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Air Crash and Pressure

By Tian-Quan Yun
Technical University (MADI)

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I. Introduction

Air crash has many investigations[1]. This paper studies hat air crash would be caused by deference of pressures between outside and inside of passenger's cargo. Such deference of pressures breaks the window or door and thus the plane crashes.

In order to calculate pressure outside the passenger's cargo, we study three type equations,: A "wind – temperature equation" ("w-t eq." in short), B "wind – pressure /density equation"("w - P/ρeq." in short). C "jet – pressure /density equation" ("J - P/ρeq." in short).

In Section 2, we derive the "w – T eq.". It states that the derivative of wind speed respect to time proportions to the derivative of temperature respect to track (space).

In Section 3, the solution of "w – T eq." is obtained by method of separating variables. And it has been checked by weather forecasting, by dimensional check, and by other model check.

In Section 4, the "w - P/ρeq." is derived by the combination of Boyles law and Charles' law.

In Section 5, the solution of "w - P/ρeq." can be obtained directly from the solution of "w – T eq." in Section 3. In Section 5.1, the air density is calculated. Where the traditional method and engineering tool bar considered the air density is the functions of temperature and highness, but no connected to wind speed. Here, the air density is connected with wind speed.

In section 6, set up the "J - P/ρeq.". The absolute motion (the jet plane motion relative a referenced point P on Earth) is equal to the linking motion (wind speed motion relative P) and the relative motion (jet speed motion relative wind motion).

In Section 7, the solution of "J - P/ρeq." is directly obtained from section 3. Two cases: 1, density of jet plane is a constant; 2, the density of the jet plane is variable.

In Section 8, as an example of calculation of pressure outside the passenger's cargo, MH370 has been used. Where it had been studied in cases of non-powered flying[[6]].

In Section 10, a conclusion is made.

II. Wind-Temperature Equation of a Point in Air ("W -T eq." in Short)

The derivation of "w – T equation" follows [3].

According to combination of Boyles' law and Charles' law (B-C law, in short), we have:

$$pV = cT, \tag{2-1}$$

Where p=the air pressure, accuracy. p is a stress tensor, with the same normal stress and zero shearing stress

$$\boldsymbol{p} = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix} = p[i \quad j \quad k]^T, \tag{2-2}$$

$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = p =$ compressive normal stress; shearing stress $\sigma_{xy} = \sigma_{yz} = \sigma_{zx} = 0$. i, j, k are unit vectors in Cartesian co-ordinates. $V = volume\ of\ a\ point = dxdydz = A_x dx = A_y dy = A_z dz$, with $dx, dy, dz \to 0$. A_x, A_y, A_z are the cross section areas of the element (point), respectively. c is a constant, T is the absolute temperature. (2-2) is the matrix form of p.

Let operator ∇ be an an inner products on both sides of (2-1), then, we have

$$\nabla \cdot (pV) = \boldsymbol{c}\nabla \cdot T, \tag{2-3}$$

Where

$$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k, \tag{2-4}$$

Author: Tian-Quan. Yun, no Affiliation. Address: # 1710 185 Roehampton Ave, Toronto, ON, M4P 0C6, Canada.
e-mail: tianquanyun@gmail.com

$$pV = p[i+j+k](dxdydz) = (pA_x dx)i + \left(pA_y dy\right)j + (pA_z dz)k =$$

$$F_x dxi + F_y dyj + F_z dzk = F \cdot ds, \quad (2\text{-}5)$$

Where F=the applied force, S=the displacement

$$F = F_x i + F_y j + F_z k, \quad (2\text{-}6)$$

$$s = s_x i + s_y j + s_z k, \quad (2\text{-}7)$$

$$\nabla(pV) = \nabla.F.ds = F.(\nabla.ds) = F = c\nabla.T, \quad (2\text{-}8)$$

By Newton's second law, we have

$$F = m\dot{u} = m\left(\dot{u}_x i + \dot{u}_y j + \dot{u}_z k\right), \quad (2\text{-}9)$$

Where m is the mass, $\boldsymbol{u}$is the wind speed, $\dot{u} = \partial u/\partial t$. Substituting (2-9) into (2-8), we have

$$\dot{u} = (c/m)\nabla \cdot T, \quad (2\text{-}10)$$

(2-10) is called the "Wind –Temperature Equation of A Point (Mass) in Air", or ("W – T equation" in short). It states that $\dot{u}$ is in proportion to ∇T, i.e., the derivative of wind speed u respect to time t proportions to the derivative of temperature T respect to track (space).

III. Solution of "W – T eq." by Method of Separating Variables

(2-10) is a vector PDE. It is hard to solve. Changing (2-10) to scalar function (its component form) is an easy way to solve. By (2-2) -- (2-4), we have:

$$k\frac{\partial}{\partial x}T = kT_x' = \dot{u}_x, \quad (3\text{-}1)$$

$$k\frac{\partial}{\partial y}T = kT_y' = \dot{u}_y, \quad (3\text{-}2)$$

$$k\frac{\partial}{\partial z}T = kT_z' = \dot{u}_z, \quad (3\text{-}3)$$

Where $k = c/M.\ T_x' = \frac{\partial}{\partial x}T,\ T_y' = \frac{\partial}{\partial y}T,\ T_z' = \frac{\partial}{\partial z}T.$

Looking at (3-3):

Where $T = T(z,t),\ \dot{u}_z = \dot{u}_z(z,t),\ \dot{u} = \partial u/\partial t.$

Suppose that variables of kT_z' and $\dot{u}_z$ can be separated:

$$kT_z'(z,t) = a'(z)b(t), \quad (3\text{-}4)$$

$$\dot{u}_z(z,t) = e(z)\dot{f}(t), \quad (3\text{-}5)$$

Where $a'(z) = da(z)/dz$, $b(t)$, $e(z)$ and $\dot{f}(t) = df(t)/dt$ are unknown functions and have continuous derivatives. We need sufficient relations to determine these unknowns.

The dimensions of kT_z' and $\dot{u}_z$ are the same (see the following red part), therefore, let (3-4) = (3-5),and separating variables, we have

$$\frac{a'(z)}{e(z)} = z^0 = k = const = \frac{\dot{f}(t)}{b(t)} = t^0, \quad (3\text{-}6)$$

Where the function of the left term is z, while the function of the middle term is t, if they are equal to each other, they must be a constant--- k.
For simple, we choose two relations:

$$e(z) = a(z), \quad (3\text{-}7)$$

$$b(t) = -f(t), \quad (3\text{-}8)$$

then, integrating both sides of (3-6) for t and by (3-8), we have:

$$\int \frac{\dot{f}(t)}{b(t)}dt = \ln f(t) = -\int Kdt + C_t = -Kt + C_t \quad (3\text{-}9)$$

Where C_t is a constant, determined by initial condition, i.e., $t=0,\ \ln f(0)=1,\ \ln\left|1\right|=0, \rightarrow C_t=0$. Then, (3-9) gives:

$$b(t) = -f(t) = exp[-Kt], \quad (3\text{-}10)$$

$$f(t) = -kexp[-kt^2], \quad (3\text{-}11)$$

Substituting (3-7) into (3-6), then, integrating both sides of (3-6) for z, we have:

$$\int \frac{a'(z)}{a(z)}dz = ln[a(z)] = \int kdz + C_z = kz + C_z, \quad (3\text{-}12)$$

Where C_z is an integral constant, determined by the boundary condition, i.e., $z = 0, T_z = T_z(0) = 1(℃) \rightarrow a(0) = 1, \rightarrow C_z = 0$. Therefore,

$$a(z) = e(z) = exp[kz], \quad (3\text{-}13)$$

$$a'(z) = kexp[kz], \quad (3\text{-}14)$$

Substituting (3-14), (3-10), into (3-4), we have

$$kT_z'(z,t) = kexp[kz]\{-kexp[-kt^2]\} = exp[k(z-t^2)], \quad (3\text{-}15)$$

Substituting (3-11), (3-13), into (3-5), we have

$$\dot{u}_z(z,t) = exp[kz]exp[-kt^2] = exp[k(z-t^2)] =$$

$$kT_z'(z,t), \quad (3\text{-}16)$$

Similar to (3-16), for (3-1), (3-2), we have:

$$\dot{u}_x(x,t) = exp[k(x-t^2)] = KT'(x,t),\ \ k=\frac{c}{m} \qquad (3\text{-}17)$$

$$\dot{u}_y(y,t) = exp[k(y-t^2)] = KT'(y,t),\ \ k=\frac{c}{m}, \qquad (3\text{-}18)$$

Expressed by vector form:

$$\dot{u}(s,t) = exp[k(\boldsymbol{s}-t^2] = k\nabla T(s,t)],\ \ k=\frac{c}{m}, \qquad (3\text{-}19)$$

Dimensional check: (3-19)

Dimensional check is a tool often used to check the correctness of calculation. The dimensions of each term of an equality must be the same.

$$c=\frac{pV}{T},\ k=\frac{c}{m}=\frac{pV}{mT}.\ dim[\nabla T]=\frac{d}{ds}J=N=\frac{kg.cm}{sec^2},$$

$$dim[k]=\frac{Ncm^{-2}cm^3}{kg..N;cm}=\frac{1}{kg},$$

$$dim\,\dot{u}=\frac{cm}{sec^2}, \qquad (3\text{-}20)$$

$$dim\,k\nabla T=\frac{1}{kg}\frac{kg.cm}{sec^3}=\frac{cm}{sec^2}. \qquad (3\text{-}21)$$

$$dim\ exp[k(\boldsymbol{s}-t^2)]=\frac{cm/kg}{sec^2/kg}=\frac{\boldsymbol{cm}}{\boldsymbol{sec^2}}, \qquad (3\text{-}22)$$

The dimensional check (red part) shows that the dimensions on both sides of the equality of (3-19) are the same. The dimensions in (3-19) is correct.

a) *Checking the "w – T eq." by weather forecasting (China Weather Net), and motion equation of mushroom cloud.*

Every winter, the weather forecast(China Weather Net) alerts people be wear that cold wave *comes with strong wind companied with temperature sharp dropping.* These description on temperature sharp dropping company with strong wind confirms that the difference of temperature between positions in track causes strong wind. Which agrees with (2-10) very well.

Moreover, a motion equation of mushroom cloud[3] derived by different model based on modifying of Navier-Stock equation , with result well agreed to (2-10). Which again confirms that (2-10) is believable.

IV. The "Wind – Pressure/Density Equation"("W– p/ρ Eq."in Short)

Dividing both sides of B – C law(combination of Boyles law and Charles law) (2-1) by M (mass), we have

$$p\frac{\boldsymbol{V}}{M}=p\frac{1}{\boldsymbol{\rho_{air}}}=\frac{\boldsymbol{c}}{M}T, \qquad (4\text{-}1)$$

Where $\rho_{air}=M/V$ is the density of air. Substituting (4-1) into (2-10), we have:

$$\dot{u}=\nabla(p/\rho_{air}), \qquad (4\text{-}2)$$

(4-2) is called the "W –P/ρ equation". (4-2) shows that $\dot{u}$ is in proportion to $\nabla(p/\rho_{air})$, i.e., the derivative of wind speed u respect to time t proportions to the derivative of (p/ρ_{air}) respect to track (space).

V. Solution of "W – p/ρ Eq." (4-2)

The solution of scalar form of (4-2) can be obtained from (3-17), (3-18), and (3-16) by changing $k=c/m$ to $k=\rho_{air}^{-1}$, i.e.,

$$Kp'(s,t)=\dot{u}_x(s,t)=exp[k(x-t^2)],\ \ (k=\rho_{air}^{-1}) \quad (5\text{-}1)$$

$$Kp'(s,t)=\dot{u}_y(s,t)=exp[k(y-t^2)],\ (k=\rho_{air}^{-1}) \quad (5\text{-}2)$$

$$Kp'(s,t)=\dot{u}_z(s,t)=exp[k(z-t^2)],\ (k=\rho_{air}^{-1}) \quad (5\text{-}3)$$

The solution of vector form of (4-2) is:

$$Kp'(s,t)=\dot{u}(s,t)=exp[k(\boldsymbol{s}-t^2)],\ \ (k=\rho_{air}^{-1}) \quad (5\text{-}4)$$

The solution of vector form of (4-2) can be obtained from (3-19) by changing k = c/m to $k=\rho_{air}^{-1}$, i.e.,

$$\dot{u}(s,t)=exp[k(s-t^2)]=k\nabla p(s,t)],\ (k=\rho_{air}^{-1}), \quad (5\text{-}5)$$

a) *Calculation of ρ_{air}*

Traditionally, methods, tools for calculation of $\rho_{air}=\rho_{air}(p,z)$ as functions of p and z [4,5], based on Boyles law and Charles law for static description, i.e., they have no connection with wind speed. However, our treatment of ρ_{air} is different to that of traditional. It connects with wind speed by "w – p/ρequation". (4-2), based on Boyles law , Charles' law and together with the Newton's second law. The calculation of ρ_{air}, can be found in Appendix.

VI. Set up the "Jet – Pressure/Density Equation" ("J- p/ρ Eq." in Short)

A jet plane flies in a atmospheric environment. In which the absolute motion (the jet plane motion relative a referenced point P on Earth) is equal to the linking motion (wind speed motion relative P) and the relative motion (jet speed motion relative wind motion).

Let the relative motion of the jet plane speed, density of air and pressure be $\boldsymbol{v}, p_v$, and ρ_{air}. Then, set up the "J-p/ρ Equation" similar to (4-2), we have:

$$\dot{u}+\dot{\boldsymbol{v}}=\rho_{air}^{-1}[\nabla p]+\left[\nabla(p_v/\rho_{jp})\right], \qquad (6\text{-}1)$$

$$\dot{v}=\nabla(p_v/\rho_{jp}), \qquad (6\text{-}2)$$

Where $\rho_{jp} = \rho_{jp}(z,t) = M(t)/V_{jp}$ = Density of jet plane; $M(t)$ is the mass of the jet plane. it varies with time t (jet fuel consuming); V_{jp} is the volume of the jet plane.

(6-2) shows that $\dot{\boldsymbol{v}}$ is in proportion to $\nabla(p_v/\rho_{jp})$ i.e., the derivative of jet plane speed $\boldsymbol{v}$ respect to time t, proportions to the derivative of pressure (p_v/ρ_{jp}) respect to track (space)of the jet plane.

VII. The Solution of "J – p/ρ Equation" (6-2)

1. If ρ_{jp} = const, then, the solution of (6-2) is the same as (5-5) just by $\dot{v}$, p_v instead of $\dot{u}$, p, respectively. That is:

$$\dot{\boldsymbol{v}}(\boldsymbol{s},t) = exp[k(\boldsymbol{s}-t^2)] = k\nabla p_v(\boldsymbol{s},t)],\ k = \rho_{air}^{-1}) \quad (7\text{-}1)$$

The solutions of similar (5-1) – (5-4) are:

$$Kp\text{'}_v(\boldsymbol{s},\boldsymbol{v}) = v_x(x,t) = exp[k(x-t^2)],\quad (k = \rho_{air}^{-1}) \quad (7\text{-}2)$$

$$Kp\text{'}_v(\boldsymbol{s},\boldsymbol{v}) = v_y(y,t) = exp[k(y-t^2)].\quad (k = \rho_{air}^{-1}) \quad (7\text{-}3)$$

$$Kp\text{'}_v(\boldsymbol{s},\boldsymbol{v}) = v_z(s,t) = exp[k(z-t^2)],\quad (k = \rho_{air}^{-1}) \quad (7\text{-}4)$$

$$Kp\text{'}_v(\boldsymbol{s},\boldsymbol{v}) = \boldsymbol{v}(\boldsymbol{s},t) = exp[k(\boldsymbol{s}-t^2)].\quad (k = \rho_{air}^{-1}) \quad (7\text{-}5)$$

2. If $\rho_{jp} = \rho_{jp}(z,t) = M(t)/V_{jp}$, is a known function. Where $M(t)$ depends on flying status, e.g. flying in a constant speed, then

$$M(t) = c_1 - c_2 t.\ (c_1, c_2, = \text{known const}). \quad (7\text{-}6)$$

Where for $t = 0$, $c_1 = M(0)$ = jet plsne mass of full fuel.; for

$t = t_1, M(t_1) = C_1 - t_1 c_2$ = jet plane mass of empty fuel

The solution of (6-2) is the same as (7-1) with $\rho_{jp} = M(t)/V_{jp}$ inside ∇,i.e.,

$$\dot{\boldsymbol{v}} = exp[k(\boldsymbol{s}-t^2)] = \nabla p_v(s,t)/(M(t)/V_{jp})], \quad (7\text{-}7)$$

Where $\boldsymbol{s}$ is instead of by z.

Similar treatment can be used for the scalar form, e.g., (7-5).

$$Kp\text{'}_v(\boldsymbol{s},\boldsymbol{v}) = v(z,t) = exp[k(z-t^2)],\ (k = (M(t)/V_{jp})^{-1}) \quad (7\text{-}8)$$

VIII. Example

MH370—a missing plane [5]is used as an example to show the calculation . Suppose that the plane was crashed due to pressures difference between inside and out side of passenger's cargo. Here, we calculate the pressure out side the cargo. The Boeing 747 cruise speed $\boldsymbol{v} =$ 1000 km/hour, cruise high $z =$ 10,000 m, The masses of jet plane of full fuel and empty fuel are $M(0)$ and $M(t_1)$, respectively.

$$\boldsymbol{v} = \int \dot{\boldsymbol{v}} dt = exp[k(10000m - t)] = 1000km/hour,$$
$$(k = (M(t)/V_{jp})^{-1}) \quad (8\text{-}1)$$

From (81), we get t.
Calculation of $\rho_{\boldsymbol{dryair}}$.

By (A-2), $\rho_{\boldsymbol{dryair}} = 31.91 \times 9.91 = 313.0371(kg/m^3)$. (8-2)

Substituting (8-2) into (5-3), we have:

$$p(z,t) = \dot{u}_z(z,t) = exp[k(z-t^2)], \quad (8\text{-}3)$$

Substituting $M(0)$, $M(t_1)$, V_{jp} and (7-6) into (7-8), we have:

$$p_v(\boldsymbol{z},t) = \dot{\boldsymbol{v}}(\boldsymbol{z},t) = exp[k(\boldsymbol{z}-t^2)],\ (k = (M(t)/V_{jp})^{-1}) \quad (8\text{-}4)$$

Substituting (6-2) and (8-4) into relative motion and linking motion, We have:

$$p(z,t) + p_v(z,t) = u_z(z,t) + v(z,t) = exp[k(z-t^2)] +$$
$$exp[k_1(z-t^2)],(k_1 = (M(t)/V_{jp})^{-1}),$$
$$(k = (313.03)^{-1}(m^3/kg) \quad (8\text{-}5)$$

If the moist air instead of the dray air, the density of moist air can be calculated by (A-3) with weight of 18 of water molecular.

Now, the pressure out side the passenger's cargo is calculated by (8-5), and the difference of pressure between inside and out side of passenger's cargo is known.

IX. Appendix

a) *Density of Dry air* ρ_{dryair}

In the atmosphere around us, 78% Nitrogen (N_2), 21% Oxygen (O_2), and 1% other gases. The N has a molecular weight of 14, so N_2 has a molecular weight of 28, Oxygen has a molecular weight of 16, so O_2 has molecular weight of 32. Given the mixture of gases of molecules weight is around 29. The total weight of 100% air of $1m^3$ is: w $=[0.78 \times 28 + 0.21 \times 32 + 0.01 \times 29] \times 1m^3 = 31.91(N)$.

$$\rho_{\boldsymbol{dryair}} = 31.91g, \quad (A\text{-}1)$$

Where $g ==$ gravity acceleration. *Assumption:* g is independent with position s, $g = 9.81(m/sec^2)$. Then, by (A-1), we have:

$$\rho_{\boldsymbol{dryair}} = 31.91 \times 9.91 = 313.0371(kg/m^3). \quad (A\text{-}2)$$

b) Density of moist air

Water H_2O, Hydrogen H is the lightest element and has a molecular weight of 1. So a water molecular weight is $(2 \times 1 + 16 = 18)$. Which shows that the water molecular weight is much lighter than the average weight of the molecular found in air.

The density of moist air can be calculated as the sum of two gases: dry air and water vapor in proportion with their partial proportion c.

$$\rho_{moistair} = c \times \rho_{dryair} + (1 - c)\rho_{vapor} \ (c \leq 1), \quad \text{(A-3)}$$

X. Conclusion

1. The "w – T eq." is basic. The solution of "w - P/ρeq." and "J - P/ρeq.". can be obtained directly from "w – T eq.".
2. The calculation of pressure outside passenger's cargo is used by(6-1) and (6-2).

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