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Certain Oberhettinger's Integrals Associated with Product of General Polynomials and Incomplete H- Function

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Abstract- In this paper, we developed Oberhettinger's integral formulas containing the product of general polynomials and incomplete H-functions. These integral formulas are very useful to obtain the Mellin transform of various simpler special functions. The Mellin transform of special functions find their applications in mathematical statistics, number theory and the theory of asymptotic expansions. The main findings of the present work are very useful in solving the problems arising in digital signals, image processing, finance and ship target recognition by sonar system and radar signals.

Keywords: *incomplete H-functions, gamma functions, improper integrals, general polynomials.*

GJSFR-F Classification: *MSC 2010: 33C60, 33C99*

Ref

6. Brychkov, Y.A. Handbook of Special Functions, Derivatives, Integrals, Series and Other formulas, CRC Press, Boca Raton, FL, USA,2008.

Certain Oberhettinger's Integrals Associated with Product of General Polynomials and Incomplete H- Function

Harshita Garg

Abstract- In this paper, we developed Oberhettinger's integral formulas containing the product of general polynomials and incomplete H-functions. These integral formulas are very useful to obtain the Mellin transform of various simpler special functions. The Mellin transform of special functions find their applications in mathematical statistics, number theory and the theory of asymptotic expansions. The main findings of the present work are very useful in solving the problems arising in digital signals, image processing, finance and ship target recognition by sonar system and radar signals.

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I. Introduction

Several integral formulas containing generalized special functions have been explored by many authors [1-5]. These integrals play a focal role in solving scientific and engineering problems. Here we develop Oberhettinger's integral formulas containing the product of general polynomials and incomplete H- functions. Many authors established several unified integral formulas containing a various kind of special functions [6-8]. The findings of this work are general in nature and very useful in science, engineering and finance. Here we find some special cases by specializing the parameters of general polynomials and incomplete H-functions (for example, Fox's H-function, Incomplete Fox-Wright functions, Fox-Wright functions and incomplete generalized hypergeometric functions) and also listed few known results. The main results of this work are very useful in solving the problems arising in digital signals, image processing, finance and ship target recognition by sonar system and radar signals [9-12].

The incomplete Gamma functions $\Gamma(\xi, z)$ and $\Upsilon(\xi, z)$ are defined as following:

$$\Upsilon(\xi, z) = \int_0^z t^{\xi-1} e^{-t}\, dt \qquad (\Re(\xi) > 0\,;\, z \geq 0) \qquad \ldots (1)$$

And

$$\Gamma(\xi, z) = \int_z^\infty t^{\xi-1} e^{-t}\, dt \qquad (z \geq 0\,;\ \Re(\xi) > 0 \text{ when } z = 0) \qquad \ldots (2)$$

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Respectively, satisfy the decomposition formula given by:

$$\Gamma(\xi, z) + \Upsilon(\xi, z) = \Gamma(\xi) \qquad (\Re(\xi) > 0) \qquad \dots (3)$$

The condition which we have used on the parameter z in and anywhere else in this paper is unrestricted of $\Re(x)$ $(x\epsilon\ C)$.

The incomplete generalized hypergeometric functions ${}_e\Gamma_f$ and ${}_e\gamma_f$ are defined by Srivastava et al. [13] in terms of incomplete Gamma functions $\Gamma(s, z)$ and $\Upsilon(s, z)$ as following:

Notes

$${}_e\Gamma_f\left[\begin{matrix}(u_1, z), u_2, \dots, u_e; \\ v_1, \dots, v_f;\end{matrix}\, x\right] = \frac{\prod_{j=1}^{f}\Gamma(v_j)}{\prod_{j=1}^{e}\Gamma(u_j)}\sum_{\ell=0}^{\infty}\frac{\Gamma(u_1+\ell, z)\prod_{j=2}^{e}\Gamma(u_j+\ell)}{\prod_{j=1}^{f}\Gamma(v_j+\ell)}\frac{x^\ell}{\ell!}$$

$$= \frac{1}{2\pi\iota}\frac{\prod_{j=1}^{f}\Gamma(v_j)}{\prod_{j=1}^{e}\Gamma(u_j)}\int_{\pounds}\frac{\Gamma(u_1+s, z)\prod_{j=2}^{e}\Gamma(u_j+s)}{\prod_{j=1}^{f}\Gamma(v_j+s)}\Gamma(-s)(-x)^s\, ds \qquad \dots (4)$$

(|arg (-x)|< π,

And

$${}_e\gamma_f\left[\begin{matrix}(u_1, z), u_2, \dots, u_e; \\ v_1, \dots, v_f;\end{matrix}\, x\right] = \frac{\prod_{j=1}^{f}\Gamma(v_j)}{\prod_{j=1}^{e}\Gamma(u_j)}\sum_{\ell=0}^{\infty}\frac{\gamma(u_1+\ell, z)\prod_{j=2}^{e}\Gamma(u_j+\ell)}{\prod_{j=1}^{f}\Gamma(v_j+\ell)}\frac{x^\ell}{\ell!}$$

$$= \frac{1}{2\pi\iota}\frac{\prod_{j=1}^{f}\Gamma(v_j)}{\prod_{j=1}^{e}\Gamma(u_j)}\int_{\pounds}\frac{\gamma(u_1+s, z)\prod_{j=2}^{e}\Gamma(u_j+s)}{\prod_{j=1}^{f}\Gamma(v_j+s)}\Gamma(-s)(-x)^s\, ds \qquad \dots (5)$$

(|arg (-x)|< π,

Where $\mathscr{L}$ is the Mellin- Barnes type contour having ϲ-ί∞ as the starting point and ϲ+ί∞ (ϲ ϵ R) as the end point with the usual indentations to separate a set of poles from another of the integrand in each and every case.

The incomplete H- function $\gamma_{e,f}^{c,d}(x)$ and $\Gamma_{e,f}^{c,d}(x)$ are introduced by Srivastava et. Al. [14] as following:

Ref

17. Mathai, A.M., Saxena, R.K., Haubold, H.J. The H Function: Theory and Applications; Springer; New York, NY, USA, 2009.

$$\Gamma_{e,f}^{c,d}(x)=\Gamma_{e,f}^{c,d}\left[x\left|\begin{matrix}(u_1,U_1,z),(u_j,U_j)_{2,e}\\(v_j,V_j)_{1,f}\end{matrix}\right.\right]=\Gamma_{e,f}^{c,d}\left[x\left|\begin{matrix}(u_1,U_1,z),(u_2,U_2),\dots,(u_e,U_e)\\(v_1,V_1),(v_2,V_2),\dots,(v_f,V_f)\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi\iota}\int_{\pounds} f(s,z)x^{-s}ds \qquad \dots (6)$$

Where $f(s,z)=\dfrac{\Gamma(1-\mathrm{u}_1-U_1s,z)\prod_{\mathrm{j=1}}^{\mathrm{c}}\Gamma(\mathrm{v_j}+V_js)\prod_{\mathrm{j=2}}^{\mathrm{d}}\Gamma(1-\mathrm{u_j}-U_js)}{\prod_{\mathrm{j}=c+1}^{\mathrm{f}}\Gamma(1-\mathrm{v_j}-V_js)\prod_{\mathrm{j}=d+1}^{\mathrm{e}}\Gamma(\mathrm{u_j}+U_js)}$

And

$$\gamma_{e,f}^{c,d}(x)=\gamma_{e,f}^{c,d}\left[x\left|\begin{matrix}(u_1,U_1,z),(u_j,U_j)_{2,e}\\(v_j,V_j)_{1,f}\end{matrix}\right.\right]=\gamma_{e,f}^{c,d}\left[x\left|\begin{matrix}(u_1,U_1,z),(u_2,U_2),\dots,(u_e,U_e)\\(v_1,V_1),(v_2,V_2),\dots,(v_f,V_f)\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi\iota}\int_{\pounds} F(s,z)x^{-s}ds \qquad \dots (7)$$

Where

$$F(s,z)=\frac{\gamma(1-\mathrm{u}_1-U_1s,z)\prod_{\mathrm{j=1}}^{\mathrm{c}}\Gamma(\mathrm{v_j}+V_js)\prod_{\mathrm{j=2}}^{\mathrm{d}}\Gamma(1-\mathrm{u_j}-U_js)}{\prod_{\mathrm{j}=c+1}^{\mathrm{f}}\Gamma(1-\mathrm{v_j}-V_js)\prod_{\mathrm{j}=d+1}^{\mathrm{e}}\Gamma(\mathrm{u_j}+U_js)}$$

The incomplete H- functions $\gamma_{e,f}^{c,d}(x)$ and $\Gamma_{e,f}^{c,d}(x)$ in (6) and (7) respectively exists for all z≥0 under the same set of conditions and same set of contour stated in the articles presented by Kilbas et al. [15], Mathai and Saxena [16] and Mathai et al. [17].

Some special cases of incomplete H-function are as following:

(i)

If we take z=0 in (6), then the incomplete H- function $\Gamma_{e,f}^{c,d}(x)$ reduces to Fox's H-function [18].

$$\Gamma_{e,f}^{c,d}\left[x\left|\begin{matrix}(u_1,U_1,0),(u_2,U_2),\dots,(u_e,U_e)\\(v_1,V_1),(v_2,V_2),\dots,(v_f,V_f)\end{matrix}\right.\right]=H_{e,f}^{c,d}\left[x\left|\begin{matrix}(u_1,U_1),(u_2,U_2),\dots,(u_e,U_e)\\(v_1,V_1),(v_2,V_2),\dots,(v_f,V_f)\end{matrix}\right.\right] \qquad \dots (8)$$

(ii) If we take c=1, d=e and replace f by f+1 and take suitable parameter, then the function (6) and (7) reduces to incomplete Fox-Wright function ${}_e\Psi_f^{(\Gamma)}$ and ${}_e\Psi_f^{(\gamma)}$ (for details see [14]).

$$\Gamma_{e,f+1}^{1,e}\left[-x\left|\begin{matrix}(1-u_1,U_1,z),(1-u_j,U_j)_{2,e}\\(0,1),(1-v_j,V_j)_{1,f}\end{matrix}\right.\right]={}_e\Psi_f^{(\Gamma)}\left[\begin{matrix}(u_1,U_1,z),(u_j,U_j)_{2,e};\\(v_j,V_J)_{1,f};\end{matrix}\,x\right] \quad \ldots (9)$$

And

$$\gamma_{e,f+1}^{1,e}\left[-x\left|\begin{matrix}(1-u_1,U_1,z),(1-u_j,U_j)_{2,e}\\(0,1),(1-v_j,V_j)_{1,f}\end{matrix}\right.\right]={}_e\Psi_f^{(\gamma)}\left[\begin{matrix}(u_1,U_1,z),(u_j,U_j)_{2,e};\\(v_j,V_J)_{1,f};\end{matrix}\,x\right] \quad \ldots (10)$$

(iii)

If we take z=0 in (9), then incomplete Fox-Wright function ${}_e\Psi_f^{(\Gamma)}$ reduces to well-known Fox-Wright function ${}_e\Psi_f$ (for details see [18]).

$${}_e\Psi_f^{(\Gamma)}\left[\begin{matrix}(u_1,U_1,0),(u_j,U_j)_{2,e};\\(v_j,V_j)_{1,f};\end{matrix}\,x\right]={}_e\Psi_f\left[\left|\begin{matrix}(u_1,U_1),(u_j,U_j)_{2,e};\\(v_j,V_j)_{1,f};\end{matrix}\,x\right.\right] \quad \ldots (11)$$

(iv) If we take $U_j = V_k = 1$ (j=1,…,e, k=1,…,f) in (9) and (10), then incomplete Fox-Wright function reduces to the incomplete generalized hypergeometric functions ${}_e\gamma_f$ and ${}_e\Gamma_f$ (see [13]).

$${}_e\Psi_f^{(\Gamma)}\left[\begin{matrix}(u_1,1,z),(u_j,1)_{2,e};\\(v_j,1)_{1,f};\end{matrix}\,x\right]={}_e\Gamma_f\left[\left|\begin{matrix}(u_1,z),u_2,\ldots,u_e;\\v_1,\ldots v_f;\end{matrix}\,x\right.\right] \quad \ldots (12)$$

And

$${}_e\Psi_f^{(\gamma)}\left[\begin{matrix}(u_1,1,z),(u_j,1)_{2,e};\\(v_j,1)_{1,f};\end{matrix}\,x\right]={}_e\gamma_f\left[\left|\begin{matrix}(u_1,z),u_2,\ldots,u_e;\\v_1,\ldots v_f;\end{matrix}\,x\right.\right] \quad \ldots (13)$$

The general polynomials are defined by Srivastava [19] as following:

$$S_{n_1,\ldots,n_s}^{m_1,\ldots,m_s}[t_1,\ldots,t_s]=\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}.\ A[n_1,\alpha_1;\ldots;n_s,\alpha_s]t_1^{\alpha_1}\ldots t_s^{\alpha_s} \quad \ldots (14)$$

Where $n_i = 0, 1, 2\ldots\ \forall i=(1,\ldots,s)$; $m_1,\ldots,m_s$ are arbitrary positive integers and the coefficients $A[n_1,\alpha_1;\ldots;n_s,\alpha_s]$ are arbitrary constants, real or complex.

In this paper we use the following integral formula [20],

$$\int_0^\infty y^{\delta-1}\left(y+k+\sqrt{y^2+2ky}\right)^{-\tau}dy=2\tau k^{-\tau}\left(\frac{k}{2}\right)^{\delta}\frac{\Gamma(2\delta)\Gamma(\tau-\delta)}{\Gamma(1+\tau+\delta)} \quad \ldots (15)$$

Global Journal of Science Frontier Research (F) Volume XXII Issue I Version I Year 2022

Ref

18. Srivastava, H.M., Gupta, K.C., Goyal, S.P. The H- Functions of One and Two Variables with Applications, South Asian Publishers, New Delhi, India, 1982.

Notes

II. The Main Integral Formula

In this section we obtain Oberhettinger's integral formulas containing product of incomplete $\overline{H}$-function and general polynomials. These integral formulas are very useful to obtain the Mellin transform of several simpler special functions.

Theorem 1: If τ, δ ∈ C with 0 ≤ R (δ) < R (τ) and y>0, then the following integral formula holds:

$$\int_0^\infty y^{\delta-1}\left(y+k+\sqrt{y^2+2ky}\right)^{-\tau} . S_{n_1,\dots,n_s}^{m_1,\dots,m_s}\left[\frac{t_1}{\left(y+k+\sqrt{y^2+2ky}\right)},\dots,\frac{t_s}{\left(y+k+\sqrt{y^2+2ky}\right)}\right]$$

$$.\Gamma_{e,f}^{c,d}\left[\frac{x}{\left(y+k+\sqrt{y^2+2ky}\right)}\left|\begin{matrix}(u_1,U_1,z),(u_j,U_j)_{2,e}\\(v_j,V_j)_{1,f}\end{matrix}\right.\right]dy = 2^{1-\delta}k^{\delta-\tau-\sum_{i=1}^{s}\alpha_i}\Gamma(2\delta)$$

$$.\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}.A[n_1,\alpha_1;\dots;n_s,\alpha_s]t_1^{\alpha_1}\dots t_s^{\alpha_s}$$

$$.\Gamma_{e+2,f+2}^{c,d+2}\left[\frac{x}{k}\left|\begin{matrix}(u_1,U_1,z),(u_j,U_j)_{2,e},\left(-\tau-\sum_{i=1}^{s}\alpha_i,1\right),\left(1-\tau-\sum_{i=1}^{s}\alpha_i+\delta,1\right)\\(v_j,V_j)_{1,f},\left(1-\tau-\sum_{i=1}^{s}\alpha_i,1\right),\left(-\tau-\sum_{i=1}^{s}\alpha_i-\delta,1\right)\end{matrix}\right.\right] \quad \dots (16)$$

All the conditions of incomplete H- function $\Gamma_{e,f}^{c,d}(x)$ in (6) are satisfied.

Proof: Let the left hand side of the assertion (16) is denoted by Δ and using (6) and (14) in the left hand side of (16), we get

$$\Delta = \int_0^\infty y^{\delta-1}\left(y+k+\sqrt{y^2+2ky}\right)^{-\tau} . . \sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\cdots\frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}A[n_1,\alpha_1;\dots;n_s,\alpha_s]$$

$$.\left(\frac{t_1}{\left(y+k+\sqrt{y^2+2ky}\right)}\right)^{\alpha_1},\dots,\left(\frac{t_s}{\left(y+k+\sqrt{y^2+2ky}\right)}\right)^{\alpha_s}\frac{1}{2\pi\iota}\int_L\left(\frac{x}{\left(y+k+\sqrt{y^2+2ky}\right)}\right)^{-\xi}f(\xi,z)d\xi dy$$

Where f(ξ , z) is defined in (6).

Now changing the order of summation, integration and contour integral involved therein (which is permissible under the stated conditions), we get

Global Journal of Science Frontier Research (F) Volume XXII Issue I Version I Year 2022

$$\Delta = \sum_{\alpha_1=0}^{[n_1/m_1]} \cdots \sum_{\alpha_s=0}^{[n_s/m_s]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1 !} \cdots \frac{(-n_s)_{m_s\alpha_s}}{\alpha_s !} A[n_1,\alpha_1;\dots;n_s,\alpha_s].t_1^{\alpha_1}\dots t_s^{\alpha_s}.\frac{1}{2\pi\iota}\int_L f(\xi,z)x^{-\xi}$$

$$.\int_0^\infty y^{\delta\text{-}1}\left(y+k+\sqrt{y^2+2ky}\right)^{-(\tau+\alpha_1+\dots+\alpha_s-\xi)} dy d\xi$$

Now by using (15) in above integral and reinterpreting it in the form of incomplete H-function $\Gamma_{e,f}^{c,d}(x)$, we get the result (16).

Theorem- 2

If τ, δ ∈ C with 0 <R (δ) < R (τ) and y > 0, then the following integral formula holds:

$$\int_0^\infty y^{\delta\text{-}1}\left(y+k+\sqrt{y^2+2ky}\right)^{-\tau}.S_{n_1,\dots,n_s}^{m_1,\dots,m_s}\left[\frac{t_1}{\left(y+k+\sqrt{y^2+2ky}\right)},\dots,\frac{t_s}{\left(y+k+\sqrt{y^2+2ky}\right)}\right].$$

$$.\gamma_{e,f}^{c,d}\left[\frac{x}{\left(y+k+\sqrt{y^2+2ky}\right)}\left|\begin{matrix}(u_1,U_1,z),(u_j,U_j)_{2,e}\\(v_j,V_j)_{1,f}\end{matrix}\right.\right]dy = 2^{1-\delta}k^{\delta-\tau-\sum_{i=1}^{s}\alpha_i}\Gamma(2\delta)$$

$$.\sum_{\alpha_1=0}^{[n_1/m_1]} \cdots \sum_{\alpha_s=0}^{[n_s/m_s]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1 !} \cdots \frac{(-n_s)_{m_s\alpha_s}}{\alpha_s !} .A[n_1,\alpha_1;\dots;n_s,\alpha_s]t_1^{\alpha_1}\dots t_s^{\alpha_s}$$

$$.\gamma_{e+2,f+2}^{c,d+2}\left[\frac{x}{k}\left|\begin{matrix}(u_1,U_1,z),(u_j,U_j)_{2,e},\left(-\tau-\sum_{i=1}^{s}\alpha_i,1\right),\left(1-\tau-\sum_{i=1}^{s}\alpha_i+\delta,1\right)\\(v_j,V_j)_{1,f},\left(1-\tau-\sum_{i=1}^{s}\alpha_i,1\right),\left(-\tau-\sum_{i=1}^{s}\alpha_i-\delta,1\right)\end{matrix}\right.\right] \quad \dots (17)$$

All the conditions of incomplete H- function $\gamma_{e,f}^{c,d}(x)$ in (7) are satisfied.

Proof: Let the left hand side of the assertion (17) is denoted by Δ and using (7) and (14) in the left hand side of (17), we get

$$\Delta = \int_0^\infty y^{\delta\text{-}1}\left(y+k+\sqrt{y^2+2ky}\right)^{-\tau}..\sum_{\alpha_1=0}^{[n_1/m_1]} \cdots \sum_{\alpha_s=0}^{[n_s/m_s]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1 !} \cdots \frac{(-n_s)_{m_s\alpha_s}}{\alpha_s !} A[n_1,\alpha_1;\dots;n_s,\alpha_s]$$

$$.\left(\frac{t_1}{\left(y+k+\sqrt{y^2+2ky}\right)}\right)^{\alpha_1},\dots,\left(\frac{t_s}{\left(y+k+\sqrt{y^2+2ky}\right)}\right)^{\alpha_s}\frac{1}{2\pi\iota}\int_L\left(\frac{x}{\left(y+k+\sqrt{y^2+2ky}\right)}\right)^{-\xi}F(\xi,z)d\xi dy$$

Notes

Where F (ξ, z) is defined in (7)

Now changing the order of summation, integration and contour integral involved therein (which is permissible under the stated conditions), we get

$$\Delta = \sum_{\alpha_1=0}^{[n_1/m_1]} \cdots \sum_{\alpha_s=0}^{[n_s/m_s]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!} \cdots \frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!} A[n_1,\alpha_1;\ldots;n_s,\alpha_s].t_1^{\alpha_1}\ldots t_s^{\alpha_s}.\frac{1}{2\pi\iota}\int_L F(\xi,z)x^{-\xi}$$

$$.\int_0^\infty y^{\delta-1}\left(y+k+\sqrt{y^2+2ky}\right)^{-(\tau+\alpha_1+\ldots+\alpha_s-\xi)} dy d\xi$$

Now by using (15) in above integral and reinterpreting it in the form of incomplete H-function $\gamma_{e,f}^{c,d}(x)$, we get the result (17).

III. Special Cases

In this section, we obtain some interesting special cases of main results (16) and (17).

(i) If τ, δ Є C with 0 < R(δ) < R(τ) and y > 0 and incomplete H-function reduces into incomplete hypergeometric function with the help of (12) and (13), then the following integral formula occurs:

$$\int_0^\infty y^{\delta-1}\left(y+k+\sqrt{y^2+2ky}\right)^{-\tau} .S_{n_1,\ldots,n_s}^{m_1,\ldots,m_s}\left[\frac{t_1}{\left(y+k+\sqrt{y^2+2ky}\right)},\ldots,\frac{t_s}{\left(y+k+\sqrt{y^2+2ky}\right)}\right]$$

$$._e\Gamma_f\left[\begin{matrix}(u_1,z),u_2,\ldots,u_e; \\ v_1,\ldots,v_f;\end{matrix} \frac{x}{\left(y+k+\sqrt{y^2+2ky}\right)}\right]dy = 2^{1-\delta} k^{\delta-\tau-\sum_{i=1}^{s}\alpha_i}\Gamma(2\delta)$$

$$.\sum_{\alpha_1=0}^{[n_1/m_1]} \cdots \sum_{\alpha_s=0}^{[n_s/m_s]} \frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!} \cdots \frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!} .A[n_1,\alpha_1;\ldots;n_s,\alpha_s]t_1^{\alpha_1}\ldots t_s^{\alpha_s}$$

$$._{e+2}\Gamma_{f+2}\left[\frac{x}{k}\left|\begin{matrix}(u_1,z),u_2,\ldots,u_e,-\tau-\sum_{i=1}^{s}\alpha_i,1-\tau-\sum_{i=1}^{s}\alpha_i+\delta \\ v_1,\ldots,v_f,1-\tau-\sum_{i=1}^{s}\alpha_i,-\tau-\sum_{i=1}^{s}\alpha_i-\delta\end{matrix}\right.\right] \quad \ldots (18)$$

And

$$\int_0^\infty y^{\delta-1}\left(y+k+\sqrt{y^2+2ky}\right)^{-\tau} .S_{n_1,\ldots,n_s}^{m_1,\ldots,m_s}\left[\frac{t_1}{\left(y+k+\sqrt{y^2+2ky}\right)},\ldots,\frac{t_s}{\left(y+k+\sqrt{y^2+2ky}\right)}\right]$$

Notes

$$\cdot_e\gamma_f\left[\begin{matrix}(u_1,z),u_2,\dots,u_e;\\ v_1,\dots,v_f;\end{matrix}\frac{x}{\left(y+k+\sqrt{y^2+2ky}\right)}\right]dy=2^{1-\delta}k^{\delta-\tau-\sum_{i=1}^{s}\alpha_i}\Gamma(2\delta)$$

$$\cdot\sum_{\alpha_1=0}^{[n_1/m_1]}\dots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\dots\frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}.A[n_1,\alpha_1;\dots;n_s,\alpha_s]t_1^{\alpha_1}\dots t_s^{\alpha_s}$$

$$\cdot_{e+2}\gamma_{f+2}\left[\frac{x}{k}\left|\begin{matrix}(u_1,z),u_2,\dots,u_e,-\tau-\sum_{i=1}^{s}\alpha_i,1-\tau-\sum_{i=1}^{s}\alpha_i+\delta\\ v_1,\dots,v_f,1-\tau-\sum_{i=1}^{s}\alpha_i,-\tau-\sum_{i=1}^{s}\alpha_i-\delta\end{matrix}\right.\right] \quad \dots (19)$$

Given that both integrals exist.

Proof: Again if we take $U_j = V_j = 1$ in (16) and (17), we get the required results.

(ii) If τ, $\delta \in C$ with $0 < R(\delta) < R(\tau)$ and $y > 0$ and incomplete H-function reduces into incomplete Wright functions, then the following integral formula occurs:

$$\int_0^\infty y^{\delta-1}\left(y+k+\sqrt{y^2+2ky}\right)^{-\tau}.S_{n_1,\dots,n_s}^{m_1,\dots,m_s}\left[\frac{t_1}{\left(y+k+\sqrt{y^2+2ky}\right)},\dots,\frac{t_s}{\left(y+k+\sqrt{y^2+2ky}\right)}\right]$$

$$\cdot_e\Psi_f^{(\Gamma)}\left[\begin{matrix}(u_1,U_1,z),(u_j,U_j)_{2,e};\\ (v_j,V_j)_{1,f};\end{matrix}\frac{x}{\left(y+k+\sqrt{y^2+2ky}\right)}\right]dy=2^{1-\delta}k^{\delta-\tau-\sum_{i=1}^{s}\alpha_i}\Gamma(2\delta)$$

$$\cdot\sum_{\alpha_1=0}^{[n_1/m_1]}\dots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-n_1)_{m_1\alpha_1}}{\alpha_1!}\dots\frac{(-n_s)_{m_s\alpha_s}}{\alpha_s!}.A[n_1,\alpha_1;\dots;n_s,\alpha_s]t_1^{\alpha_1}\dots t_s^{\alpha_s}$$

$$\cdot_{e+2}\Psi_{f+2}^{(\Gamma)}\left[\frac{x}{k}\left|\begin{matrix}(u_1,U_1,z),(u_j,U_j)_{2,e},(1+\tau+\sum_{i=1}^{s}\alpha_i,1),(\tau+\sum_{i=1}^{s}\alpha_i-\delta,1)\\ (v_j,V_j)_{1,f},(\tau+\sum_{i=1}^{s}\alpha_i,1),(1+\tau+\sum_{i=1}^{s}\alpha_i+\delta,1)\end{matrix}\right.\right] \quad \dots (20)$$

And

$$\int_0^\infty y^{\delta-1}\left(y+k+\sqrt{y^2+2ky}\right)^{-\tau}.S_{n_1,\dots,n_s}^{m_1,\dots,m_s}\left[\frac{t_1}{\left(y+k+\sqrt{y^2+2ky}\right)},\dots,\frac{t_s}{\left(y+k+\sqrt{y^2+2ky}\right)}\right]$$

Notes

$$ {}_e\Psi_f^{(\gamma)}\left[\begin{matrix}(u_1,U_1,z),(u_j,U_j)_{2,e};\\(v_j,V_j)_{1,f};\end{matrix}\ \frac{x}{\left(y+k+\sqrt{y^2+2ky}\right)}\right]dy = 2^{1-\delta}k^{\delta-\tau-\sum_{i=1}^{s}\alpha_i}\Gamma(2\delta) $$

$$.\sum_{\alpha_1=0}^{[n_1/m_1]}\cdots\sum_{\alpha_s=0}^{[n_s/m_s]}\frac{(-\mathrm{n}_1)_{\mathrm{m}_1\alpha_1}}{\alpha_1!}\cdots\frac{(-\mathrm{n}_s)_{\mathrm{m}_s\alpha_s}}{\alpha_s!}.A[\mathrm{n}_1,\alpha_1;\ldots;\mathrm{n}_s,\alpha_s]\,\mathrm{t}_1^{\alpha_1}\ldots\mathrm{t}_s^{\alpha_s} $$

$$.{}_{e+2}\Psi_{f+2}^{(\gamma)}\left[\frac{x}{k}\left|\begin{matrix}(u_1,U_1,z),(u_j,U_j)_{2,e},(1+\tau+\sum_{i=1}^{s}\alpha_i,1),(\tau+\sum_{i=1}^{s}\alpha_i-\delta,1)\\(v_j,V_j)_{1,f},(\tau+\sum_{i=1}^{s}\alpha_i,1),(1+\tau+\sum_{i=1}^{s}\alpha_i+\delta,1)\end{matrix}\right.\right] \quad \ldots (21) $$

Given that each member of assertion (20) and (21) are exist.

Proof: with the help of (9) and (10), we get the above results.

(iii) If τ, δ ∈ C with $0 < R(\delta) < R(\tau)$, $y > 0$ and incomplete H-function reduces into Fox-Wright generalized hypergeometric function and general polynomials reduces into unity, then the following integral formula holds:

$$ \int_0^\infty y^{\delta-1}\left(y+k+\sqrt{y^2+2ky}\right)^{-\tau}\ {}_e\Psi_f\left[\begin{matrix}(u_j,U_j)_{1,e};\\(v_j,V_j)_{1,f};\end{matrix}\ \frac{x}{\left(y+k+\sqrt{y^2+2ky}\right)}\right]dy = 2^{1-\delta}k^{\delta-\tau}\Gamma(2\delta) $$

$$.{}_{e+2}\Psi_{f+2}\left[\frac{x}{k}\left|\begin{matrix}(u_j,U_j)_{1,e},(1+\tau,1),(\tau-\delta,1)\\(v_j,V_j)_{1,f},(\tau,1),(1+\tau+\delta,1)\end{matrix}\right.\right] \quad \ldots (22) $$

Given that each member of assertion (22) is exist.

Proof: Let general polynomials reduces into unity and with the help of (11), we get the above result.

IV. Conclusions

In this paper, we obtained some engrossing integrals containing the product of incomplete H-function and general polynomials, which are expressed in terms of incomplete H-functions. We have also given some special cases by specializing the parameters of general polynomials and incomplete H-functions (Incomplete Fox-Wright functions, incomplete hypergeometric functions, Fox - Wright generalized hypergeometric functions). These results are general in nature and very useful in science, engineering and finance.

Notes

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