The Universal Gravitational Constant(G) in an Expanding Universe

By John A. T. Bye
The University of Melbourne

Abstract- This paper indicates how the inclusion of dark matter, which is shown in Bye (2021) to have a constant density ($\rho_o$) throughout the Universe, together with the velocity of light (c), which is also a constant, leads to the expression, $G = \frac{3c^2}{4\pi\rho_o} / R^2$, for the universal gravitational constant in which $R$ is the radius of the Universe. As the Universe ages $G$ decreases.

Keywords: dark matter, universal gravitational constant (G).

GJSFR-A Classification: DDC Code: 530.1 LCC Code: QC6
The Universal Gravitational Constant (G) in an Expanding Universe

John A. T. Bye

Abstract- This paper indicates how the inclusion of dark matter, which is shown in Bye (2021) to have a constant density (ρD) throughout the Universe, together with the velocity of light (c), which is also a constant, leads to the expression, G = [3c^2/4πρD]/R^2, for the universal gravitational constant in which R is the radius of the Universe. As the Universe ages G decreases.

Keywords: dark matter, universal gravitational constant (G).

I. Introduction

Theoretical cosmology has been traditionally underpinned by two universal constants, the speed of light (c) and the universal gravitational constant (G). A recent investigation of dark matter (Bye 2021) has found that there is a third universal constant, which is the density of dark matter (ρD). This note assumes that c and ρD are absolute constants, i.e. they are independent of the evolutionary state of the Universe, from which an expression for the universal gravitational constant (G) is derived.

II. The Key Relations

(i) The azimuthal velocity at the edge of the Universe is,

\[ c = (G M/R)^{1/2} \]  

where c is the velocity of light, and M and R are respectively the mass and the radius of the Universe. From (1),

\[ 2 \pi R / T = c \]  

where T is the orbital period of the dark matter. On substituting (2) in (1) we obtain Newton’s Law for the mass (M),

\[ GM = 4\pi^2 R^3 / T^2 \]  

in which

(ii) The mass of the universe (M) is,

\[ M = 4/3 \pi \rho_0 R^3 \]  

where \( \rho_0 \) is the density of the dark matter, which the planetary data indicate is a universal quantity [1].

III. The Universal Gravitational Constant

On eliminating M between (3) and (4), we find that the universal gravitational constant (G) is,

\[ G = 3 \pi / (\rho_0 T^3) \]  

Eq. (5) is a general expression for G, which, on using (2) yields,

\[ G = [3c^2/4\pi\rho_0]/R^2 \]  

Hence the universal gravitational constant (G) is inversely proportional to the square of R. At the birth of the Universe (R \( \to \) 0), M \( \to \) 0 and T \( \to \) 0, and G \( \to \) \( \infty \), whereas at the death of the Universe (R \( \to \) \( \infty \)), M \( \to \) \( \infty \) and T \( \to \) \( \infty \) and G \( \to \) 0. The intermediate phase between these two limits may be regarded as the mature Universe, of which we are a part.

Planetary data indicate that \( \rho_0 = 2.1 \times 10^{-6} \text{ kg m}^{-3} \) and also that R \( \approx \) Ro where Ro = \( 1.25 \times 10^{16} \text{ m} \) (Bye 2021). On substituting in (6) we obtain G = \( 6.54 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2} \), which is very similar to the observed value of 6.674 \( \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2} \) (Wikipedia 2022) and well within the likely error bounds for \( \rho_0 \) and Ro. On evaluating (6) for an arbitrary R, we obtain,

\[ G = A R^{-2} \]  

in which for \( \rho_0 = 2.1 \times 10^{-6} \text{ kg m}^{-3}, A=1.02 \times 10^{-22} \text{ kg}^{-1} \text{ m}^5 \text{ s}^{-2}. \]

We suggest that (6) should be used for G in cosmic models in which R is evolving, rather than the traditional relation in which G = \( 6.674 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2} \).

IV. The Expanding Universe

Eq. (6) shows that the universal gravitational constant is a function of the size of the Universe (R) as might have been expected a priori, and the properties of the present Universe predict a value for G(6.54 \( \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2} \), which is similar to the observed experimental value of G = \( 6.674 \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2} \). This gives confidence in the use of (6). Eq. (6) has already been incorporated implicitly in the universal energy balance expressions due to dark matter in Bye (2021) through Eq. (15). Here it is shown to be a seminal expression for the evolving Universe, which in particular, relates the time variability of G to that of R.

Author: School of Earth Sciences, The University of Melbourne, Victoria 3010, Australia. e-mail: jbye@unimelb.edu.au
V. Conclusion

The most important conclusion is that as the Universe ages, the universal gravitational constant reduces according to (6). We propose that this reduction of G must be fully included in cosmological modelling.

In broad brush terms the decrease of the universal gravitational constant (G) with time is 'a secular relativity' in which, (1) shows that as the Universe ages, in order to maintain an azimuthal velocity which is equal to the velocity of light (c), the reduction in the universal gravitational constant (G) is compensated by an increase in mass density (M/R). Within the Universe, however, as the universal gravitational constant (G) decreases, the orbital velocity about a principal mass (M_0 = M) at a radius (R) slows, arguably promoting planetary formation.

References Références Referencias