right)\Gamma\left(k+\frac{3}{2}\right)} \left(\frac{z}{2}\right)^{2k+l+1}, \lambda > 0, \varepsilon > 0 \dots \dots\dots\dots\dots (1.5)$$

The generalized Struve function of the first kind $H_{p,b,c}(z)$ [see 7] defined for complex $z \in \mathbb{C}$ and $b, c, p \in \mathbb{C}$, $(Re(p) > -1$ by:

$$H_{p,b,c}(z) = \sum_{K=0}^{\infty} \frac{(-1)^k c^k}{\Gamma\left(p+1+\frac{b}{2}+k\right)\Gamma\left(k+\frac{3}{2}\right)} \left(\frac{Z}{2}\right)^{2k+p+1} \quad \dots\dots\dots\dots\dots\dots (1.6)$$

Where Γ is the classical gamma function whose Euler's integral is given by Srivastava and Choi(see [11])

$$\Gamma(y) = \int_0^{\infty} e^{-t}\, t^{y-1}\, dt \quad Re(y) > 0 \quad \dots\dots\dots\dots\dots\dots (1.7)$$

The special cases of Struve function are as follows[7]:

$$H_{1-\frac{b}{2},b,c^2} = \frac{1}{c^2\sqrt{\pi}}\left[\frac{2}{z}\right]^{b/2}(1-\cos(cz))b \in \mathbb{R}, \& \quad c \neq 0 \quad \dots \quad \dots\dots\dots (1.8)$$

$$H_{1-\frac{b}{2},b,-c^2} = \frac{1}{c^2\sqrt{\pi}}\left[\frac{2}{z}\right]^{b/2}(\cosh(cz)-1) \quad b \in \mathbb{R}, \& \quad c \neq 0 \quad (1.9)$$

$$H_{-\frac{b}{2},b,c^2} = \frac{1}{c\sqrt{\pi}}\left[\frac{2}{z}\right]^{b/2} \sin(cz) \qquad b \in \mathbb{R}, \& \quad c \neq 0 \quad \dots\dots (1.10)$$

$$H_{-\frac{b}{2},b,-c^2} = \frac{1}{c\sqrt{\pi}}\left[\frac{2}{z}\right]^{b/2} \sinh(cz) \qquad b \in \mathbb{R}, \& \quad c \neq 0 \quad \dots\dots\dots (1.11)$$

The generalized Wright hypergeometric function $r\Psi s\,(z)$ defined for a_i , $b_j \in \mathbb{C}$, and real α_i , $\beta_j \in \mathbb{R}$ (α_i , $\beta_j \neq 0$;i = 1,2, . . . , r ; j = 1,2, . . . s) is given by the series :

$$r\Psi s\,(z) = r\Psi s\left[\begin{matrix}(a_i + \alpha \cdot 1)_{1,r} \\ ((a_i + \alpha \cdot 1)_{1,s})\end{matrix}\, z\right] = \sum_{k=0}^{\infty} \frac{\prod_{i=1}^{r} \Gamma(a_i+\alpha_i k) z^k}{\prod_{j=1}^{s} \Gamma(b_j+\beta_j k)\, k!} \quad \dots\dots\dots (1.12)$$

where Γz is the Euler gamma function and the asymptotic behavior of this function for large values of argument of $z \in \mathbb{C}$ were studied in [3] and under the condition:

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13. Singh, R.P; Some integral representation of generalized Struve function, *Math Ed(Siwan)*, 22,(1988), 91-94.

$$\sum_{j=1}^{r} \beta_j - \sum_{i=1}^{s} \alpha_i > -1 \qquad \ldots\ldots\ldots\ldots (1.13)$$

For detailed Study of various properties, generalization and applications of wright function and generalized wright function, we refer to paper (for instance see [19] [20] [21]). The generalized hypergeometric function represented as fallows [8]:

$$r\Psi s \left[\begin{matrix} (\alpha_r) ; \\ (\beta_s) ; \end{matrix} \; z \right] = \sum_{R=0}^{\infty} \frac{\prod_{i=1}^{r} (\alpha_i)_n z^n}{\prod_{j=1}^{s} (\beta_i)_n \, n!} \qquad \ldots\ldots\ldots\ldots (1.14)$$

Provided $r \leq s ; r = s + 1$ and $|z| < 1$ where $(\lambda)_n$ is well known pochhammer symbol defined for$\lambda \in \mathbb{C}$ [3][8]

$$(\lambda)_n = \begin{Bmatrix} 1 & (n = 0) \\ \lambda(\lambda+1)\ldots(\lambda+n-1) & (n \in N = 1,2,3,\ldots) \end{Bmatrix} \qquad \ldots\ldots (1.15)$$

$$= \frac{\Gamma(\lambda + n)}{\Gamma(\lambda)} (\lambda \in \mathbb{C}/Z_0^-)$$

where $Z_0^-$is the set of non – positive integers. If we put $\alpha_1 = \ldots = \alpha_r \ldots = \beta_1 = \ldots = \beta_s$ in equation (1.8), then (1.10) is a special case of the generalized wright function [16]

$$r\Psi s = r\Psi s \left[\begin{matrix} (\alpha_1, 1), \ldots, (\alpha_r, 1); \\ (\beta_1, 1), \ldots, ; (\beta_s, 1); \end{matrix} \; z \right] = \frac{\prod_{i=1}^{r} (\alpha_i)_n z^n}{\prod_{j=1}^{s} (\beta_i)_n \, n!} rFs \left[\begin{matrix} (\alpha_1), \ldots, (\alpha_r) ; \\ (\beta_1), \ldots, (\beta_s); \end{matrix} \; z \right] \quad (1.16)$$

The Srivastava polynomial defined by Srivastava [18](pp. 1, eq.1), [5](pp. 11, eq. 7) in the following manner

$$S_w^u[x] = \sum_{s=0}^{(w/u)} \frac{(-W)u\,,s}{s!} A_{w,s} \, x^s, \qquad w = 0,1,2, \ldots\ldots (1.17)$$

where w is an arbitrary positive intenger and the coefficient $A_{w,s}\,(w\,,s\,) > 0$ are the arbitrary constant real or complex. This polynomial provide a large number spectrum of well-known polynomial as one of its particular cases on appropriately specializing the coefficient $A_{w,s}$ particularly by setting$u = 1$,$A_{w,s}$ for s = k and $A_{w,s} = 0$ for s ≠ k the above polynomial leads to a power function.

$$S_w^u[x] = x^k \qquad (\, k \in Z^+ \, with \, k \leq w) \ldots\ldots\ldots\ldots (1.18)$$

II. Main Results

Theorem 1: Let $\eta, \rho, \beta, \gamma, \mu, \delta \in \mathbb{C}$, $R\left(1 + \frac{\eta}{(1-\alpha)}\right) > 0$ min$\{Re(\rho), Re(\beta), Re(\gamma), Re(\mu), Re(\delta), Re(\eta)\} > 0$ and $p_i, q_i > 0$, $\alpha < 1$. Then the pathaway fractional integral operator$(p_{0^+}^{(n,\alpha)})$ defined by (1.2) then the following formula holds:

Ref

8. Rainville, E. D; Special function, Macmillan, New York, 1960; Reprinted by Chelsea publishing Company, New York(1971).

$$P_{0^+}^{(\eta,\alpha)}\{t^{\mu-1}H_{l,\varepsilon}^{\lambda}(t)S_w^u[\sigma t^\rho]\}(x) =$$

$$\frac{x^{\eta+l+\mu+1}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{[a(1-\alpha)]^{l+\mu+1}}\left(\frac{1}{2}\right)^{l+1}\sum_{s=0}^{(w/u)}\frac{(-W)u,s}{s!}A_{w,s}\,[\sigma\left(\frac{x}{[a(1-\alpha)]}\right)^{\rho}]^s$$

$$\times\,{}_1\Psi_3\left[\begin{matrix}(l+\mu+\rho s+1,2)\\ \left(\frac{l}{3}+\frac{3}{2},\lambda\right),\left(\frac{3}{2},1\right),\left(l+\frac{\eta}{(1-\alpha)}+\mu+\rho s+2,2\right)\end{matrix};-\frac{x^2}{4[a(1-\alpha)]^2}\right]\quad\ldots\ldots\ldots\ldots\ldots(2.1)$$

Proof: Making the use of (1.2), (1.5) and (1.17) in LHS of the theorem first and then interchange the order of integration and summation, we evaluate the inner integral by making use of beta function and using (1.12) we arrive at the desired result RHS of (2.1).

Theorem 2: Let η, μ, p, b, c ϵ $\mathbb{C}$ and $\alpha<1$ such that $\{Re(\eta), Re(\mu), Re(\mu+p)\}>0$, $Re(p+1+\frac{b}{2})>-1$ and $Re(\frac{\eta}{1-\alpha})>-1$ then the following formula hold:

$$P_{0^+}^{(\eta,\alpha)}\{t^{\mu-1}H_{p,b,c}(t)S_w^u[\sigma t^\rho]\}(x) =$$

$$\frac{x^{\eta+p+\mu+1}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{[a(1-\alpha)]^{p+\mu+1}}\left(\frac{1}{2}\right)^{p+1}\sum_{s=0}^{(w/u)}\frac{(-W)u,s}{s!}A_{w,s}\,[\sigma\left(\frac{x}{[a(1-\alpha)]}\right)^{\rho}]^s$$

$$\times\,{}_1\Psi_3\left[\begin{matrix}(p+\mu+\rho s+1,2)\\ \left(p+\frac{b}{2}+1,1\right),\left(\frac{3}{2},1\right),\left(p+\frac{\eta}{(1-\alpha)}+\mu+\rho s+2,2\right)\end{matrix}\,|\,-\frac{cx^2}{4[a(1-\alpha)]^2}\right]\ldots\ldots\ldots\ldots\ldots(2.2)$$

Proof: Making the use of (1.2), (1.6) and (1.18) in LHS of the theorem 2 and then interchange the order of integration and summation, we evaluate the inner integral by making use of beta function and using (1.12) we arrive at the desired result RHS of (2.2).

Theorem 3: Let η,μ, p, b, c ϵ $\mathbb{C}$ and $\alpha<1$ such that $\{Re(\eta), Re(\mu), Re(\mu+p)\}>0$, and $Re(\frac{\eta}{1-\alpha})>-1$ then the following formula hold:

(i) $P_{0^+}^{(\eta,\alpha)}\{t^{\mu-1}\sin(ct)S_w^u[\sigma t^\rho]\}(x) =$

$$\frac{1}{4}c\sqrt{\pi}\frac{x^{\eta+\mu+1}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{[a(1-\alpha)]^{\mu+1}}\sum_{s=0}^{(w/u)}\frac{(-W)u,s}{s!}A_{w,s}\,[\sigma\left(\frac{x}{[a(1-\alpha)]}\right)^{\rho}]^s$$

$$\times\,{}_1\Psi_3\left[\begin{matrix}(\mu+\rho s+1,2)\\ \left(\frac{3}{2},1\right),(1,1),\left(\frac{\eta}{(1-\alpha)}+\mu+\rho s+2,2\right)\end{matrix};-\frac{(cx)^2}{4[a(1-\alpha)]^2}\right]\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.3)$$

Proof: Making the use of (1.2), (1.10) and (1.18) in LHS of the theorem 3 (part I) and then interchange the order of integration and summation, we evaluate the inner integral

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by making use of beta function and using (1.12) we arrive at the desired result RHS of (2.3)

(ii) $P_{0^+}^{(\eta,\alpha)}\{t^{\mu-1}\sinh(ct)S_w^u[\sigma t^\rho]\}(x) =$

$$\frac{1}{4}c\sqrt{\pi}\frac{x^{\eta+\mu+1}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{[a(1-\alpha)]^{\mu+1}}\sum_{s=0}^{(w/u)}\frac{(-W)u,s}{s!}A_{w,s}[\sigma\left(\frac{x}{[a(1-\alpha)]}\right)^{\rho}]^{s}$$

$$\times\, {}_1\Psi_3\left[\begin{matrix}(\mu+\rho s+1,2)\\ \left(\frac{3}{2},1\right),(1,1),\left(\frac{\eta}{(1-\alpha)}+\mu+\rho s+2,2\right)\end{matrix};-\frac{(cx)^2}{4[a(1-\alpha)]^2}\right] \quad\ldots\ldots\ldots\ldots\ \ldots.\ (2.4)$$

Proof: Making the use of (1.2), (1.11) and (1.18) in LHS of the theorem 3 (part II) and then interchange the order of integration and summation, we evaluate the inner integral by making use of beta function and using (1.12) we arrive at the desired result RHS of(2.4).

Theorem 4: Let $\eta,\mu,p,b,c\in\mathbb{C}$ and $\alpha<1$ such that $Re(\alpha)>0,$ and $\mathrm{Re}(\beta-\eta)>2$ then the following formula hold:

(i) $P_{0^+}^{(\eta,\alpha)}\{t^{\mu-1}(1-\cos(ct))S_w^u[\sigma t^\rho]\}(x) =$

$$\frac{1}{4}c^2\sqrt{\pi}\frac{x^{\eta+\mu+2}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{[a(1-\alpha)]^{\mu+2}}\sum_{s=0}^{(w/u)}\frac{(-W)u,s}{s!}A_{w,s}[\sigma\left(\frac{x}{[a(1-\alpha)]}\right)^{\rho}]^{s}$$

$$\times\, {}_1\Psi_3\left[\begin{matrix}(\mu+\rho s+2,2);\\ \left(\frac{3}{2},1\right),(2,1),\left(\frac{\eta}{(1-\alpha)}+\mu+\rho s+3,2\right);\end{matrix};-\frac{(cx)^2}{4[a(1-\alpha)]^2}\right] \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (2.5)$$

Proof: Making the use of (1.2), (1.8) and (1.18) in LHS of the theorem 4 (part I) and then interchange the order of integration and summation, we evaluate the inner integral by making use of beta function and we arrive at the desired result RHS of (2.5)

(ii) $P_{0^+}^{(\eta,\alpha)}\{t^{\mu-1}(\cosh(ct)-1)S_w^u[\sigma t^\rho]\}(x) =$

$$\frac{1}{4}c^2\sqrt{\pi}\frac{x^{\eta+\mu+2}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{[a(1-\alpha)]^{\mu+2}}\sum_{s=0}^{(w/u)}\frac{(-W)u,s}{s!}A_{w,s}[\sigma\left(\frac{x}{[a(1-\alpha)]}\right)^{\rho}]^{s}$$

$$\times\, {}_1\Psi_3\left[\begin{matrix}(\mu+\rho s+2,2);\\ \left(\frac{3}{2},1\right),(2,1),\left(\frac{\eta}{(1-\alpha)}+\mu+\rho s+3,2\right);\end{matrix};\frac{(cx)^2}{4[a(1-\alpha)]^2}\right] \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.6)$$

Proof: Making the use of (1.2), (1.9) and (1.18) in LHS of the theorem 4 (part II) and then interchange the order of integration and summation, we evaluate the inner integral by making use of beta function and using (1.12) we arrive at the desired result RHS of (2.6)

Notes

III. Special Cases

1. If we take $\alpha = 0, a = 1$ and η is replaced by $\eta - 1$ in (2.1), then pathway fractional integral operator will reduce Riemann – Liouville fractional integral defined in (1.1). Then we get the following result:

$$I_{0^+}^{\alpha}\{t^{\mu-1} H_{l,\varepsilon}^{\lambda}(t) S_w^u[\sigma t^{\rho}]\}(x) = x^{\eta+\mu+1}\,\Gamma(\eta)\left(\frac{1}{2}\right)^{l+1} \sum_{s=0}^{(w/u)} \frac{(-W)u,s}{s!} A_{w,s}(\sigma x^{\rho})^s$$

$$\times\ {}_1\Psi_3\left[\begin{matrix} (l+\mu+\rho s+1,\ 2);\\ \left(\frac{l}{3}+\frac{3}{2},\lambda\right), \left(\frac{3}{2},1\right),\ (l+\eta+\mu+\rho s+1, 2); \end{matrix} (-x^2)\right]$$

2. If we take $\alpha = 0, a = 1$, η is replaced by $\eta - 1$ and also on setting $w = 0$, $A_{0,0} = 1$, then $S_0^u[x] \to 1$ in (2.1), then pathway fractional integral operator will reduce Riemann – Liouville fractional integral defined in (1.1) and general class of polynomial will reduce to 1 defined in (1.13). then we get the following result:

$$I_{0^+}^{\alpha}\{t^{\mu-1} H_{l,\varepsilon}^{\lambda}(t)\}(x)$$

$$= x^{\eta+\mu+1}\,\Gamma(\eta)\left(\frac{1}{2}\right)^{l+1}$$

$$\times\ {}_1\Psi_3\left[\begin{matrix} (l+\mu+1,\ 2);\\ \left(\frac{l}{3}+\frac{3}{2},\lambda\right), \left(\frac{3}{2},1\right),\ (l+\eta+\mu+1, 2); \end{matrix} (-x^2)\right]$$

3. On setting $w = 0$, $A_{0,0} = 1$, then $S_0^u[x] \to 1$ in (2.2), we arrive at the known result given by Nisar K .S [10, pp. 66, eq. 13]
4. On setting $w = 0$, $A_{0,0} = 1$, then $S_0^u[x] \to 1$ in (2.3), we arrive at the known result given by Nisar K .S [10, pp. 67, theorem (3.1)(part I)].
5. On setting $w = 0$, $A_{0,0} = 1$, then $S_0^u[x] \to 1$ in (2.4), we arrive at the known result given by Nisar K .S [10, pp. 67, theorem (3.1)(part II)].
6. On setting $w = 0$, $A_{0,0} = 1$, then $S_0^u[x] \to 1$ in (2.5), we arrive at the known result given by Nisar K .S [10, pp. 68 theorem (3.2), (part I)].
7. On setting $w = 0$, $A_{0,0} = 1$, then $S_0^u[x] \to 1$ in (2.6), we arrive at the known result given by Nisar K .S [10, pp. 68 theorem (3.2), (part II)].

IV. Conclusion

In this paper, we have presented Struve function, generalized Struve function and Srivastava polynomial via pathway fractional integral operator. As in this operator α establishes a path of going from one distribution to another and to different classes of distribution. we conclude this investigation by remarking that the result obtained here are general in character and useful in deriving various integral formulas in the theory of the pathway fractional integration operator and also our result will help to extend some

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classical statistical distribution to wider classes of distribution, useful in practical application.

Notes

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