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# Physics and Space Science


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# Two-Stage Model of Silicate Glass Transition

By Shangcong Cheng

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*Keywords: silicate glasses, medium range ordering structure, glass transition, phase diagram, critical temperature, second-order phase transition.*

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## I. Introduction

As a liquid is cooled from its freezing temperature, it may either crystallize or become glass. The transition from liquid to crystal is well-studied and understood. It is a first-order phase transition in the Paul Ehrenfest classification scheme because the first-order deviations of the Gibbs energy function, such as volume and entropy, exhibit discontinuity with respect to temperature. The transition process from liquid to glass is much less straightforward. In the glass transition, as the temperature decreases, the molecules in the liquid move more and more slowly. Without crystallization, the viscosity of the liquid increases rapidly by more than ten orders of magnitude. Eventually, molecular motion is frozen on the time scale of laboratory experiments, and the liquid is transformed into glass. The structural changes that occur during this cooling process are small and hard to detect. The glass seems to have the liquid-like disordered structure but holds solid-like mechanical properties. Understanding the complexity of glass formation remains one of the major intellectual challenges of solid-state physics[1-6]. The question concerning the nature of glass state and glass transition is so challenging that it was selected by the journal Science in 2005 as one of 125 key, unanswered scientific questions[7,8]. The central issue is whether the physics of glass formation can be understood on the basis of purely dynamical origin with no thermodynamic signature, or thermodynamic and structural explanations are required. Over several decades, due to the outstanding work of many scientists, various models and theories that describe the phenomena and mechanism of glass transition have continuously emerged. The Free-volume model, Configurational-entropy model, Mode-coupling theory, Random first-order transition theory are the more well-known examples among them. These models explain some contradictory data, but not all of them. Details of the various models and their relative merits and problems can be found in books and comprehensive review articles[9-17].

This work proposes a new two-stage model for silicate glass transition based on recent results on studies of the medium-range ordering structure of silica glass [18-20]. The newly gained knowledge about the formation and evolution of the medium-range ordering structure is the core of this two-stage model for silicate glass transition. The two-stage model recognizes that there are two different temperature regions in the glass transition process. The coherent structures in these two regions change along different paths, and all observed dynamic features of the glass transition can be explained according to the structural rearrangements in this transition process. This work will first describe the two-stage model, and then apply this model to silica glass and other silicate glass transitions.

## II. The Two-Stage Model

Figure 1 shows the entropy S as a function of temperature for a typical liquid at atmospheric pressure. As the liquid cools down from a high temperature, it may become glass or crystal. To form a crystal, the cooling rate must be low, and the entropy decreases along the path (I). Tm is the melting temperature of the crystal, and Tc is the polymorphic inversion temperature of the crystals. One material may have several polymorphic inversion temperatures. Tc is the one closest to Tm. If the cooling rate is high and crystallization is avoided, the entropy decreases along the path (II) or (II') to form glasses. Both paths (II) and (II') avoid crystallization but the cooling rate of the path (II') is lower than that of the path (II). The two-stage model defines the glass formation in two temperature regions separated by Tc. The region from Tm to Tc is the first or high-temperature region. The region from Tc down to Tg, which will be defined later, is the second or low-temperature region. Glass transitions during the cooling of liquid in these two regions have very different characters, as the term two-stage model implies and will be presented separately.

The first stage of glass transition occurs in the high-temperature region from Tm to Tc. In this

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temperature region, the liquidis not stable and is called supercooled liquid. Because of the Gibbs free energy difference between the two states, there is a driving force to transfer the liquid to the crystal state. In the following equation (1) of Gibbs free energy (G) at atmospheric pressure,

$$G = U - ST \quad (1)$$

there are two competing terms, internal energy (U) and entropy (ST). For the temperature T is lower than Tm, the entropy term ST is low and G is dominated by the internal energy term U. Because the free energy of a crystal is lower than that of a liquid, the super cooled liquid constantly reorganizes to form a crystal to minimize the free energy of the system. In the case of low cooling rate, crystal nuclei will form and distribute inside the super cooled liquid. These nuclei will then grow to large crystals. The transformation from liquid to high temperature polymorphic crystal is a well-defined first-order phase transition in the Paul Ehrenfest classification scheme because the first-order derivative of the Gibbs energy function, such as volume and entropy, exhibit discontinuity with respect to temperature.

Due to the high cooling rate of glass formation, crystal nucleation is avoided. Although no crystal nuclei form in the super cooled liquid, the embryonic clusters can still form inside. These clusters have slightly higher densities than the surrounding regions. Their sizes may be smaller or closer to that of nuclei. These clusters are randomly distributed in the super cooled liquid and intend to grow to nuclei and further to large-sized crystals. As the liquid cooled from Tm to Tc, the number of the embryonic clusters in the super cooled liquid continue to increase, reducing the entropy of the system. The clusters in super cooled liquids would not all dissolve if the temperature did not rise to a temperature higher than Tm. Thus, the super cooled liquid has a heterogeneous structure. The above view on the cluster formation before nucleation in super cooled liquid is not described in the classical nucleation theory [21], but is consistent with the more recent two-step nucleation theory and is verified by advanced experimental results [22-25].

To understand the glass transition, it is necessary to recognize the existence of embryonic clusters in the super cooled liquid. It is more important to find out how these clusters transform in the second stage of the cooling process from Tc to Tg. Tc is a special temperature for the corresponding crystals. At Tc all high temperature polymorphic crystals, regardless of sizes, start to convert to the low temperature polymorphic crystals. In glass transition, there are no crystals but only embryonic clusters in the super cooled liquid. These clusters in the super cooled liquid cannot simply switch their growing pathway from formation of high temperature polymorphic nuclei to that of low temperature polymorphic nuclei. The reason is that during cooling, the viscosity of the super cooled liquid has significantly increased at Tc; and the molecules in the super cooled liquid are difficult to move from one place to another. What the molecules can do to minimize the system's free energy is to form more ordered structures locally by breaking the bonds and changing the orientations of molecules, as what happens in the crystal polymorphic inversion.

In the second stage of the glass transition from Tc to Tg, the internal energy term and the entropy term in equation (1) continue to compete against each other. As the temperature decreases, the entropy term decreases, and more clusters are allowed to transform into more ordered structures. Assume N is the number of clusters with more ordered structures at a given temperature and M is the number of total clusters. At Tc the number N is equal to zero. As the temperature decreases from Tc, the number N increases, until all M clusters transform to a more ordered structure at Tg. The ratio of N to M is defined by

$$\zeta = N/M \quad (2)$$

In equation (2), $\zeta$ is the order parameter of glass transition. The fact that the symmetry starts to broke at Tc and the order parameter $\zeta$ changes from 0 at Tc to 1 at Tg indicates that the second stage of glass transition has the typical characteristics of a second-order phase transition [26].The ending temperature of the second-order phase transition Tg can be experimentally determined by the uprising point in Cp measurement from low to high temperature. All physical properties of glass in the temperature range from Tc to Tg should reflect the nature of the second-order phase transition. Because the second stage of transition immediately follows the first stage of transition, the total number of clusters M in the system depends on the cooling rate of the first stage. For slower cooling, the system has a longer time period to complete the first stage of transition and hence there are more embryonic clusters formed in the system. The system with more clusters needs a larger temperature range to complete the second-order phase transformation. Thus, temperature Tg is lower for the glass with a slower cooling rate, as shown in Fig. 1.The cooling rate dependence of Tg and other physical properties is a special feature of the glass transition, which is distinct from other second-order phase transitions, such as super ordering structure transformation in alloys and ferromagnetic phase transitions [27,28].

## III. Silica Glass Transition

Silica glass is an archetypical glass. Over the years, significant experimental data and observations have been acuminated for the silica glass transition process. Any viable model of the glass transition must

be able to explain these phenomena observed in the silica glass transition.

Applying the two-stage model for silica glass transition, Tc is the polymorphic phase transition temperature between ß-cristobalite and ß-tridymite, which is 1470°C, as shown in Fig. 2(a). The first stage of silica glass transition is the cooling process in the temperature range from Tm, 1723°C, to Tc, 1470°C. Although the super cooled liquid avoids ß-cristobalite crystallization, embryonic clusters are formed inside. Recent studies on the medium-range structure of silica glass indicate that embryonic clusters in the super cooled liquid are 1-2 nm in size and have an approximately octahedral shape [19]. Because of the formation of clusters, the configuration entropy of the system significantly declines at 1470°C, as illustrated in Fig. 2(a). Furthermore, due to increase in number and average size of the clusters, the internal friction of super cooled silica liquid continuously increases and causes the activation energy of the liquid to increase. This is the reason that viscosity-temperature behavior of super cooled liquid silica changes at Tm from Arrhenius type in temperatures above Tm to super-Arrhenius type in the temperature range from Tm to Tc, as shown in Fig. 2(b).

At Tc, 1470°C, the first stage transition ends and the second stage starts. In the second stage of glass transition, the continuous decrease in the configuration entropy of the system is due to the formation of the one-dimensional ordering structure on the clusters' facets, called nanoflakes for convenience [18]. The reduction in configuration entropy resulting from the formation of nanoflakes is much slower compared to the direct formation of clusters. Thus, the entropy vs. temperature curve in Fig. 2(a) bends at Tc, (1470°C), the curve would not cross over the crystal entropy curve, and the so called "Kauzmann catastrophe" would not happen [19,31].

Also, in the second stage of glass transition in the temperature region below Tc, the population of the ordered clusters increases, and the order parameter $\zeta$ changes from 0 at Tc to 1 at Tg. Published experimental data on specific heat Cp of silica glass as a function of temperature has a hump at around 1200°C [29]. Thus, Tg of silica glass is located at around 1200°C. However, Tg is influenced by the cooling rate: the lower the cooling rate, the lower the temperature Tg. Hence, it is possible to find silica glass with Tg lower than 1200°C from different specimens [19, 30]. The cooling rate dependence of Tg does not change the nature of the second-order phase transition of glass in the low temperature range. This is because the critical temperature Tc for silica glass is always 1470°C, regardless of cooling rate.

The formation of nanoflakes affects various physical properties of glass, such as optical transmission, X-rays cattering, etc. However, it does not influence the activation energy of the liquid flow. This is because the size and shape of the clusters are not affected by the ordering on the facets, and the energy barriers in the liquid flow stay the same. Thus, after the activation energy reaches the maximum value at Tc, it stays unchanged in the whole low temperature region, and the behavior of viscosity of silica reverts back to Arrhenius type, as shown in Fig. 2(b) [32-34].

## IV. Sodium Silicate Glass Transitions

The two-stage model is not limited to pure silica glass. It can also describe binary silicate glass transition, provided that the liquidus temperature Tm and critical temperature Tc of the silicate glasses are known. For most binary silicate glasses, Tm and Tc can be identified from the corresponding phase diagram. For sodium silicate glasses with various contents of sodium, Tm and Tc can be determined from the $Na_2O$-$SiO_2$ phase diagram, as shown in Fig. 3(a) [35].This phase diagram indicates that for sodium silicate glasses with 0 - 11.3 weight% $Na_2O$ concentration, Tm is in the range from 1713°C to 1470°C and Tc is at 1470°C; for glasses with 11.3 – 24.5 weight% $Na_2O$ concentration Tm is in the range from 1470°C to 870°C and Tc is at 870°C. Taking a glass with 20% $Na_2O$ as an example, its position on the phase diagram is at the arrow-pointed B in Fig. 3(a). From the position of B, two characteristic temperatures Tm at 1140°C and Tc at 870°C can be found in the figure. For this sodium silica glass transition, Tc is the polymorphic phase transition temperature between ß-tridymite and ß-quartz, which is 870°C.

The first stage of the glass transition is the cooling process in the temperature range from 1140°C to 870°C. Although the super cooled liquid avoids the ß-tridymite crystallization, embryonic clusters are formed inside and cause a significant decrease of the system's entropy. At Tc, 870°C, the first stage of transition ends and the second stage starts. The second stage of transition is a continuous disorder-order transition. The decreasing rate of entropy in the second stage is lower than that in the first stage. The second stage ends at Tg, which can be found from the specific heat Cp data of the glass as a function of temperature. The heat capacity Cp data of sodium silicate glass with 15 mol.% of $Na_2O$ as a function of temperature are available from references [36,37]. The sharp rising of Cp between 480°C and 560°C indicates that heat absorption increases sharply in this temperature range, and Tg of the glass is hence found at around 500°C. Since the heat absorption properties of sodium silicate glasses with 15 and 20 mol. % of $Na_2O$ are similar, the Tg of the sodium silicate glass with 20 mol. % $Na_2O$ is also estimated at around 500°C. Thus, similar to silica glass, the entropy-temperature relation in the sodium silicate (20% $Na_2O$) glass transition can also be illustrated by Fig. 1, in which polymorph 1 and 2 are ß-tridymite and ß-

quartz, respectively, and the value of Tm is 1140°C, Tc is 870°C, and Tg is around 500°C. Note that in the $Na_2O$-$SiO_2$ system, the compositions expressed in wt. % and in mol. % differ very little.

In addition to entropy, the viscosity-temperature behavior of the sodium silicate glass is also similar to that of silica glass, as described by the two-stage model. Experimental viscosity data of the sodium silicate glass are readily available. As shown in Fig. 3(b), the logarithm of viscosity of this glass is plotted as a function of the reciprocal of temperature [32]. The results are similar to that of silica glass as presented in Fig. 2(b): there are two straight lines showing the Arrhenius types of this sodium silicate viscosities, in the temperature ranges higher than Tm and lower than Tc. In the temperature range from Tm to Tc, the behavior is a super-Arrhenius type.

## V. Sodium Borosilicate Glass Transition

The two-stage model can also describe a more complicated sodium borosilicate glass transition. The ternary glasses are broadly used for cookware, chemical laboratory ware, TV screens, etc., due to their thermal shock resistance and excellent chemical durability. Substantial physical property data of these glasses have been accumulated. These data can be explained from the internal structures of these glasses by the two-stage model. Here the glass with 71.56% $SiO_2$, 8.28% $B_2O_3$ and 18.76% $Na_2O$ is used as an example. This particular sodium borosilicate glass was defined as 602 glass by S. English in 1924 [38]. Using experimental viscosity data measured by English, the logarithm of the viscosity of the glass as a function of reciprocal temperature is shown in Fig. 4(a). To understand the viscosity-temperature behavior of the glass, two characteristic temperatures Tm and Tc are needed. These can be found from the phase diagram of the ternary system $Na_2O$-$B_2O_3$-$SiO_2$, as shown in Fig. 4(b) [39]. In Fig. 4(b), point A represents 602 glass defined by English [38]. The liquidus temperature of glass, represented by point A, is about 1100°C. Because the corresponding crystal of the glass is tridymite, Tc is the polymorphic phase transition temperature between ß-tridymite and ß-quartz, which is 870°C. It is found that the viscosity-temperature behavior of the sodium borosilicate glass is similar to that of silica glass described by the two-stage model. In Fig. 4(a), there are two straight lines showing the Arrhenius type of this sodium borosilicate viscosity, in both the temperature range higher than Tm (1100°C) and lower than Tc (870°C). In the temperature range from Tm to Tc, the behavior is a super-Arrhenius type.

The temperature Tg of this sodium borosilicate glass can be found from the existing experimental data. Nearly a hundred years ago, A. A. Lebedev already reported that sodium borosilicate glasses exhibited characteristic heat absorption between 555°C and 610°C [40]. A recent study indicated that Tg of this sodium borosilicate glass should be close to 570°C [37].Therefore, similar to silica and sodium (20%) silicate glasses, Fig. 1 can also be applied to describe entropy changes in the transition of sodium borosilicate glass (71.56% $SiO_2$, 8.28% $B_2O_3$ and 18.76% $Na_2O$) with polymorph 1 and 2 being ß-tridymite and ß-quartz, respectively, and with the value of Tm at 1100°C, Tc at 870°C, and Tg at around 570°C.

## VI. Concluding Remarks

The two-stage model is based on the studies of formation and evolution of the medium-range order structure in silicate glasses. It is recognized that there are two stages in the glass transition process, separated by the critical temperature Tc, which is the polymorphic inversion temperature of the corresponding crystals. The first transition stage is in the high-temperature range from the melting temperature Tm to the critical temperature Tc. In the first stage, nano-clusters form. The number of clusters increases with decreasing temperature and reaches a maximum value at Tc. The second transition stage is in the temperature range from Tc to Tg, in which the clusters transform into more ordered structures. Temperature Tg, the ending temperature of the second stage, is cooling rate dependent and can be determined experimentally. According to the two-stage model, the silicate glass transition can be recognized as a second-order phase transition following an incomplete first-order phase transition. The model correctly predicts the physical properties, such as entropy and viscosity as a function of temperature in a wide temperature range. The application of the new model is valid for silica glass, as well as for complex silicate glasses. However, its validity to other types of glasses, such as metallic glasses and organic glasses, etc., needs to be further investigated.

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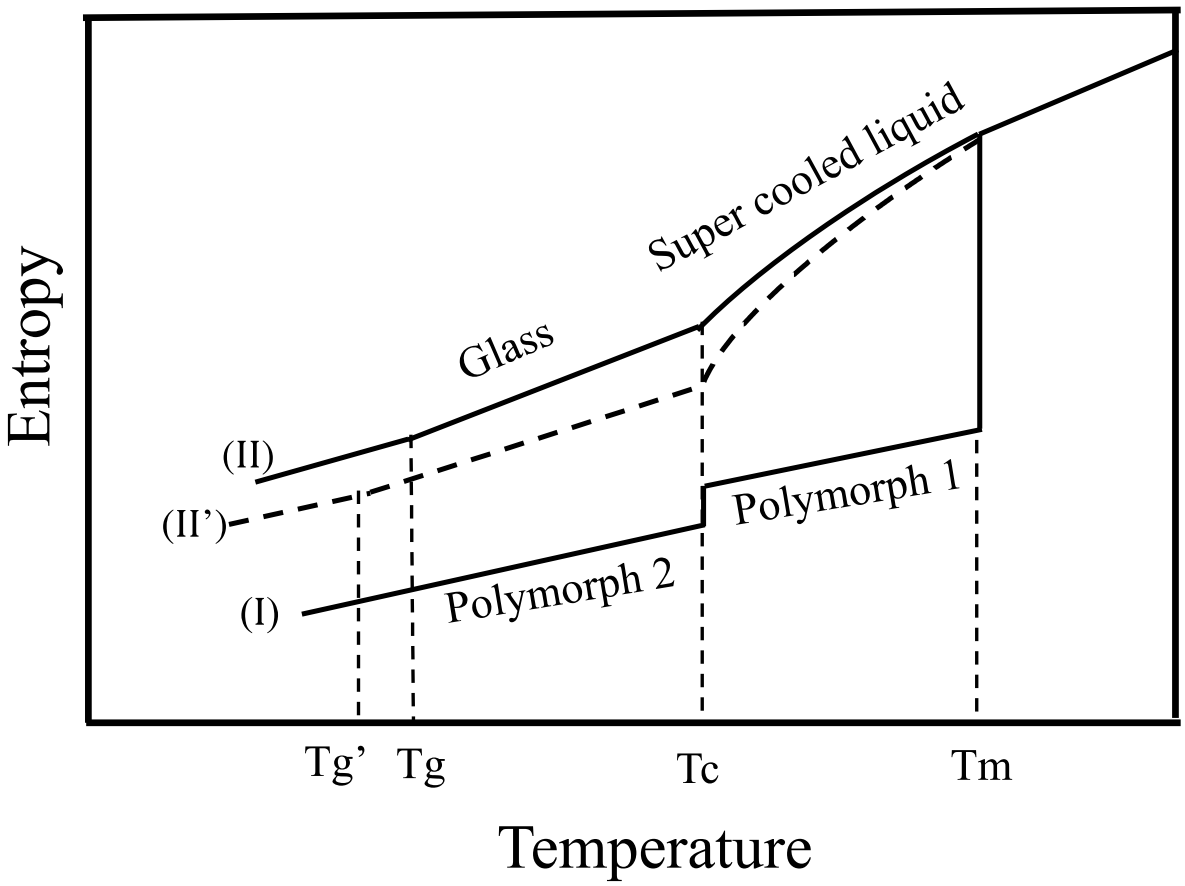
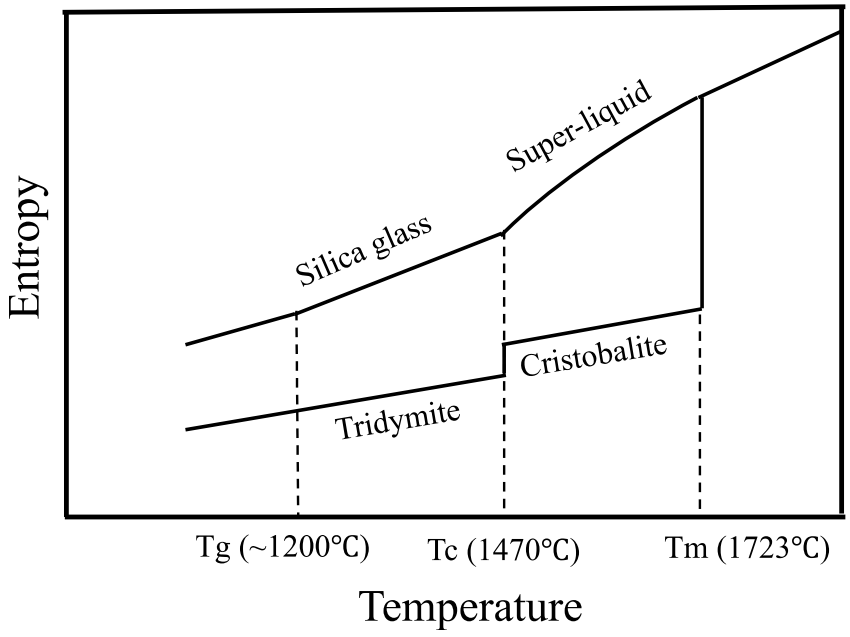
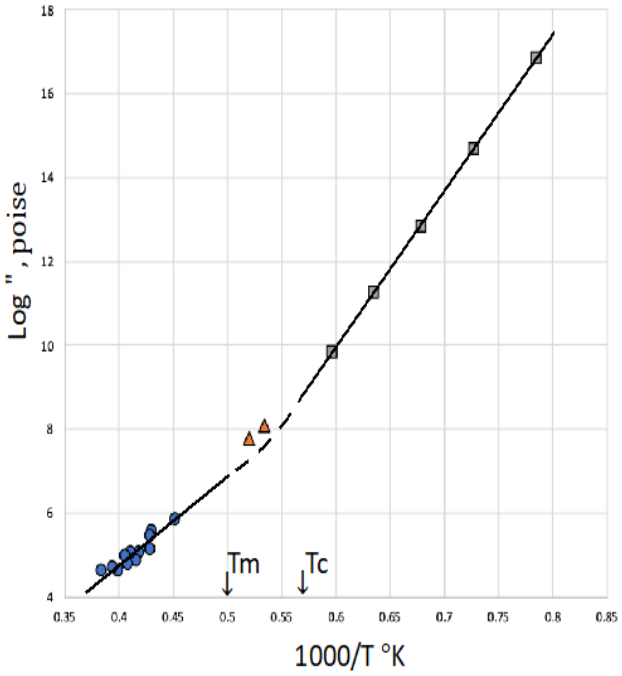



*Fig. 1:* The entropy-temperature relation of a typical silicate glass transition. As the liquid cools down from high temperature, it may become glass or crystal. The entropy of crystal decreases along the path (I). Tm is the melting temperature of the crystal and Tc is the polymorphic inversion temperature of the crystals. The entropy of glasses decreases along path (II) or (II'). The cooling rate of path (II') is lower than path (II). The two-stage model defines the glass formation for two temperature regions separated by Tc. From Tm to Tc is the first or high temperature region. From Tc down to Tg is the second or low temperature region.

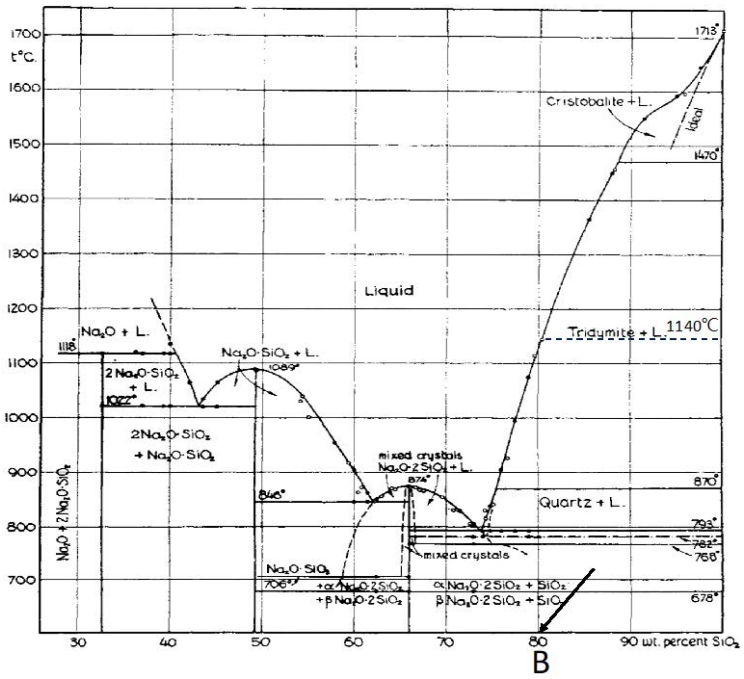



(a)

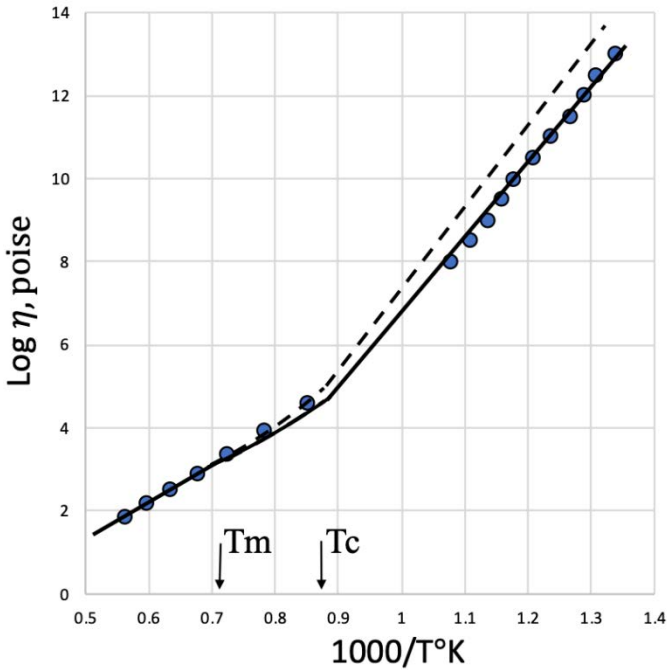



(b)

*Fig. 2:* Two-stage model for silica glass transition. (a) The entropy-temperature relation of the silica glass transition. (b) The logarithm of silica viscosity as a function of reciprocal temperature. Reproduced from ref. 32.

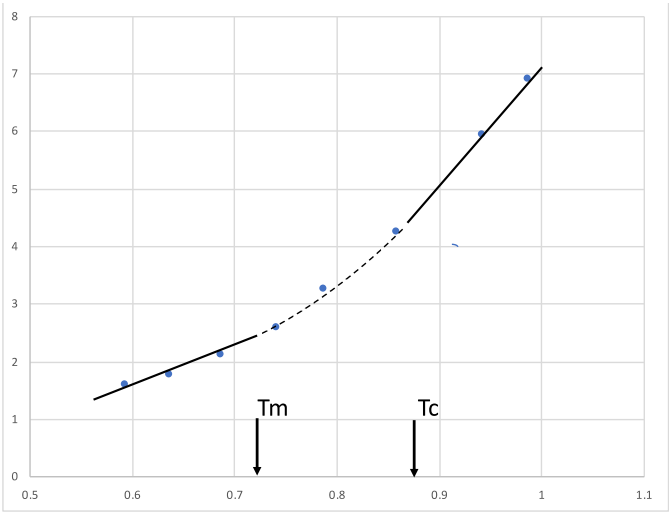



(a)

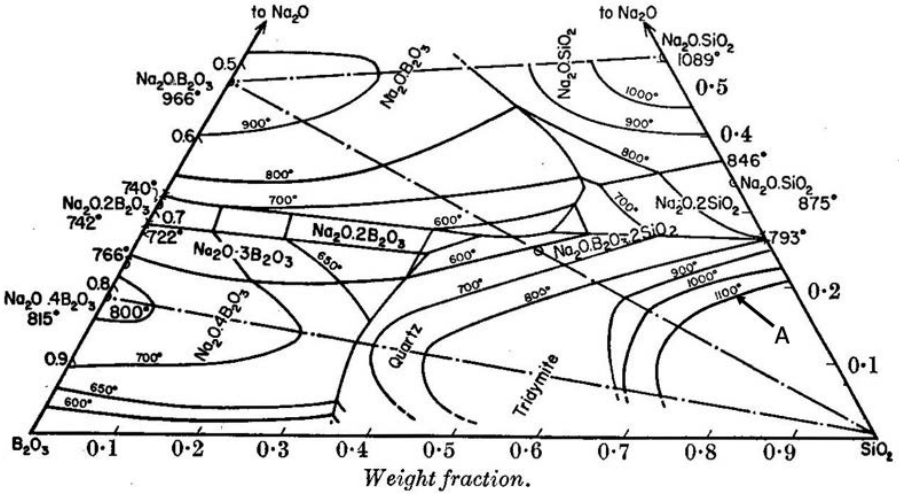



(b)

*Fig. 3:* (a) The $Na_2O$-$SiO_2$ phase diagram. Reproduced from ref. 35. (b) The logarithm viscosity of sodium silicate with 20 mol% $Na_2O$ as a function of reciprocal temperature. The solid line represents the existed experimental data. The dash line represents assumed experimental data with lower cooling rate. Reproduced from ref. 32.




(a)




(b)

*Fig. 4:* (a) The logarithm viscosity of 602 glass as a function of reciprocal temperature. (b) The phase diagram of the ternary system $Na_2O$-$B_2O_3$-$SiO_2$. Point A represents the 602 glass with 71.56% $SiO_2$, 8.28% $B_2O_3$ and 18.76% $Na_2O$. Reproduced from ref. 39.

Global Journal of Science Frontier Research ( A ) Volume XXII Issue VII Version I Year 2022
10

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# Improving our Understanding of the Klein-Gordon Equation

By P. J. Bussey

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*Abstract-* A detailed consideration of the Klein-Gordon equation in relativistic quantum mechanics is presented in order to offer more clarity than many standard approaches. The equation is frequently employed in the research literature, even though problems have often been raised regarding its second-order nature, the status of its negative-energy solutions and the formulation of particle density and flux. Most of these problems can be avoided by dismissing the negative energy solutions. An application of the equation to a broad wave-packet shows that a small amendment to the usual relativistic formalism can be helpful to demonstrate continuity with the non-relativistic case, although difficulties remain when the proposed quantum state has a broad relativistic energy distribution.

*Keywords:* *quantum mechanics, Klein-Gordon equation, Schrödinger equation, relativistic.*

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*Strictly as per the compliance and regulations of:*

# Improving our Understanding of the Klein-Gordon Equation

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*Abstract-* A detailed consideration of the Klein-Gordon equation in relativistic quantum mechanics is presented in order to offer more clarity than many standard approaches. The equation is frequently employed in the research literature, even though problems have often been raised regarding its second-order nature, the status of its negative-energy solutions and the formulation of particle density and flux. Most of these problems can be avoided by dismissing the negative energy solutions. An application of the equation to a broad wave-packet shows that a small amendment to the usual relativistic formalism can be helpful to demonstrate continuity with the non-relativistic case, although difficulties remain when the proposed quantum state has a broad relativistic energy distribution.

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## I. Introduction

The simplest approach to a relativistic description of a quantum particle was proposed in 1926 by Klein and by Gordon [1], among others [2]; a historical survey has been given by Kragh [3]. The familiar relativistic equation $E^2 = p^2c^2 + (mc^2)^2$ is quantised by replacing the classical observables $E$ and $p^2$ by quantum observables $i\hbar\partial/\partial t$ and $-\hbar^2[(\partial/\partial x)^2 + (\partial/\partial y)^2 + (\partial/\partial z)^2]$. Thus, the Klein-Gordon (KG) equation for the wave function $\psi$ is

$$-\hbar^2\frac{\partial^2\psi}{\partial t^2} = \left(-\hbar^2c^2\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right] + m^2c^4\right)\psi. \tag{1}$$

Since the energy eigenstate of a free quantum particle is represented by a plane wave of the form $\exp i(\mathbf{k}.\mathbf{x} - \omega t)$, with $\omega$ and $\mathbf{k}$ proportional to energy and momentum, the KG equation must hold unavoidably for such states of a spinless relativistic quantum particle. Being linear in $\psi$, it remains valid for linear combinations of these plane-wave eigenstates, just as does the similarly linear (non-relativistic) Schrödinger equation. Although it was first proposed for massive scalar particles, it should apply to other massive particles that are in a definite spin state.

Soon after the equation's proposal, objections were raised that were not fully answered, resulting in uncertainty that is still present about the validity of relativistic quantum mechanics, and of the KG equation in particular. These objections are repeated without resolution in most textbooks on the subject. However, they are ignored in much of the research literature and there has been no shortage of publications, far too many to refer to here, in which the KG equation is accepted, modified, applied in a large variety of contexts, or employed in various interpretations of quantum theory. Particular examples would be the evaluation of CP violation properties of meson states and entanglement in particle systems. The KG equation is certainly in use, and the situation can be puzzling to those encountering the subject.

The purpose of the present paper is to examine to what extent the claimed difficulties with the equation can be overcome, with the aim of clarifying how the equation should be better understood and where it can be confidently applied. We will concentrate on the more basic quantum mechanical issues and, in particular, we will not discuss field theory at any length.

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Global Journal of Science Frontier Research ( A ) Volume XXII Issue VII Version I Year 2022
11

## II. Traditional Difficulties with the Klein-Gordon Equation

*(i) The use of a second-order differential equation, and the negative-energy problem*

A problem raised early in the KG equation's history was that a second-order differential equation, such as (1), does not normally allow the future values of its argument to be predicted from a given set of starting values alone. Thus the wave-function $\boldsymbol{\psi}$ at an initial reference time would apparently be insufficient to allow the KG equation to determine the particle's later state, and this was seen as unsatisfactory. This criticism was used to cast doubt on the KG equation and to support a need for a first-order equation, such as that of Dirac, but it can be countered. If $\boldsymbol{\psi}(\mathbf{x})$ is Fourier analysed at an initial time in terms of eigenstates of momentum $\mathbf{k}$, then its future behaviour is well determined from this information alone, using the KG equation for each component, provided that there is no ambiguity in specifying $\omega$ for a given $\mathbf{k}$. This requirement becomes the key to understanding many of the issues raised regarding the KG equation, which formally allows solutions with both positive and negative values of $\omega$.

A contrast can be made with a scalar classical wave, for example a sound wave. If, at a given time, the amplitude of such a wave is specified as a function of position, this does not differentiate between components travelling in opposite directions, and for a wave of this type a given $\boldsymbol{k}$ can have either sign of $\omega$. For the KG equation, however, this mathematical possibility is to be discounted on the grounds of unphysicality. A relativistic massive particle always has positive energy, which means that for a given $\mathbf{k}$, only a positive value of $\omega$ can be assigned. The negative-energy solutions must be rejected, and by doing this we can ensure that the KG wave equation determines the particle's future state unambiguously from its original state, as desired.

Dirac's first-order differential equation for spin-half fermions did not remove the mathematical existence of negative-energy states, but placed them in a new part of the spinor. As is well known, his interpretation of these states was as a "sea" of mostly filled states in which unoccupied states or "holes" behave as antiparticles, whose energy is positive compared to that of an *occupied* negative-energy state. This viewpoint led to the successful expectation that the positron should exist, but it subsequently fell out of favour although it was still supported by Pauli in 1955 [4]. However it will not work at all for bosons, because the entire negative-energy sea could acquire unlimited numbers of bosonic particles, and the vacuum would be unstable. Thus the negative-energy solutions remain problematic.

*(ii) The question of "incompleteness"*

The apparent incompleteness of a set of solutions that lacks negative-energy contributions has been criticised [5]. But it is clear that the Fourier components in terms of $E$ or $\omega$ cannot in any case comprise a formally complete set for a massive real particle, because $|E|$ cannot be less than $mc^2$; the energy spectrum is obliged to have a cutoff at this value. The set of states in $\omega$ is in this sense incomplete, but we may instead rely on the states specified by $\mathbf{k}$, which constitute a complete set of physical states of a positive-mass particle. This seems perfectly satisfactory.

To take a classical example, there is no problem in insisting that Pythagoras' equation shall give only positive solutions for the hypotenuse of a triangle, even though the square root of $x^2+y^2$ can mathematically take negative values. We state that these solutions are geometrically invalid and ignore them. An analogous approach can be applied to the KG equation. Indeed,

it does not seem to disturb us that Einstein's original equation $E^2 = p^2c^2 + (mc^2)^2$ bears negative-energy solutions; these are likewise ignored. It might seem anomalous to retain them in the quantum context.

*(iii) Feynman's solution*

Feynman's well-known response was to say that negative-energy solutions to (1) (and also to the Dirac equation) correspond to antiparticle states with positive energy, but with the sign of $t$ reversed [6]. The antiparticle states, he supposed, are to be included within the same set of solutions as the particle states. A problem here, however, is that two different treatments of time are then implied within the same equation and its solutions. While this interpretation is generally accepted in connection with Feynman diagrams, it is therefore very untidy with regard to free particles and the KG equation, and it is unclear what to do when a particle is its own antiparticle, as for example the $\pi^0$ meson – only one set of states is wanted in this case.

A much clearer and more transparent approach to the KG equation is to treat particles and their antiparticles as solutions to separate but identical equations, with negative-energy solutions for free particles always disallowed. When appropriate, quantum superpositions of particle and antiparticle states may then be constructed for a given system, as is common practice with neutral meson systems. On the other hand, a superposition of positive-charge and negative-charge states is never observed. The issue of combining particle and antiparticle states thus needs to be settled on its own terms in a given physical situation and is not a simple implication of the relativistic quantum formalism.

Feynman's proposal has been applied principally in connection with Feynman diagrams in field theory, in which propagators – that is, virtual particles to which the KG equation does not apply – are employed to indicate the transfer of particle characteristics between different vertices in a scattering process: "streams of influence", we may say. Amongst other things, propagators denote energy flow, which can be positive or negative, since positive energy flowing out of a vertex is the same as negative energy flowing into it. Feynman's picture is therefore in essence dynamical, whereas quantum mechanics (starting from de Broglie's equations) has a foundation in free real-particle states. But an understanding of these is necessary as a basis for Feynman's theory.

As pointed out above, the basic objection to the use of Feynman's idea with the KE equation is that it requires one symbol $t$ to denote two different signs of physical time simultaneously in the same differential equation, and within its one set of solutions. This connotational ambiguity cannot reasonably be accepted.

*(iv) The probability density question*

A third and more serious criticism of the KG equation concerns the particle's spatial probabiity density $\rho$ and current $\boldsymbol{j}$, as discussed in many textbooks; that by Desai [7] has been referenced here. For both the non-relativistic and relativistic cases these quantities should be related by the continuity equation

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{\nabla} \cdot \boldsymbol{j}. \tag{2}$$

For a non-relativistic particle, $\rho = \boldsymbol{\psi}^*\boldsymbol{\psi}$, while the current is given by

$$\boldsymbol{j} = \frac{\hbar}{2im}\left[\psi^*(\boldsymbol{\nabla}\psi) - \psi(\boldsymbol{\nabla}\psi^*)\right]. \tag{3}$$

The same equation for $\boldsymbol{j}$ is conventionally taken also in the relativistic case, where $m$ is again the rest mass, but the probability density is now written as

$$\rho = \frac{-\hbar}{2imc^2}\left[\psi^*\frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\right]. \tag{4}$$

This equation is counterintuitive, however, although it satisfies (2), and it generates suspicions about the KG equation. For a plane wave, it gives $\rho = (E/mc^2)\psi^*\psi$, which is negative for negative $E$. Once more, negative energy values are deeply problematic, since a probability density cannot be negative. An alternative suggestion from Pauli and Weisskopf was that we are no longer really discussing particle density but charge density, which can take negative values [8]. This proposal has been taken up by other authors [9], and it evades some of the difficulties, but it does not really address the original task of evaluating particle density and current; the KG equation says nothing about electric charge. Neutral particles remain problematic,[1] and an unsatisfactory discontinuity is introduced between the relativistic and non-relativistic interpretations; after all, a relativistic particle may be just a non-relativistic particle viewed in a different reference frame. In the end, this suggestion would seem to introduce more problems than it solves.

## III. The Case of a Broad Wave-Packet

It is instructive to examine the situation in more detail by considering a broad wave-packet moving in one dimension $x$ with mean positive energy $E = \hbar\omega$ and group velocity $v_g$.[2] To a good approximation its wave function can be represented as

$$\psi = Af(x - \bar{x})e^{i(kx-\omega t)}, \tag{5}$$

where $A$ is a normalisation constant. The function $f$ describes the envelope in $x$ of the wave-packet, whose mean value is $\bar{x} = x_0 + v_g t$ and is $x_0$ at time $t = 0$. The partial differentials of $f$ with respect to $x$ and $t$ are thus $f'$ and $-f'v_g$ respectively. To avoid the introduction of further momentum components, $f$ is taken to be real. For example, a Gaussian wave-packet with half-width $\sigma$ has

$$\psi = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^{\frac{1}{2}} e^{-(x-\bar{x})^2/4\sigma^2} e^{i(kx-\omega t)}. \tag{6}$$

As usual, $v_g$ equals $\partial\omega/\partial k$ and is given by $\hbar k/m = p/m$ for the Schrödinger equation and $c^2k/\omega = c^2p/E$ for the KG equation. This equals $p/m_{\text{rel}}$, where $m_{\text{rel}} = \gamma m$ is the relativistic mass and $E = m_{\text{rel}}c^2$. (As expected, the relativistic group velocity has an upper bound of $c$.)

[1] There have been attempts to solve this issue. Greiner [10], following Bjorken and Drell [11] and Feshbach and Villars [12], gives an argument whose conclusion is that a particle such as a $\pi^0$ must have a wave-function that is a real mathematical function. But this would have to be a real sinusoidal wave, which cannot be correct, since this is not an eigenstate of the momentum operator $-i\hbar\boldsymbol{\nabla}$ !

We consider a sufficiently broad wave-packet such that the values of energy and momentum have narrow widths and the group velocity is well defined. Broadening effects with time are neglected at this level of description.

For a packet given by (5), the non-relativistic case gives

$$\frac{\partial \rho}{\partial t} = \boldsymbol{\psi}^* \frac{\partial \psi}{\partial t} + \boldsymbol{\psi} \frac{\partial \psi^*}{\partial t} = -2v_g |A|^2 f f' \tag{7}$$

and

$$j = \frac{\hbar}{2im} \left[ \boldsymbol{\psi}^* \frac{\partial \psi}{\partial x} - \boldsymbol{\psi} \frac{\partial \psi^*}{\partial x} \right] = \frac{\hbar k}{m} |A|^2 f^2. \tag{8}$$

As intuitively expected, $j$ is $v_g$ times the local value of $\boldsymbol{\psi}^* \boldsymbol{\psi}$. Its gradient in $x$ is $j' = 2v_g |A|^2 f f'$, in required agreement with (7) and (2).

In the relativistic case, we start from the representation for $\rho$ given by (4). The wave-packet (5) now gives

$$\rho = \frac{\hbar}{mc^2} A^* A f^2 \omega = \frac{E}{mc^2} \boldsymbol{\psi}^* \boldsymbol{\psi} = \gamma \psi^* \psi, \tag{9}$$

since the terms in $f f'$ cancel; $\gamma$ is the usual relativistic factor for the particle and $\rho$ is clearly positive. We obtain

$$\frac{\partial \rho}{\partial t} = -2 \frac{\hbar \omega}{mc^2} |A|^2 f f' v_g = -2\gamma v_g |A|^2 f f'. \tag{10}$$

Taking the expressions for $j$ unchanged from (3) and (8), we obtain the required result

$$j' = 2 \frac{\hbar k}{m} |A|^2 f f' = 2\gamma v_g |A|^2 f f' = -\partial \rho / \partial t. \tag{11}$$

The KG equation thus gives a well-defined account of a relativistic particle whose wave function has the form of a broad wave-packet, but the conventionally adopted expressions for probability density and current gradient now give results that contain a factor of $\gamma$. Since (4) and (3) are purely conventional, and the probability density must be normalised to unity anyway, it would be convenient to redefine them to remove this factor. In this way, equations (4) and (3) may be written as

$$\rho_{\text{rel}} \;=\; \gamma^{-1} \frac{-\hbar}{2imc^2} \left[ \boldsymbol{\psi}^* \frac{\partial \psi}{\partial t} - \boldsymbol{\psi} \frac{\partial \psi^*}{\partial t} \right], \text{ which reduces to } \boldsymbol{\psi}^* \boldsymbol{\psi}, \tag{12}$$

[2]This topic has been treated by Mosley [13] with a different mathematical perspective from that offered here.

and
$$\boldsymbol{j}_{\rm rel} \;=\; \gamma^{-1}\frac{\hbar}{2im}\left[\psi^*(\boldsymbol{\nabla}\psi)-\psi(\boldsymbol{\nabla}\psi^*)\right]. \tag{13}$$

These amended equations are valid provided that the wave packet has an acceptably well-defined value of $\gamma$. They represent the probability density and current for a relativistic wave-packet more naturally than (4) and (3), since the quantity $\psi^*\psi$ is non-negative and may now be given its usual interpretation, while (2) continues to hold. The rest mass $m$ in (4) and (3) is to be replaced by the relativistic mass $m_{\rm rel} = m\gamma$, which clearly expresses the continuity between the non-relativistic and relativistic accounts.

## IV. A More General Case

The example of the previous section does not give a complete answer to the probability density problem. To investigate further, we examine the case of a wave function that is the sum of components with different energy. Let

$$\psi = \sum_j \psi_j, \text{ where } \psi_j = a_j e^{i(\mathbf{k_j}.\mathbf{x}-\omega_j t)}. \tag{14}$$

Thus $\partial\psi_j/\partial t = -i\omega_j\psi_j$. The terms in the sum may have arbitrary positive energies $\hbar\omega_j$ and arbitrary amplitudes $a_j$, subject to unitarity. Then (4) gives

$$\begin{aligned}\rho \;&=\; \frac{\hbar}{2m}\sum_j \omega_j \sum_k (\psi_k^*\psi_j + \psi_k\psi_j^*)\\ &=\; \frac{\hbar}{m}\sum_j \omega_j\,(|a_j|^2 + \sum_{k\neq j}|a_j a_k|\cos\phi_{jk}),\end{aligned} \tag{15}$$

where $\phi_{jk}$ is the phase difference between $\psi_j$ and $\psi_k$ at a given spacetime point $(\mathbf{x}, t)$. Even with the $\omega_j$ positive, $\rho$ is not necessarily positive in this situation. This is easily seen by considering the case of just two terms, which gives

$$\rho = \frac{\hbar}{m}\left[\omega_1\left(|a_1|^2 + |a_1a_2|\cos\phi_{1\,2}\right) + \omega_2\left(|a_2|^2 + |a_1a_2|\cos\phi_{1\,2}\right)\right]. \tag{16}$$

If the two amplitudes are equal in magnitude, then both terms are non-negative. Otherwise, the term with smaller amplitude can become negative and does not have to be cancelled everywhere by the other term if $\omega_1 \neq \omega_2$. Thus $\rho$ can become negative, and this may be expected to be true in general for more than two components and for the case of a broad continuous distribution in $\omega$.

## V. Discussion

We have identified the most major source of difficulty with the KG equation as associated with solutions with negative energy; once these are dismissed, many of the claimed problems disappear. In practice, many applications of the theory ignore such states implicitly, but it is interesting to observe just how troublesome they turn out to be.[3]

The question of apparently negative probability densities is more complex. Most practical applications (including field treatments) concern particles in plane-wave states, although a more realistic approach requires a wave-packet model. Within the KG equation, such states do not give a probability problem. (A plane wave has $f = 1$ in eq. (5).) The state should have a well-defined central positive energy and a narrow energy width. A non-relativistic particle wave function is also well described in this way when viewed in a boosted reference frame, because its spread in $\gamma$ remains small. This does not imply that the Schrödinger equation and its dynamics can be similarly transformed. A proposal for a more relativistic adaptation of the Schrödinger equation has been given by Grave de Peralta et al. [15].

Baym, unusually among textbook authors, says that the KG equation is "quite useful", while repeating the problems that we have already discussed [18]. He shows that particle wave-packets that are narrowly confined in space are problematic. This is no doubt true, and such particles also have a broad energy spectrum. However the states discussed in section 4.4 do not necessarily have narrow spatial wave-packets, and so the difficulty appears to be more general than Baym indicates.

A particle state with a substantial spread of relativistic energy possesses no proper rest frame and thus no non-relativistic counterpart. In the absence of well defined energy, the redefinitions (12) and (13) cannot be applied. In this case (4) can give probability densities that are not always positive, although the case of two contributing amplitudes of equal magnitude remains well behaved. Here, therefore, there apppear to be unresolved issues regarding the interpretation of relativistic quantum theory; the main problem is found with the probability density and current rather than with the KG equation itself. A similar problem might also arise with regard to a single-particle excited state of a quantum field, if it had poorly defined relativistic energy. Such states are not commonly discussed, and this is a topic that invites further examination.

One simple suggestion would be that while the probability density seems clearly measurable, corresponding to the position of a particle, its current may not always be a valid measurable variable. Its interpretation, we have seen, is plausible in the case of a broad wave packet. In general, however, it does not correspond to a Hermitian quantum variable and so is not a property of a particle that is measurable in the normal quantum scheme. Perhaps, then, it should not be considered as fundamentally important.

A limitation to any first-quantised relativistic theory is that dynamical problems can be treated only approximately. It is perfectly possible to solve the KG equation with a potential energy term included, for example to evaluate a pionic atom [16, 17]. Here however, as with the Dirac equation, a major issue is well known, namely that the use of a simple potential does not give precise results in a relativistic context. Field theories are set up to avoid negative energies of real particles, and in the end provide a more comprehensive physical account, but we may still wish to explore the limits of the more basic quantum method.

[3]Removing such states is also a feature of the Foldy-Wouthuysen treatment of the Dirac equation [10]. Some aspects of the present discussion can also be applied to the Dirac equation, but we do not pursue this in detail here. Another approach, with doubtful success, is to modify the KG equation so as to produce only positive-energy solutions [14].

## VI. Conclusions

Special relativity relies in an essential way on the use of well-defined frames of reference. A quantum particle with a broad energy spread lacks a proper frame of reference, and so it may be no surprise if such states present interpretational difficulties. However if a free particle is in a state with positive energy that is sufficiently well-defined to provide a usable proper reference frame, the Klein-Gordon equation gives an acceptable account of its basic quantum features, and the most frequent criticisms are overcome. With a minor amendment to the standard notation, the relativistic mass can now be used in the probability current equation, giving a natural continuity between the non-relativistic and relativistic treatments. Despite the objections of many distinguished practitioners of the subject, the negative-energy solutions to the equation are not physically usable in describing single particles; real antiparticles should be treated separately and in a parallel way to the particles. For many neutral meson systems, the two classes of state can then be combined in an extended quantum formulation.

In the end, a relativistic equation of the Klein-Gordon type cannot be avoided for the description of free spinless quantum particles and those with fixed-spin states. However, its application may be restricted to states in which the particle has a proper relativistic rest frame. With due allowance for this constraint, the Klein-Gordon equation is able to retain an important role in quantum mechanics. Its place is assured, provided that limitations such as discussed above are kept in mind.

The author has no conflicts of interest.

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# Noncommutative Quantum Gravity and Symmetry of Klein-Gordon Equation

By Gang Lee

*Abstract-* In the paper 'A New Approach to Quantum Gravity'[1], we suggest a new approach to quantum gravity. Using this theory, we can study the noncommutative gravitational field in momentum space. In this paper, we obtain the general form of the Klein-Gordon equation in noncommutative gravitational field. Then we find the symmetry associated with noncommutative gravity from the Klein-Gordon equation. We study black hole in momentum space and conclude that the event horizon of black holes is formed by the dipoles in momentum space with limit state.

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# Noncommutative Quantum Gravity and Symmetry of Klein-Gordon Equation

Gang Lee

*Abstract-* In the paper 'A New Approach to Quantum Gravity'[1], we suggest a new approach to quantum gravity. Using this theory, we can study the noncommutative gravitational field in momentum space. In this paper, we obtain the general form of the Klein-Gordon equation in noncommutative gravitational field. Then we find the symmetry associated with noncommutative gravity from the Klein-Gordon equation. We study black hole in momentum space and conclude that the event horizon of black holes is formed by the dipoles in momentum space with limit state.

## I. Introduction

In the paper 'A New Approach to Quantum Gravity'[1], we suggest a new theory of quantum gravity, give the propagator of the graviton, solve the difficulty of the Feynman integral divergence, and give evidence to prove that this theory is classical equivalent to the general theory of relativity. In this paper, we discuss the multi-graviton system and the self-interaction between gravitons. In momentum space, we obtain the general form of the Klein-Gordon equation in the gravitational field and find the symmetry associated with noncommutative gravity. We give the metric of the gravitational field of the multi-graviton system. There are singularities in this metric, which can produce black holes.

In section 2, we give a brief review of the quantum gravity theory suggested in the paper[1]. In section 3, we discuss the multiple-graviton system with self-interaction, giving the metric of the multiple-graviton system. In section 4, we calculate the Klein-Gordon equation in gravitational field. Due to the specificity of the metric of curved space caused by gravitational field in momentum space, from the Klein-Gordon equation in curved space, we obtain the general form of the Klein-Gordon equation in gravitational field, which is exactly the usual form of the Klein-Gordon equation in quantum field theory. Then we find the symmetry associated with noncommutative gravity from the Klein-Gordon equation. In section 5, by transforming the metric of gravitational field from coordinate space to momentum space, we get the isolated singularities, which is the event horizon of the black hole. This type of isolated singularity means that the horizon is formed by limit state dipoles.

## II. A Brief Review of Quantum Gravity

In this section, we briefly review the theory of quantum gravity suggested in the paper[1]. More details can be found in [1].

Since the introduction of the uncertainty principle into the general theory of relativity, we get a wave packet approximate to the Dirac $\delta$-function as follows

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$$\xi^i(x,r) = \begin{cases} \xi^r = r + C^r(x)\exp(-\dfrac{r}{l_P}) \\ \xi^\theta = \theta(x) \\ \xi^\phi = \phi(x) \\ \xi^t = t + C^t(x)\exp(-\dfrac{|t|}{t_P}) \end{cases} \tag{2.1}$$

It can be explained as a semiclassical graviton. The Lagrangian density can be written as

$$\mathscr{L} = -\frac{\eta^{\mu\nu}}{2}\frac{\partial\xi^i(x,r)}{\partial x^\mu}\frac{\partial\xi^j(x,r)}{\partial x^\nu}\eta_{ij} \tag{2.2}$$

The free field equation is

$$\partial^\mu\partial_\mu\xi^i = 0 \tag{2.3}$$

From the free field equation, we obtain Green's function

$$\tilde{G}^i(k) = \begin{cases} \tilde{G}^r(k) = -\dfrac{1}{(k^r)^2}\cdot\delta\Big(k^r - \dfrac{i}{l_P}\Big) \\ \tilde{G}^\theta(k) = -\dfrac{1}{(k^\theta)^2} \\ \tilde{G}^\phi(k) = -\dfrac{1}{(k^\phi)^2} \\ \tilde{G}^t(k) = -\dfrac{1}{\omega^2}\cdot\delta\Big(\omega - \dfrac{i}{t_P}\Big) \end{cases} \tag{2.4}$$

Compare Green's function (2.4) with the usual Feynman propagator, it can be seen that the generalized functions $\delta\Big(k^r - \frac{i}{l_P}\Big)$ and $\delta\Big(\omega - \frac{i}{t_P}\Big)$ give the regularization to the integral over $k^r$ and $\omega$ of the usual Feynman propagator. According to the properties of the Dirac $\delta$-function, we just need to give singularity on the integral paths without calculating specific integrals when calculating the Feynman diagrams. So that the difficulty of divergence of the Feynman integral over large virtual momenta of graviton has been solved.

The energy-momentum tensor of graviton is

$$\begin{aligned} T_{\mu\nu} &= \eta_{\mu\nu}\mathscr{L} - \frac{\partial\mathscr{L}}{\partial(\partial^\mu\xi^i)}\partial_\nu\xi^i \\ &= -\frac{\eta_{\mu\nu}}{2}\partial^\lambda\xi^i\partial_\lambda\xi^j\eta_{ij} + \partial_\mu\xi^i\partial_\nu\xi^j\eta_{ij} \end{aligned} \tag{2.5}$$

In the general theory of relativity, the energy-momentum tensor of gravitational field itself is

$$t_{\mu\nu} = \frac{1}{8\pi G}\left(\frac{1}{2}\eta_{\mu\nu}R^{(1)} - R^{(1)}_{\mu\nu}\right)$$

$$= \frac{1}{8\pi G\cdot C}\left(\frac{1}{2}\eta_{\mu\nu}\frac{\partial\xi^i}{\partial x^\kappa}\frac{\partial\xi_i}{\partial x_\kappa} - \frac{\partial\xi^i}{\partial x^\mu}\frac{\partial\xi_i}{\partial x^\nu}\right) \tag{2.6}$$

Up to a factor of a constant, Eq.(2.5) and Eq.(2.6) are equivalent. This shows that the quantum gravity theory established by a completely different method in the paper[1] is classical equivalent to the general theory of relativity.

## III. Multi-Graviton System with Self-Interaction

According to the model of graviton suggested in the paper[1], the noncommutative space made up of $\vee$ posets is flat. If we depend on traditional differential geometry to explain the spacetime limited by the uncertainty principle, the 4-dimensional space made up of mathematical points can be interpreted as curved, this is the gravitational field in the sense of the general theory of relativity. Let $g_{\mu\nu}$ be the metric of the gravitational field. To describe a graviton, we used both coordinate systems $x^\mu$ and $X^\mu$. If a graviton is excited at point $x$, the locally inertial coordinate system $\xi^\alpha$ at point $x$ can be written as

$$\xi^\alpha(x,X)|_x = X + C^\alpha(x)\cdot\exp(-\left|\frac{X}{L_P(x)}\right|)\Big|_{X=0} \tag{3.1}$$

where $L_P(x) \equiv L^\mu_P(x)$, $X \equiv X^\mu$.

In the case of multiple-graviton, due to the ductility of gravitons, gravitons elsewhere in a multi-graviton system will act on a point $x$ together. Therefore, we must also consider the self-interaction between gravitons caused by the ductility of gravitons. Specifically, if another graviton is excited at a distance of $l \equiv l^\mu = (l^1, l^2, l^3, l^4)$ to point $x$, the locally inertial coordinate system $\xi$ at point $x$ caused by this graviton can be written as

$$\lambda(\xi^\alpha) = \xi^\alpha\big((x+l), |l|\big)$$

$$= X + C^\alpha(x+l)\cdot\exp(-\left|\frac{l}{L_P(x+l)}\right|) \tag{3.2}$$

Then in multi-graviton system the locally inertial coordinate system $\xi^\alpha$ at point $x$ have to written as

$$\begin{aligned}\lambda(\xi^\alpha) =& X + \int d^4l\,\xi^\alpha\big((x+l), |l|\big)\\ =& X + \int d^4l\,\left(C^\alpha(x+l)\cdot\exp(-\left|\frac{l}{L_P(x+l)}\right|)\right)\end{aligned} \tag{3.3}$$

This expression shows that there is the self-interaction between gravitons.

For the gravitational field in vacuum, the field $C^\alpha(x+l)$ in Eq.(3.3) satisfy the free field equation Eq.(2.3), the solution is

$$C^\alpha(x+l) = \int d^4k\left(C^\alpha(k)\exp\big(ik(x+l)\big) + (C^\alpha(k))^*\exp\big(-ik(x+l)\big)\right)$$

$$= \int d^4k \left( C^\alpha(k) \exp(ikx) \exp(ikl\,) + (C^\alpha(k))^* \exp(-ikx) \exp(-ikl\,) \right) \tag{3.4}$$

where $k \equiv k_\mu$ is the energy-momentum conjugate to $x^\mu$.
From Eq.(3.3) and Eq.(3.4) we have

$$\frac{\partial \lambda(\xi^\alpha)}{\partial x^\mu} = \frac{\partial \int d^4l\, \xi^\alpha\big((x+l\,), |l\,|\big)}{\partial x^\mu} = \frac{\partial \int d^4l \left( C^\alpha(x+l\,) \cdot \exp(-\left|\frac{l}{L_P(x+l\,)}\right|) \right)}{\partial x^\mu}$$

$$= \int d^4l\, d^4k \left[ \left( \frac{\partial \big( C^\alpha(k) \exp(ik(x+l\,)) \big)}{\partial x^\mu} \right.\right.$$

$$\left.\left. + \frac{\partial \big( (C^\alpha(k))^* \exp(-ik(x+l\,)) \big)}{\partial x^\mu} \right) \cdot \exp(-\left|\frac{l}{L_P(k)}\right|) \right]$$

$$= \int d^4l\, d^4k \left[ \left( ik_\mu C^\alpha(k) \exp(ikx) \exp(ikl\,) \right.\right. \tag{3.5}$$

$$\left.\left. - ik_\mu \, (C^\alpha(k))^* \exp(-ikx) \exp(-ikl\,) \right) \cdot \exp(-\left|\frac{l}{L_P(k)}\right|) \right]$$

$$= \int d^4l\, d^4k \left( ik_\mu C^\alpha(k) \exp(ikx) \exp(\frac{\pm ikL_P(k) - 1}{|L_P(k)|} \cdot |l\,|) \right.$$

$$\left. - ik_\mu \, (C^\alpha(k))^* \exp(-ikx) \exp(\frac{\mp ikL_P(k) - 1}{|L_P(k)|} \cdot |l\,|) \right)$$

$$= \int d^4k \left( \frac{2|L_P|}{1 \mp ikL_P} ik_\mu C^\alpha(k) \exp(ikx) - \frac{2|L_P|}{1 \pm ikL_P} ik_\mu \, (C^\alpha(k))^* \exp(-ikx) \right)$$

Note that in Eq.(3.5), $L_P^\mu(x+l\,)$ in coordinate space has been transformed to $L_P^\mu(k)$ in momentum space, and we denote $L_P^\mu(k)$ as $L_P(k)$ for short. The modulus of $L_P(k)$ is $(l_P, t_P)$, $L_P(k)$ has only 3 degrees of freedom, the phase angle $(\theta, \phi, \pm t_P)$.

Then the metric in momentum space can be written as follows

$$g_{\mu\nu} = \frac{\partial \lambda(\xi^\alpha)}{\partial x^\mu} \frac{\partial \lambda(\xi^\beta)}{\partial x^\nu} \eta_{\alpha\beta}$$

$$= \frac{\partial \int d^4l\, \xi^\alpha\big((x+l\,), |l\,|\big)}{\partial x^\mu} \frac{\partial \int d^4l\, \xi^\beta\big((x+l\,), |l\,|\big)}{\partial x^\nu} \eta_{\alpha\beta}$$

$$= \int d^4k \left( \frac{2|L_P|}{1 \mp ikL_P(k)} ik_\mu C^\alpha(k) \exp(ikx) - \frac{2|L_P|}{1 \pm ikL_P(k)} ik_\mu \, (C^\alpha(k))^* \exp(-ikx) \right)$$

$$\cdot \int d^4k' \left( \frac{2|L_P|}{1 \mp ik'L_P(k')} ik'_\nu C_\alpha(k') \exp(ik'x) - \frac{2|L_P|}{1 \pm ik'L_P(k')} ik'_\nu \left(C_\alpha(k')\right)^* \exp(-ik'x) \right) \tag{3.6}$$

$$= \int d^4k d^4k' \quad 4|L_P|^2 \Big( - \frac{k_\mu k'_\nu}{(1 \mp ikL_P)(1 \mp ik'L_P)} C^\alpha(k) C_\alpha(k') \exp\left[i(k+k')x\right]$$

$$- \frac{k_\mu k'_\nu}{(1 \pm ikL_P)(1 \pm ik'L_P)} \left(C^\alpha(k)\right)^* \left(C_\alpha(k')\right)^* \exp\left[-i(k+k')x\right]$$

$$+ \frac{k_\mu k'_\nu}{1 + kL_P \cdot k'L_P} \left[C^\alpha(k) \left(C_\alpha(k')\right)^* + \left(C^\alpha(k)\right)^* C_\alpha(k')\right] \Big) \Big)$$

It can be written as follows

$$g_{\mu\nu} = \int d\vec{k} d\omega \, d\vec{k'} d\omega' \left( 4|L_P|^2 \Big( - \frac{C^\alpha(k) C_\alpha(k') \exp\left[i(k+k')x\right]}{(1 \mp ikL_P)(1 \mp ik'L_P)} \right.$$

$$- \frac{\left(C^\alpha(k)\right)^* \left(C_\alpha(k')\right)^* \exp\left[-i(k+k')x\right]}{(1 \pm ikL_P)(1 \pm ik'L_P)} \tag{3.7}$$

$$\left. + \frac{C^\alpha(k) \left(C_\alpha(k')\right)^* + \left(C^\alpha(k)\right)^* C_\alpha(k')}{1 + kL_P \cdot k'L_P} \Big) \cdot k_\mu k'_\nu \right)$$

where $k \equiv k^\mu = (\vec{k}, \omega)$.

## IV. Symmetry of Klein-Gordon Equation

Let's study the real scalar particles in the gravitational field. Complex scalar fields are completely similar.

In curved spacetime, the Lagrangian density of real scalar particle with spin 0 is

$$\mathscr{L} = g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + m^2 \phi^2 \tag{4.1}$$

where $m$ is the mass of scalar particle.
Then in the spacetime with the metric $g_{\mu\nu}$, the Klein-Gordon equation is

$$\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left( \sqrt{-g} g^{\mu\nu} \frac{\partial \Phi}{\partial x^\nu} \right) - m^2 \Phi = 0 \tag{4.2}$$

where $g = \mathrm{Det}\, g_{\mu\nu}$ is the scalar density.
It can be written as

$$g^{\mu\nu}\frac{\partial^2\Phi}{\partial x^\mu\partial x^\nu}+\frac{1}{2g}\frac{\partial g}{\partial x^\mu}g^{\mu\nu}\frac{\partial\Phi}{\partial x^\nu}+\frac{\partial g^{\mu\nu}}{\partial x^\mu}\frac{\partial\Phi}{\partial x^\nu}-m^2\Phi=0 \tag{4.3}$$

Eq.(4.2) can be written as follows

$$g^{\mu\nu}\frac{\partial^2\Phi}{\partial x^\mu\partial x^\nu}+\frac{1}{2g}\frac{\partial g}{\partial x^\mu}g^{-1}_{\mu\nu}\frac{\partial\Phi}{\partial x^\nu}-g^{-1}_{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial x^\mu}g^{-1}_{\mu\nu}\frac{\partial\Phi}{\partial x^\nu}-m^2\Phi=0 \tag{4.4}$$

The inverse of the metric $g_{\mu\nu}$ is

$$g^{-1}_{\mu\nu}=\frac{1}{g}\left[g^*\right]^{\mu\nu} \tag{4.5}$$

where $\left[g^*\right]^{\mu\nu}$ is the adjoint matrix of the metric $g_{\mu\nu}$.
Then Eq.(4.4) can be written as follows

$$g^{\mu\nu}\frac{\partial^2\Phi}{\partial x^\mu\partial x^\nu}+\left(\frac{1}{2g}\frac{\partial g}{\partial x^\mu}-\frac{1}{g}\left[g^*\right]^{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial x^\mu}\right)g^{-1}_{\mu\nu}\frac{\partial\Phi}{\partial x^\nu}-m^2\Phi=0 \tag{4.6}$$

For the metric (3.7), we have

$$\frac{1}{2g}\frac{\partial g}{\partial\vec{x}\,\partial t}=\frac{1}{g}\left[g^*\right]^{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial\vec{x}\,\partial t} \tag{4.7}$$

where $x^\mu=(\vec{x},t)$.

So that Eq.(4.6) can be written as

$$g^{\mu\nu}\frac{\partial^2\Phi}{\partial x^\mu\partial x^\nu}-m^2\mathrm{F}=0 \tag{4.8}$$

Therefore in the gravitational field, the Klein-Gordon equation (4.2) is the usual form in quantum field theory, which can be written as follows

$$\left(\Box^2-m^2\right)\mathrm{F}=0 \tag{98}$$

where $\Box^2$ is the usual D'Alembertian operator in curved spacetime as follows

$$\Box^2=g^{\mu\nu}\frac{\partial^2}{\partial x^\mu\partial x^\nu} \tag{4.10}$$

In general, in curved space, the Klein-Gordon equation is Eq.(4.2). Due to the specificity of the metric (3.7) of noncommutative gravitational field in momentum space, Eq.(4.8) has the following symmetry:

For the transformation from Minkowski spacetime to curved spacetime, the Klein-Gordon equation be invariant if the transformation of the local inertial system (or the cotangent frame of spacetime) $\lambda(\hat{\xi})$ is as follows:

$$\lambda(\hat{\xi})\big|_x\ :\ \ r^i\ \rightarrow\ r^i+C^i(x)\exp(-\frac{r^i}{L^i_P}) \tag{4.11}$$

where the pole of the spherical polar coordinate system $r^i$ is $x$.

Eq.(4.8) and Eq.(4.11) show the symmetry associated with noncommutative gravity. It can be interpreted as follows:

For the transformation from Minkowski spacetime to curved spacetime, if the Klein-Gordon equation is required to be invariant, we need to introduce the noncommutative gravitational field with ductility which in Eq.(4.11). And the noncommutative gravitational field must satisfy the field equation. In this way, the metric $g_{\mu\nu}$ of spacetime is constrained, so that not all curved spacetime is physically possible. The ductility of graviton produces the self-interaction of gravitational field, which can produce black holes, and keep the symmetry of the Klein-Gordon equation associated with noncommutative gravity. The ductility of graviton can also be understood as that the graviton itself has the gravitational charge, thus affecting the geometry of spacetime around the graviton, and also produces the self-interaction of gravitational field. The most important feature of the ductile structure is: due to the characteristics of the ductile function, by quantizing the noncommutative gravitational field with ductility, we can obtain the Feynman rule without diverged integrals.

## V. Black Hole

From the last term in the integrand of the metric (3.7), it can be seen that there are singularities as follows

$$k \cdot k' = -\frac{1}{L_P(k)} \frac{1}{L_P(k')} \tag{5.1}$$

$L_P(k)$ has 3 degrees of freedom, the phase angles $(\theta, \phi, \pm t_P)$. If the phase angles of the $L_P(k)$ and $L_P(k')$ in Eq.(5.1) are opposite, Eq.(5.1) be true, which also means

$$|k| = |k'| = \left| \frac{1}{L_P(k)} \right| = \left| \frac{1}{L_P(k')} \right| = \left( \frac{1}{l_P}, \frac{1}{t_P} \right) \tag{5.2}$$

Eq.(5.1) and Eq.(5.2) independent of the source flow of gravity. From Eq.(5.1) we can see that the phase angles of $k$ and $k'$ are the same as $L_P(k)$ or $L_P(k')$, so that the singularity in momentum space must be in pairs at the same point in coordinate space, and the phase angles of the paired singularities are opposite.

If the field $C^\alpha(x)$ given by the free field equation is an analytical function, then except for singularities shown in Eq.(5.1), the integrand in Eq.(3.6) is analytic, so these singularities are the isolated singularities. The metric be singular while the integral paths pass through these singularities. This can be interpreted that the event horizon in coordinate space is the set of isolated singularities in momentum space.

From the viewpoint of hydrodynamic, this type of isolated singularity is a drain or source of the flow in momentum space, where the complex potential is the integrand in Eq.(3.6). From Eq.(3.6), it can be seen that the isolated singularity in the integrand in Eq.(3.6) is the limiting case of a dipole in which the source and drain of the same strength are infinitely close and the strength increases infinitely.

In the paper[1], we get the energy-momentum tensor Eq.(2.5) of gravitational field itself. Recall Eq.(3.5), the expression clearly shows the self-interaction between gravitons. Therefore in the multi-graviton system, due to the self-interaction of the gravitational field, the energy-momentum tensor Eq.(2.5) should be written as follows

$$T_{\mu\nu} = -\frac{\eta_{\mu\nu}}{2}\partial^{\lambda}\lambda(\xi^{i})\partial_{\lambda}\lambda(\xi^{j})\eta_{ij} + \partial_{\mu}\lambda(\xi^{i})\partial_{\nu}\lambda(\xi^{j})\eta_{ij} \tag{5.3}$$

The eigenvalue $\lambda(\xi)$ is given by Eq.(3.3). Using Eq.(3.5) we get the density of the energy-momentum tensor as follows

$$\begin{aligned} T_{\mu\nu}(x) = -\frac{\eta_{\mu\nu}}{2}\int d^4k \left(\frac{2|L_P|}{1\mp ikL_P} ik^{\lambda} C^{\alpha}(k)\exp(ikx) - \frac{2|L_P|}{1\pm ikL_P} ik^{\lambda}\left(C^{\alpha}(k)\right)^{*}\exp(-ikx)\right) \\ \cdot \int d^4k' \left(\frac{2|L_P|}{1\mp ik'L_P} ik'_{\lambda} C_{\alpha}(k')\exp(ik'x) - \frac{2|L_P|}{1\pm ik'L_P} ik'_{\lambda}\left(C_{\alpha}(k')\right)^{*}\exp(-ik'x)\right) \\ + \int d^4k \left(\frac{2|L_P|}{1\mp ikL_P} ik_{\mu} C^{\alpha}(k)\exp(ikx) - \frac{2|L_P|}{1\pm ikL_P} ik_{\mu}\left(C^{\alpha}(k)\right)^{*}\exp(-ikx)\right) \\ \cdot \int d^4k' \left(\frac{2|L_P|}{1\mp ik'L_P} ik'_{\nu} C_{\alpha}(k')\exp(ik'x) - \frac{2|L_P|}{1\pm ik'L_P} ik'_{\nu}\left(C_{\alpha}(k')\right)^{*}\exp(-ik'x)\right) \end{aligned} \tag{5.4}$$

Notice that there are the isolated singularities $k\cdot k' = -\frac{1}{L_P(k)}\frac{1}{L_P(k')}$ in Eq.(5.4), and these singularities are unacceptable for energy-momentum. We can avoid this problem in this way: in the derivation of the energy-momentum of gravitational field itself Eq.(2.6) and Eq.(5.4), we have used the mass-shell condition, therefore it is considered that, the energy-momentum of a real graviton must satisfy $|k| < \left(\frac{1}{l_P}, \frac{1}{t_P}\right)$, the graviton with the energy-momentum $|k| \geq \left(\frac{1}{l_P}, \frac{1}{t_P}\right)$ must be off-shell. So that the state of isolated singularity $|k| = \left(\frac{1}{l_P}, \frac{1}{t_P}\right)$ is the limit state that cannot be reached by the on-shell graviton. Therefore, the event horizon is formed by dipoles composed of virtual gravitons that can reach the limit state, which cannot radiate like the gravitational wave composed of real gravitons.

## VI. Conclusion

In the paper 'A New Approach to Quantum Gravity'[1], we suggest a new theory of quantum gravity. Using this theory, we can study the noncommutative gravitational field in momentum space. The metric of this curved space caused by the gravitational field is special in momentum space, by this metric, we obtain the general form of the Klein-Gordon equation in the gravitational field. Then we find the symmetry associated with noncommutative gravity from the Klein-Gordon equation. The noncommutative gravitational field in momentum space has singularities, which means the event horizon of black holes. From this metric, we conclude that the event horizon of black holes is formed by the dipoles in momentum space with limit state.

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# Quasi-Clusters in Nuclear Photodisintegration in the Giant Dipole Resonance Region

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*Abstract-* The role of quasi-clusters (systems of a small number of nucleons) in the photodisintegration of light atomic nuclei of the $1p$-shell is considered. In these nuclei, there is a strong supermultiplet splitting of levels due to the prominent role of the Majorana forces. It is manifested in the excitation and decay of the Giant Dipole Resonance (GDR), leading to its large-scale configurational splitting. Nucleon quartets, formed inside the nucleus, have a powerful effect on the properties of the GDR of 1p-shell nuclei. The preservation or decay of such quartets during nuclear photoabsorption leads to the formation of various GDR regions. Experimental data confirming this phenomenon are presented.

*Keywords:* *giant dipole resonance, $1p$-shell nuclei, configurational splitting, supermultiplet structure, Majorana forces, Young schemes, nucleon quartets.*

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# Quasi-Clusters in Nuclear Photodisintegration in the Giant Dipole Resonance Region

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## I. Introduction

As is known, the photoabsorption cross section of atomic nuclei has a dominant maximum, called Giant Dipole Resonance (GDR), caused by the absorption of electric dipole (E1) photons. In heavy nuclei, it is located in the region of 10–20 MeV, and in light nuclei – in the area of 15–30 MeV.

Excitation of GDR usually occurs by transferring the energy of the E1-photon to an individual nucleon of the nucleus.Then this nucleon either escapes from the nucleus (semi-direct mechanism of GDR decay) or exchanges energy with other nucleons with their subsequent emission from a nucleus in pre-equilibrium decay or decay at the stage of a compound nucleus.

The study of the GDR has played an outstanding role in the formation of modern concepts of the structure and dynamics of the atomic nucleus [1]. The mechanism of the formation of collective states from the initial particle-hole configurations was discovered within the framework of the shell model [2,3], and it was shown that due to this mechanism that a collective dipole state is formed in medium and heavy nuclei, which reproduces the experimentally observed GDR. At the same time, it was shown that in medium and heavy nuclei, the GDR is determined not by an individual (structural) but by the averaged characteristics of the nuclei. Indeed, the collective particle-hole $(ph)$ dipole state underlying GDR is in massive nuclei, a coherent superposition of many $ph$-configurations, and its properties vary slightly from nucleus to the nucleus. Further, the decay properties of GDR in these nuclei are associated with the fragmentation of the dipole state over a considerable number of $2p2h$ and more complex configurations and, therefore, reflect only the averaged characteristics of the nuclei.

On the contrary, in light nuclei, the properties of the GDR are primarily determined by the individuality of the nucleus. In this regard, the most indicative is the $1p$-shell nuclei (from lithium isotopes to oxygen isotopes). In contrast to medium and heavy nuclei in $1p$-shell nuclei, GDR excitation occurs not through the transfer of E1-photon energy to individual nucleons but to systems of a small number of additionally bound nucleons that form *quasi-clusters* inside the nucleus. The specific features of this type of excitation and decay of the GDR of $1p$-shell nuclei are dictated by their supermultiplet structure. The purpose of this review is to consider the manifestation of this structure in nuclear photodisintegration.

## II. Dipole Photodisintegration of $1p$-Shell Nuclei

For $1p$-shell nuclei, the self-consistent average potential strongly depends on the quantum number of the "Young scheme," which characterizes the permutation symmetry of the spatial variables of the shell configurations. In fact, under these conditions, it is convenient to indicate the shell configuration and its Young scheme. The splitting of configurations, according to Young schemes, reaches 15–16 MeV in the nuclei of the $1p$-shell, which leads to a strong «configurational splitting of the GDR» of nuclei in this region [4].

Let us first touch upon those features of the configurational splitting of the GDR in the $1p$-shell nuclei, which are due to a deep ($1s$) hole. We will use the most advanced approach based on direct diagonalization of a particular basis and neglecting the one-particle continuum. This approach is called the Bound Shell Model (BSM). Figure 1 shows the total GDR cross sections calculated in the framework of this approach for $^{7}$Li, $^{9}$Be, $^{11}$B, $^{12\text{-}14}$C, $^{14,15}$N, and $^{16}$O nuclei [5]. The dotted line intersecting the figure separates the dipole transitions from different shells. The transitions A (to the left of the line) are due to nucleons of the outer ($1p$) shell, and the transitions B (to the right of the line) are due to nucleons of the inner ($1s$) shell.

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Groups A and B of dipole excitations in the $1p$-shell nuclei are due to $1s \rightarrow 1p$ and $1p \rightarrow 1d2s$ nucleon transitions, respectively:

$$1s^4 1p^{A-4} \rightarrow \begin{cases} 1s^4 1p^{A-5}(1d2s)^1, & \text{A} \\ 1s^3 1p^{A-3}, & \text{B.} \end{cases}$$

Analyzing Figure 1, we note that the transitions A and B in the nuclei of the $1p$-shell are mixed insignificantly, and a single dipole state is not formed. The dipole-resonance states are greatly affected by the supermultiplet character of the structure of low-lying states and will be considered below. B-transitions play the most crucial role in nuclei where the $1p$-shell is just beginning to be populated, i.e., in $^7$Li and $^9$Be nuclei. Calculations by the BSM method qualitatively correctly reproduce the main feature of the GDR in these nuclei – its strong broadening. For example, in $^9$Be, the GDR extends up to 50 MeV (Figure 6). The strengths of the transitions A and B are compared in $^7$Li and $^9$Be, and in heavier nuclei, B-transitions play a subordinate role, forming the 30-MeV and higher energy GDR region.

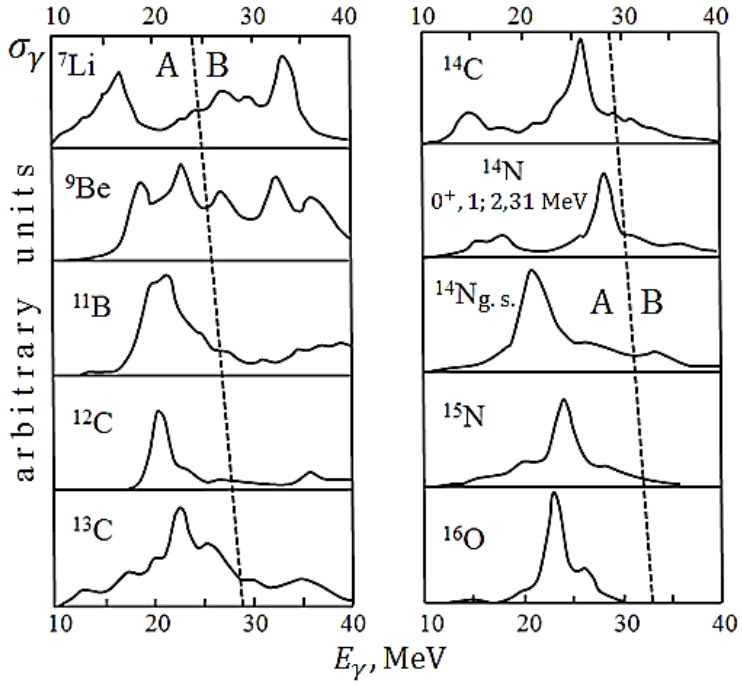


*Figure 1:* Total cross sections for the photoabsorption of the $1p$-shell nuclei calculated in the BSM approach [5]. The columns obtained in the calculations are "broadened" according to the Breit-Wigner formula with a width $\Gamma = 2$ MeV. The dotted line intersecting the figure separates transitions A (left) and B (right)

## III. Supermultiplet Structure of the $1p$-Shell Nuclei

The states of light nuclei of the $1p$-shell can be described in the $LS$-coupling scheme using the so-called supermultiplet structure of the nucleus, characterized by Young schemes defining the symmetry of the spatial part of the wave function of the nucleus. The prerequisites for the existence of such a structure are the smallness of spin-orbit splitting and the prominent role of Majorana space-exchange nucleon-nucleon forces.The structure of the A-branch of the GDR in $1p$-shell nuclei strongly depends on the supermultiplet symmetry, i.e., the spin-isospin group $SU_4$ of the $1p$-shell nuclei. Due to the antisymmetry of the nuclear wave function, $SU_4$ supermultiplets can be specified using the Young scheme of the permutation symmetry of the spatial part of the wave function. This Young scheme is the Young conjugate scheme of the $SU_4$ group. It is given by a set of quantum numbers $\{f\}LSTJ$, where $L, S, T, J$ are the orbital, spin, isospin, and total angular momenta of the state under consideration and $\{f\} = [f_1, f_2, \ldots, f_n]$ c $f_1 \geq f_2 \geq \cdots \geq f_n$, $f_i \leq 4$ and $\sum_i f_i = A$, where $A$ is the number of nucleons in the nucleus. The structure of the $1p$-shell nuclei is discussed in these terms below.

The effects of supermultiplet symmetry – supermultiplet splitting of the levels of light nuclei – arise due to the prominent role of Majorana forces, i.e., pair forces of the type $\hat{V}_{12} = -V(r)\hat{P}_{12}^X$, where $\hat{P}_{12}^X$ is the Majorana operator of permutation of spatial coordinates nucleons, $V(r)$ is a radial function that sets the interaction intensity and its radial dependence.

We will assume first that the energies of the levels can be estimated in the "diagonal" approximation, that is, as the average values of the nuclear Hamiltonian over fixed shell configurations. Denoting $\bar{V}$ the average intensity of pair interaction in a given configuration, we

obtain for the operator $\hat{M}$ of the total Majorana interaction $\hat{M} = -\bar{V}\sum_{i<j}\hat{P}_{ij}^{X}$.

It is easy to see that the operator $\sum_{i<j}\hat{P}_{ij}^{X}$ is an invariant of the group of permutations of the spatial coordinates of nucleons (more precisely, the Casimir operator of this group). The eigenvalues of this operator for the representation (supermultiplet) $\{f\}$ are given by the formula

$$\langle\{f\}\left|\sum_{i<j}\hat{P}_{ij}^{X}\right|\{f\}\rangle = \frac{1}{2}[f_1(f_1-1)+f_2(f_2-3)+\cdots].$$

Hence it follows that the states belonging to different Young schemes should be separated by a wide energy gap. Let us illustrate this with the example of the $^6$Li and $^7$Li nuclei. In $^6$Li, the levels with the scheme $\{42\}$ and the $1s^4 1p^2$-configuration, and in $^7$Li with the scheme $\{43\}$ and the $1s^4 1p^3$-configuration, are, as shell calculations show, in the energy range below 10 MeV (the energy is measured from the ground state). The levels $^6$Li and $^7$Li with Young schemes $\{411\}$ and $\{421\}$ are in the region of 10–20 MeV. A similar situation takes place in other nuclei of the $1p$-shell.

As a result of the action of Majorana exchange forces, the lowest state of the supermultiplet has the most symmetric Young scheme with the maximum possible number of rows $f_i = 4$, which allows us to speak of the effect of "quarteting" (the formation of quartets of nucleons) in the ground state of the nucleus [6,7] (non-quarteting nucleons, if any, correspond to the valence shell). In the case of E1-excitation of such a nucleus, the Majorana forces can lead to a significant difference in the average energies of the transition groups associated with a substantial change in the symmetry of the spatial part of the wave function of the nucleus. In this case, the most substantial and most stable effects of supermultiplet splitting arise with such changes in Young schemes when the number of the nucleon four-groups (quartets) in them decreases. Let us illustrate the existence of the "quarteting" effect by the example of $^8$Be and $^{12}$C nuclei. The ground state of these nuclei belongs to Young schemes $\{44\}$ and $\{444\}$. The isospin of these states should be zero (all nucleons are quarteted). The states arising in the breaking of the nucleon four-groups, i.e., the states with Young schemes $\{431\}$ and $\{4431\}$, can have an isospin $T = 1$ and, as spectroscopy shows, are at energies $E$ =15–16 MeV. This number (15–16 MeV) is the typical "quarteting energy" in all nuclei of the $1p$-shell.

## IV. Supermultiplet Symmetry Effects in the GDR of Lithium Isotopes

Let us now trace how the effects of supermultiplet splitting manifest themselves in the GDR. Let's start again with the lightest $1p$-shell nuclei –$^6$Li and $^7$Li. In these nuclei, the GDR is formed by the following configurations:

$$^6\text{Li: } 1s^4 1p(1d2s)\{411\},$$
$$1s^3 1p^3\{33\},$$
$$1s^3 1p^3\{321\},$$
$$^7\text{Li: } 1s^4 1p^2(1d2s)\{43\},\{421\},$$
$$1s^3 1p^4\{43\},$$
$$1s^3 1p^4\{331\}.$$

According to what was said above, the energies of the configurations $1s^3 1p^3\{33\}$ in $^6$Li and $1s^3 1p^4\{331\}$ in $^7$Li should be significantly (≈10 MeV) higher than the energies of the configurations $1s^4 1p(1d2s)\{411\}$, $1s^4 1p^2(1d2s)\{43\},\{421\}$ and $1s^3 1p^4\{43\}$ (a 1.5-fold decrease in the energy of quarteting in comparison with the previously indicated one is due to the fact that here the "quarteted" nucleons are in different shells). Some additional spread in the energies of configurations for $^6$Li is associated with the fact that two Young schemes, $\{321\}$ and $\{33\}$, operate. As a result, one should expect that in the Li isotopes, the absorption of γ-quanta should be concentrated in the energy intervals shown in Table 1.

*Table 1:* Energies and Young schemes for the excited configurations of Li isotopes

| Energy intervals, MeV | Young schemes |
|---|---|
| 5–15 | $\{411\},\{43\}$ |
| 15–20 | $\{33\},\{421\}$ |
| 25–40 | $\{321\},\{331\}$ |

Since the decay properties of GDR in light nuclei are largely determined by the configurations directly excited by γ-quanta, the multiplication rules for Young schemes dictate the following preferred types of GDR decays in lithium isotopes ($\mathrm{d} \equiv {}^2_1\mathrm{H}, \mathrm{t} \equiv {}^3_1\mathrm{H}$):

$$1s^4 1p(1d2s)\{411\} \to \alpha + \mathrm{p} + \mathrm{n},$$

$$1s^3 1p^3\{33\} \to \mathrm{t} + {}^3_2\mathrm{He},$$

$$1s^3 1p^3\{321\}\begin{cases}\mathrm{t}+\mathrm{d}+\mathrm{p},\\ {}^3_2\mathrm{He}+\mathrm{d}+\mathrm{n},\end{cases}$$

$$1s^3 1p^4,\ 1s^4 1p^2(1d2s)\{43\} \to \alpha + \mathrm{t},$$

$$1s^4 1p^2(1d2s)\{421\}\to\begin{cases}{}^5_2\mathrm{He}+\mathrm{d}\to\alpha+\mathrm{d}+\mathrm{n},\\ {}^6_3\mathrm{Li}^*\{42\}+\mathrm{n}\to\begin{cases}{}^6_3\mathrm{Li}+\mathrm{n}+\gamma,\\ \alpha+\mathrm{d}+\mathrm{n},\end{cases}\end{cases}$$

$$1s^3 1p^4\{331\} \to\begin{cases}\mathrm{t}+\mathrm{t}+\mathrm{p},\\ \mathrm{t}+{}^3_2\mathrm{He}+\mathrm{n}.\end{cases}$$

For greater clarity, the scheme of excitation and decay of the GDR of the $^{6}$Li nucleus is shown in Figure 2.

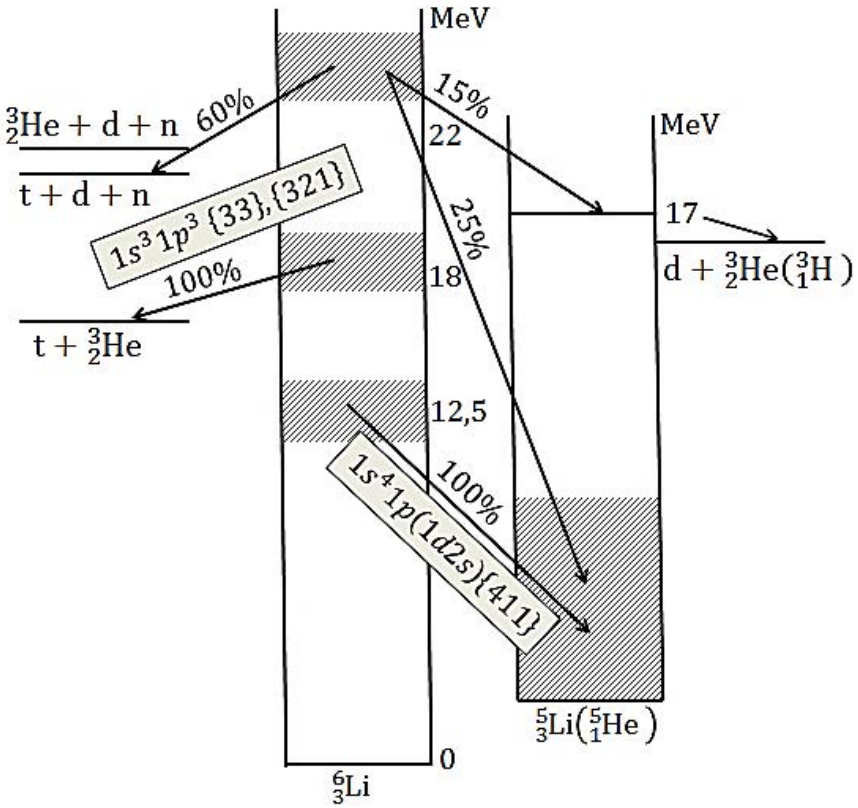


*Figure 2:* The scheme for excitation and decay of $^{6}$Li as a result of the absorption of γ-quanta, predicted by the theory in diagonal approximation [4].

Figure 3 shows the photoabsorption cross section for $^{6}$Li [8] and Young schemes that form the central maximum of the GDR and the maximum in the low-energy region (pygmy resonance).

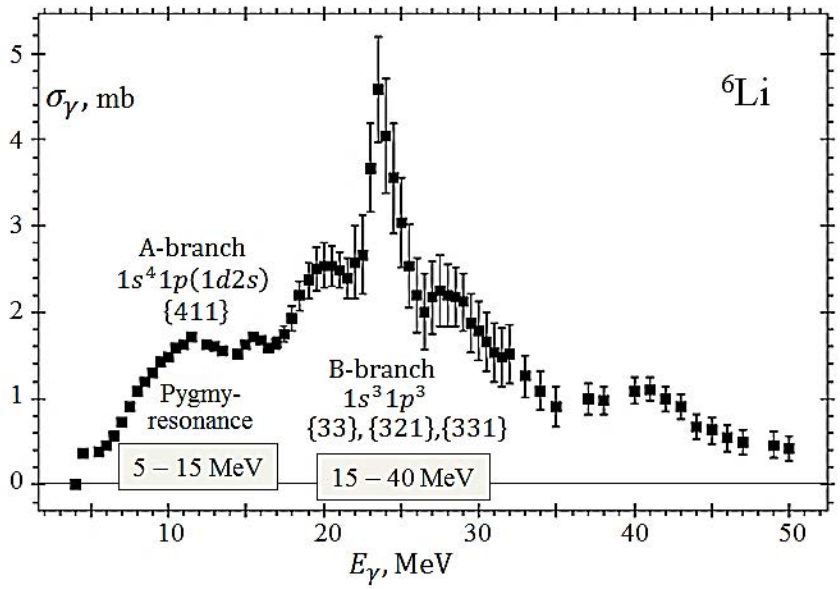


*Figure 3:* Photoabsorption cross section of the $^{6}$Li [8] and Young schemes forming different energy regions of this cross section

Figure 4 shows a diagram of nucleon E1-transitions of the $^{7}$Li nucleus, indicating the corresponding Young schemes, nucleon quartets, and energies. Figure 5 shows the photoabsorption cross section in $^{7}$Li [8] and Young schemes for different regions of this cross section.

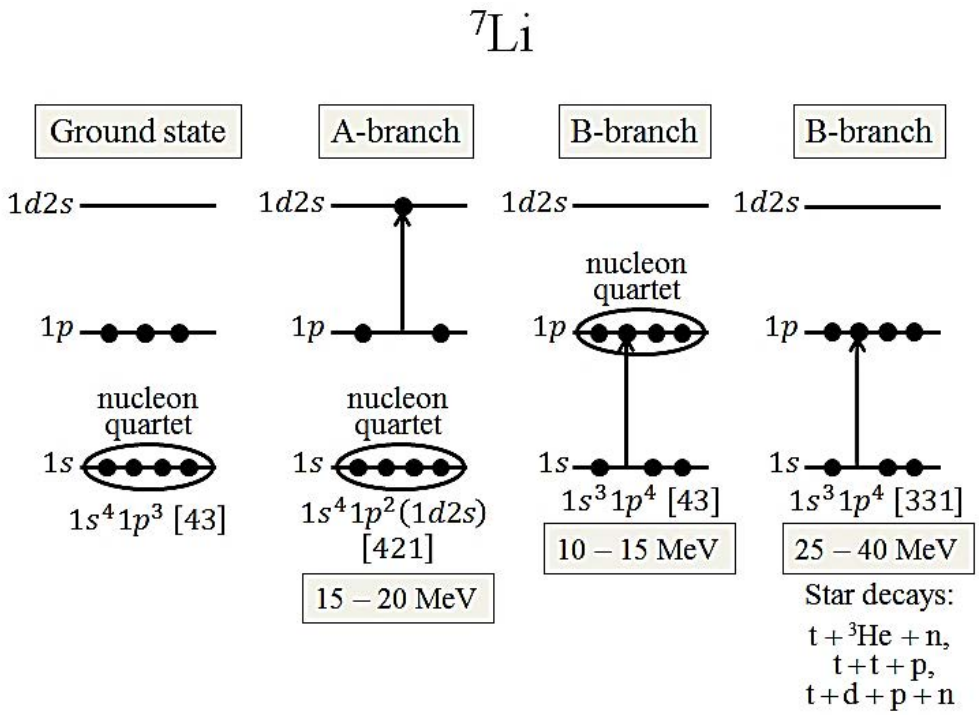


*Figure 4:* Nucleon transitions and Young schemes are forming the GDR in $^{7}$Li

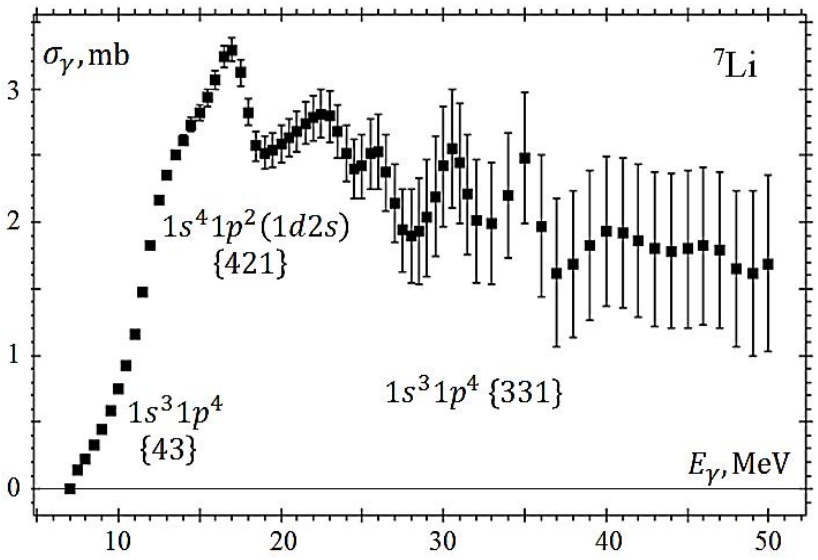


*Figure 5:* Photoabsorption cross section of $^{7}$Li [8] nucleus and Young schemes forming different energy regions of this cross section

The most striking property of the GDR in $^{6,7}$Li nuclei should be a high probability of decay through channels α–t, $^{3}$He–t, and "star-like" channels, i.e., many-particle channels, which correspond to different parts of the dipole absorption band. Experimental data confirm this [9–19].

## V. Photo Disintegration of $^{9}$Be and $^{13}$C

Photodisintegration of the $^{9}$Be nucleus can be a good test of the theory under consideration. The ground state of this nucleus has the configuration $1s^4 1p^5\{441\}$. In transitions of type A, configurations $1s^4 1p^4(1d2s)$ with

Young schemes {441} and {432}, {4311} are excited. According to the above, states with Young schemes {441} and {432}, {4311} are separated by an energy gap of approximately 15–16 MeV (quarteting effect). Accordingly, the set of states with Young schemes {432} and {4311} forms the main GDR maximum in $^{9}$Be, located in the region of 15–25 MeV [20] (Figure 6), and states with Young scheme {441} give a pygmy-resonance, clearly visible in the channel $(\gamma, n)$ [21,22] (Figure 7). The pygmy-resonance, naturally, lies at much lower energies.

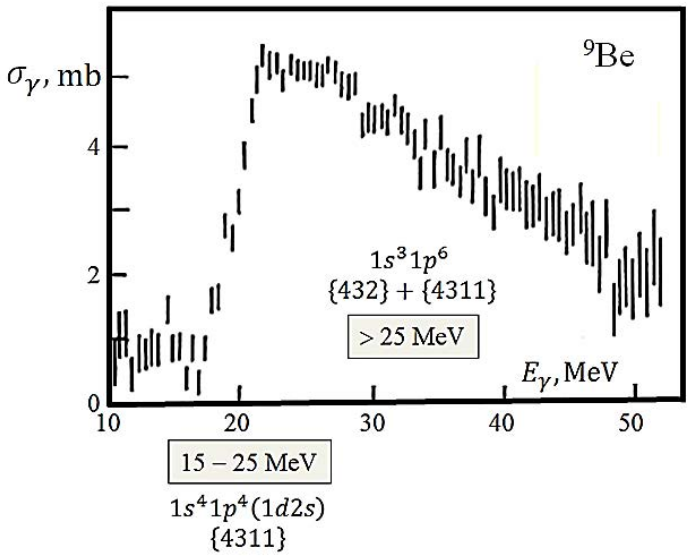


*Figure 6:* Photoabsorption cross section of $^{9}$Be nucleus [20] and Young schemes, forming different energy regions of this cross section

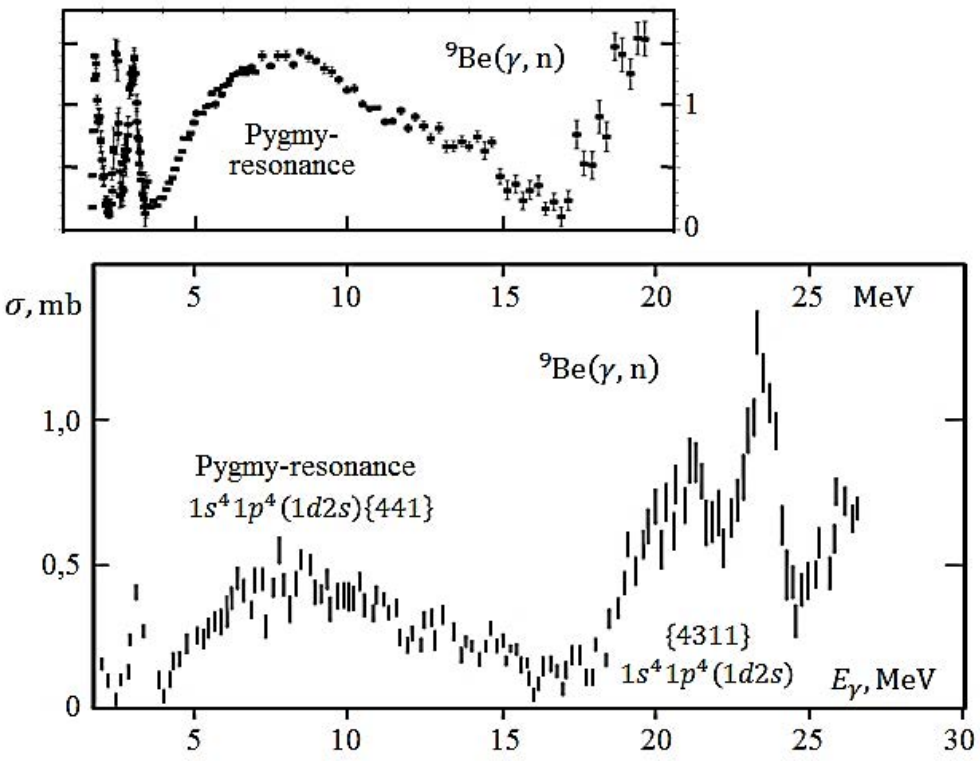


*Figure 7:* Pigmy resonance in the cross section of the reaction $^{9}$Be$(\gamma, n)$ according to works [21] (bottom) and [22] (top)

The $^{13}$C nucleus is very close in its supermultiplet properties to the $^{9}$Be nucleus under consideration. Everything that has been said about $^{9}$Be is also true about this nucleus. The only thing that needs to be changed formally is to make a substitution in Young schemes $\{441\} \rightarrow \{4441\}$, etc. Therefore, a pygmy resonance should exist in the $^{13}$C nucleus, as well as in $^{9}$Be. It is seen experimentally both in the $(\gamma, \mathrm{n})$ channel [23] (Figure 8) and in the photoabsorption cross section [24].

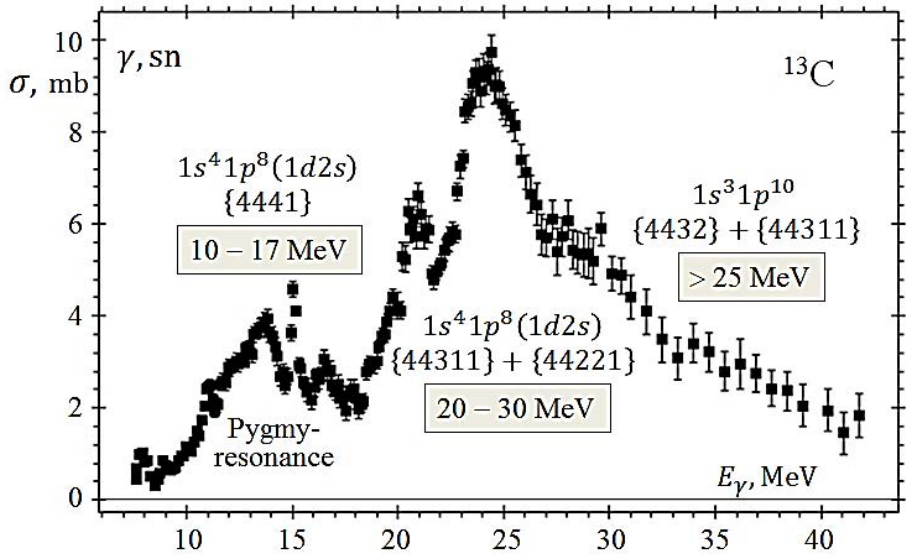


*Figure 8:* The cross section for the photoneutron reaction of the $^{13}$C nucleus [23] and Young scheme, forming different energy regions of this cross section

Pigmy resonance is also found in $^{11}$B, $^{14}$C, and $^{15}$N nuclei, which contain non-quarteting nucleons (see details in [4]). A pygmy resonance could also exist in $^{14}$N. Still, because the quarteting effect in this nucleus is much less pronounced than in neighboring odd nuclei, the residual interaction mixes the configurations so that a single broad absorption maximum is formed.

## VI. Concluding Remarks

So far, we have neglected the residual interaction and considered the supermultiplet properties of the GDR in the diagonal approximation. The residual interaction will naturally destroy, to a certain extent, the supermultiplet structure of nuclear states. However, numerous calculations of the GDR in $1p$-shell nucleiin the BSM ($1\hbar\omega$) approximation have shown that the supermultiplet gross structure of the GDR is nevertheless retained. Table 2 shows the weights of the dominant components of the wave functions of the ground states of the $1p$-shell nuclei. As seen, we can speak with reasonable accuracy of the manifestation of a supermultiplet structure.

*Table 2:* Weights of the dominant components in the wave function of the ground state of the$1p$-shell nuclei in the $LS$-representation (a variant of the Hamiltonian with Rosenfeld forces)

| Nucleus | Dominant component $\{f\}^{2T+1\ 2S+1}L_J$ | Weight, % | Nucleus | Dominant component $\{f\}^{2T+1\ 2S+1}L_J$ | Weight, % |
|---|---|---|---|---|---|
| $^{7}$Li | $\{43\}^{22}P_{3/2}$ | 97 | $^{12}$C | $\{444\}^{11}S_0$ | 71 |
| $^{8}$Be | $\{44\}^{11}S_0$ | 97 | $^{13}$C | $\{4441\}^{22}P_{1/2}$ | 64 |
| $^{9}$Be | $\{441\}^{22}P_{3/2}$ | 81 | $^{14}$N | $\{4442\}^{13}D_1$ | 90 |
| $^{10}$B | $\{442\}^{13}D_3$ | 64 | $^{14}$C | $\{4442\}^{31}S_0$<br>$\{4433\}^{33}P_0$ | 56<br>44 |
| $^{11}$B | $\{443\}^{22}P_{3/2}$<br>$\{443\}^{22}D_{3/2}$ | 41<br>32 | $^{15}$N | $\{4443\}^{22}P_{1/2}$ | 100 |

In conclusion, we note that in the nuclei of the $1d2s$-shell, the supermultiplet effects of the GDR are significantly reduced, and they can no longer be clearly distinguished. This is due to both an increase in the role of the spin-orbit interaction and a decrease in the intensity of the monopole part of the Majorana forces.

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### Get a remuneration of 15% of author fees

Fellow members are eligible to join as a paid peer reviewer at Global Journals Incorporation (USA) and can get a remuneration of 15% of author fees, taken from the author of a respective paper.

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### Become a member of the Editorial Board

Fellows may join as a member of the Editorial Board of Global Journals Incorporation (USA) after successful completion of three years as Fellow and as Peer Reviewer. Additionally, Fellows get a chance to nominate other members for Editorial Board.

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## And Much More

### Get access to scientific museums and observatories across the globe

All members get access to 5 selected scientific museums and observatories across the globe. All researches published with Global Journals will be kept under deep archival facilities across regions for future protections and disaster recovery. They get 10 GB free secure cloud access for storing research files.

## ASSOCIATE OF SCIENCE FRONTIER RESEARCH COUNCIL

ASSOCIATE OF SCIENCE FRONTIER RESEARCH COUNCIL is the membership of Global Journals awarded to individuals that the Open Association of Research Society judges to have made a 'substantial contribution to the improvement of computer science, technology, and electronics engineering.

The primary objective is to recognize the leaders in research and scientific fields of the current era with a global perspective and to create a channel between them and other researchers for better exposure and knowledge sharing. Members are most eminent scientists, engineers, and technologists from all across the world. Associate membership can later be promoted to Fellow Membership. Associates are elected for life through a peer review process on the basis of excellence in the respective domain. There is no limit on the number of new nominations made in any year. Each year, the Open Association of Research Society elect up to 12 new Associate Members.

## To the institution

### Get letter of appreciation

Global Journals sends a letter of appreciation of author to the Dean or CEO of the University or Company of which author is a part, signed by editor in chief or chief author.

## Exclusive Network

### Get access to a closed network

A ASFRC member gets access to a closed network of Tier 1 researchers and scientists with direct communication channel through our website. Associates can reach out to other members or researchers directly. They should also be open to reaching out by other.

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## Certificate

### Receive a print ed copy of a certificate

Associates receive a printed copy of a certificate signed by our Chief Author that may be used for academic purposes and a personal recommendation letter to the dean of member's university.

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## Designation

### Get honored title of membership

Associates can use the honored title of membership. The “ASFRC” is an honored title which is accorded to a person’s name viz. Dr. John E. Hall, Ph.D., ASFRC or William Walldroff, M.S., ASFRC.

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## Recognition on the Platform

### Better visibility and citation

All the Associate members of ASFRC get a badge of "Leading Member of Global Journals" on the Research Community that distinguishes them from others. Additionally, the profile is also partially maintained by our team for better visibility and citation. All associates get a dedicated page on the website with their biography.

Career | Credibility | Reputation

## Future Work

### Get discounts on the future publications

Associates receive discounts on the future publications with Global Journals up to 60%. Through our recommendation programs, members also receive discounts on publications made with OARS affiliated organizations.

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## GJ Internal Account

### Unlimited forward of Emails

Associates get secure and fast GJ work emails with unlimited forward of emails that they may use them as their primary email. For example,
john [AT] globaljournals [DOT] org.

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## Premium Tools

### Access to all the premium tools

To take future researches to the zenith, fellows receive access to almost all the premium tools that Global Journals have to offer along with the partnership with some of the best marketing leading tools out there.

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## Conferences & Events

### Organize seminar/conference

Associates are authorized to organize symposium/seminar/conference on behalf of Global Journal Incorporation (USA). They can also participate in the same organized by another institution as representative of Global Journal. In both the cases, it is mandatory for him to discuss with us and obtain our consent. Additionally, they get free research conferences (and others) alerts.

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## Early Invitations

### Early invitations to all the symposiums, seminars, conferences

All associates receive the early invitations to all the symposiums, seminars, conferences and webinars hosted by Global Journals in their subject.

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## Publishing Articles & Books

### Earn 30-40% of sales proceeds

Associates can publish articles (limited) without any fees. Also, they can earn up to 30-40% of sales proceeds from the sale of reference/review books/literature/publishing of research paper.

Exclusive | Financial

## Reviewers

### Get a remuneration of 15% of author fees

Associate members are eligible to join as a paid peer reviewer at Global Journals Incorporation (USA) and can get a remuneration of 15% of author fees, taken from the author of a respective paper.

Financial

## And Much More

### Get access to scientific museums and observatories across the globe

All members get access to 2 selected scientific museums and observatories across the globe. All researches published with Global Journals will be kept under deep archival facilities across regions for future protections and disaster recovery. They get 5 GB free secure cloud access for storing research files.

| Associate | Fellow | Research Group | Basic |
|---|---|---|---|
| $4800<br>**lifetime designation** | $6800<br>**lifetime designation** | $12500.00<br>**organizational** | APC<br>**per article** |
| **Certificate,** LoR and Momento<br>**2** discounted publishing/year<br>**Gradation** of Research<br>**10** research contacts/day<br>**1 GB** Cloud Storage<br>**GJ** Community Access | **Certificate,** LoR and Momento<br>**Unlimited** discounted publishing/year<br>**Gradation** of Research<br>**Unlimited** research contacts/day<br>**5 GB** Cloud Storage<br>**Online Presense** Assistance<br>**GJ** Community Access | **Certificates,** LoRs and Momentos<br>**Unlimited** free publishing/year<br>**Gradation** of Research<br>**Unlimited** research contacts/day<br>**Unlimited** Cloud Storage<br>**Online Presense** Assistance<br>**GJ** Community Access | **GJ** Community Access |

# Preferred Author Guidelines

**We accept the manuscript submissions in any standard (generic) format.**

We typeset manuscripts using advanced typesetting tools like Adobe In Design, CorelDraw, TeXnicCenter, and TeXStudio. We usually recommend authors submit their research using any standard format they are comfortable with, and let Global Journals do the rest.

Alternatively, you can download our basic template from https://globaljournals.org/Template.zip

Authors should submit their complete paper/article, including text illustrations, graphics, conclusions, artwork, and tables. Authors who are not able to submit manuscript using the form above can email the manuscript department at submit@globaljournals.org or get in touch with chiefeditor@globaljournals.org if they wish to send the abstract before submission.

## Before and during Submission

Authors must ensure the information provided during the submission of a paper is authentic. Please go through the following checklist before submitting:

1. Authors must go through the complete author guideline and understand and *agree to Global Journals' ethics and code of conduct,* along with author responsibilities.
2. Authors must accept the privacy policy, terms, and conditions of Global Journals.
3. Ensure corresponding author's email address and postal address are accurate and reachable.
4. Manuscript to be submitted must include keywords, an abstract, a paper title, co-author(s') names and details (email address, name, phone number, and institution), figures and illustrations in vector format including appropriate captions, tables, including titles and footnotes, a conclusion, results, acknowledgments and references.
5. Authors should submit paper in a ZIP archive if any supplementary files are required along with the paper.
6. Proper permissions must be acquired for the use of any copyrighted material.
7. Manuscript submitted *must not have been submitted or published elsewhere* and all authors must be aware of the submission.

**Declaration of Conflicts of Interest**

It is required for authors to declare all financial, institutional, and personal relationships with other individuals and organizations that could influence (bias) their research.

## Policy on Plagiarism

Plagiarism is not acceptable in Global Journals submissions at all.

Plagiarized content will not be considered for publication. We reserve the right to inform authors' institutions about plagiarism detected either before or after publication. If plagiarism is identified, we will follow COPE guidelines:

Authors are solely responsible for all the plagiarism that is found. The author must not fabricate, falsify or plagiarize existing research data. The following, if copied, will be considered plagiarism:

- Words (language)
- Ideas
- Findings
- Writings
- Diagrams
- Graphs
- Illustrations
- Lectures

- Printed material
- Graphic representations
- Computer programs
- Electronic material
- Any other original work

## Authorship Policies

Global Journals follows the definition of authorship set up by the Open Association of Research Society, USA. According to its guidelines, authorship criteria must be based on:

1. Substantial contributions to the conception and acquisition of data, analysis, and interpretation of findings.
2. Drafting the paper and revising it critically regarding important academic content.
3. Final approval of the version of the paper to be published.

**Changes in Authorship**

The corresponding author should mention the name and complete details of all co-authors during submission and in manuscript. We support addition, rearrangement, manipulation, and deletions in authors list till the early view publication of the journal. We expect that corresponding author will notify all co-authors of submission. We follow COPE guidelines for changes in authorship.

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**Appealing Decisions**

Unless specified in the notification, the Editorial Board's decision on publication of the paper is final and cannot be appealed before making the major change in the manuscript.

**Acknowledgments**

Contributors to the research other than authors credited should be mentioned in Acknowledgments. The source of funding for the research can be included. Suppliers of resources may be mentioned along with their addresses.

**Declaration of funding sources**

Global Journals is in partnership with various universities, laboratories, and other institutions worldwide in the research domain. Authors are requested to disclose their source of funding during every stage of their research, such as making analysis, performing laboratory operations, computing data, and using institutional resources, from writing an article to its submission. This will also help authors to get reimbursements by requesting an open access publication letter from Global Journals and submitting to the respective funding source.

## Preparing your Manuscript

Authors can submit papers and articles in an acceptable file format: MS Word (doc, docx), LaTeX (.tex, .zip or .rar including all of your files), Adobe PDF (.pdf), rich text format (.rtf), simple text document (.txt), Open Document Text (.odt), and Apple Pages (.pages). Our professional layout editors will format the entire paper according to our official guidelines. This is one of the highlights of publishing with Global Journals—authors should not be concerned about the formatting of their paper. Global Journals accepts articles and manuscripts in every major language, be it Spanish, Chinese, Japanese, Portuguese, Russian, French, German, Dutch, Italian, Greek, or any other national language, but the title, subtitle, and abstract should be in English. This will facilitate indexing and the pre-peer review process.

The following is the official style and template developed for publication of a research paper. Authors are not required to follow this style during the submission of the paper. It is just for reference purposes.

***Manuscript Style Instruction (Optional)***

- Microsoft Word Document Setting Instructions.
- Font type of all text should be Swis721 Lt BT.
- Page size: 8.27" x 11'", left margin: 0.65, right margin: 0.65, bottom margin: 0.75.
- Paper title should be in one column of font size 24.
- Author name in font size of 11 in one column.
- Abstract: font size 9 with the word "Abstract" in bold italics.
- Main text: font size 10 with two justified columns.
- Two columns with equal column width of 3.38 and spacing of 0.2.
- First character must be three lines drop-capped.
- The paragraph before spacing of 1 pt and after of 0 pt.
- Line spacing of 1 pt.
- Large images must be in one column.
- The names of first main headings (Heading 1) must be in Roman font, capital letters, and font size of 10.
- The names of second main headings (Heading 2) must not include numbers and must be in italics with a font size of 10.

***Structure and Format of Manuscript***

The recommended size of an original research paper is under 15,000 words and review papers under 7,000 words. Research articles should be less than 10,000 words. Research papers are usually longer than review papers. Review papers are reports of significant research (typically less than 7,000 words, including tables, figures, and references)

A research paper must include:

a) A title which should be relevant to the theme of the paper.
b) A summary, known as an abstract (less than 150 words), containing the major results and conclusions.
c) Up to 10 keywords that precisely identify the paper's subject, purpose, and focus.
d) An introduction, giving fundamental background objectives.
e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition, sources of information must be given, and numerical methods must be specified by reference.
f) Results which should be presented concisely by well-designed tables and figures.
g) Suitable statistical data should also be given.
h) All data must have been gathered with attention to numerical detail in the planning stage.

Design has been recognized to be essential to experiments for a considerable time, and the editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned unrefereed.

i) Discussion should cover implications and consequences and not just recapitulate the results; conclusions should also be summarized.
j) There should be brief acknowledgments.
k) There ought to be references in the conventional format. Global Journals recommends APA format.

Authors should carefully consider the preparation of papers to ensure that they communicate effectively. Papers are much more likely to be accepted if they are carefully designed and laid out, contain few or no errors, are summarizing, and follow instructions. They will also be published with much fewer delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and suggestions to improve brevity.

## Format Structure

***It is necessary that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.***

All manuscripts submitted to Global Journals should include:

**Title**

The title page must carry an informative title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) where the work was carried out.

**Author details**

The full postal address of any related author(s) must be specified.

**Abstract**

The abstract is the foundation of the research paper. It should be clear and concise and must contain the objective of the paper and inferences drawn. It is advised to not include big mathematical equations or complicated jargon.

Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

**Keywords**

A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy: planning of a list of possible keywords and phrases to try.

Choice of the main keywords is the first tool of writing a research paper. Research paper writing is an art. Keyword search should be as strategic as possible.

One should start brainstorming lists of potential keywords before even beginning searching. Think about the most important concepts related to research work. Ask, “What words would a source have to include to be truly valuable in a research paper?” Then consider synonyms for the important words.

It may take the discovery of only one important paper to steer in the right keyword direction because, in most databases, the keywords under which a research paper is abstracted are listed with the paper.

**Numerical Methods**

Numerical methods used should be transparent and, where appropriate, supported by references.

**Abbreviations**

Authors must list all the abbreviations used in the paper at the end of the paper or in a separate table before using them.

**Formulas and equations**

Authors are advised to submit any mathematical equation using either MathJax, KaTeX, or LaTeX, or in a very high-quality image.

**Tables, Figures, and Figure Legends**

Tables: Tables should be cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g., Table 4, a self-explanatory caption, and be on a separate sheet. Authors must submit tables in an editable format and not as images. References to these tables (if any) must be mentioned accurately.

**Figures**

Figures are supposed to be submitted as separate files. Always include a citation in the text for each figure using Arabic numbers, e.g., Fig. 4. Artwork must be submitted online in vector electronic form or by emailing it.

## Preparation of Eletronic Figures for Publication

Although low-quality images are sufficient for review purposes, print publication requires high-quality images to prevent the final product being blurred or fuzzy. Submit (possibly by e-mail) EPS (line art) or TIFF (halftone/ photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Avoid using pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings). Please give the data for figures in black and white or submit a Color Work Agreement form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution at final image size ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs): >350 dpi; figures containing both halftone and line images: >650 dpi.

Color charges: Authors are advised to pay the full cost for the reproduction of their color artwork. Hence, please note that if there is color artwork in your manuscript when it is accepted for publication, we would require you to complete and return a Color Work Agreement form before your paper can be published. Also, you can email your editor to remove the color fee after acceptance of the paper.

## Tips for Writing a Good Quality Science Frontier Research Paper

Techniques for writing a good quality Science Frontier Research paper:

***1. Choosing the topic:*** In most cases, the topic is selected by the interests of the author, but it can also be suggested by the guides. You can have several topics, and then judge which you are most comfortable with. This may be done by asking several questions of yourself, like "Will I be able to carry out a search in this area? Will I find all necessary resources to accomplish the search? Will I be able to find all information in this field area?" If the answer to this type of question is "yes," then you ought to choose that topic. In most cases, you may have to conduct surveys and visit several places. Also, you might have to do a lot of work to find all the rises and falls of the various data on that subject. Sometimes, detailed information plays a vital role, instead of short information. Evaluators are human: The first thing to remember is that evaluators are also human beings. They are not only meant for rejecting a paper. They are here to evaluate your paper. So present your best aspect.

***2. Think like evaluators:*** If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

***3. Ask your guides:*** If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

***4. Use of computer is recommended:*** As you are doing research in the field of science frontier then this point is quite obvious. Use right software: Always use good quality software packages. If you are not capable of judging good software, then you can lose the quality of your paper unknowingly. There are various programs available to help you which you can get through the internet.

***5. Use the internet for help:*** An excellent start for your paper is using Google. It is a wondrous search engine, where you can have your doubts resolved. You may also read some answers for the frequent question of how to write your research paper or find a model research paper. You can download books from the internet. If you have all the required books, place importance on reading, selecting, and analyzing the specified information. Then sketch out your research paper. Use big pictures: You may use encyclopedias like Wikipedia to get pictures with the best resolution. At Global Journals, you should strictly follow here.

***6. Bookmarks are useful:*** When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

***7. Revise what you wrote:*** When you write anything, always read it, summarize it, and then finalize it.

***8. Make every effort:*** Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

***9. Produce good diagrams of your own:*** Always try to include good charts or diagrams in your paper to improve quality. Using several unnecessary diagrams will degrade the quality of your paper by creating a hodgepodge. So always try to include diagrams which were made by you to improve the readability of your paper. Use of direct quotes: When you do research relevant to literature, history, or current affairs, then use of quotes becomes essential, but if the study is relevant to science, use of quotes is not preferable.

***10. Use proper verb tense:*** Use proper verb tenses in your paper. Use past tense to present those events that have happened. Use present tense to indicate events that are going on. Use future tense to indicate events that will happen in the future. Use of wrong tenses will confuse the evaluator. Avoid sentences that are incomplete.

***11. Pick a good study spot:*** Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

***12. Know what you know:*** Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

***13. Use good grammar:*** Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice.

Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

***14. Arrangement of information:*** Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

***15. Never start at the last minute:*** Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

***16. Multitasking in research is not good:*** Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

***17. Never copy others' work:*** Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

***18. Go to seminars:*** Attend seminars if the topic is relevant to your research area. Utilize all your resources.

***19. Refresh your mind after intervals:*** Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

***20. Think technically:*** Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

***21. Adding unnecessary information:*** Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

***22. Report concluded results:*** Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

***23. Upon conclusion:*** Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

## Informal Guidelines of Research Paper Writing

***Key points to remember:***

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

**Final points:**

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

*The introduction:* This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

**The discussion section:**

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

**General style:**

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

**To make a paper clear:** Adhere to recommended page limits.

*Mistakes to avoid:*

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

**Title page:**

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

**Abstract:** This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

*Reason for writing the article—theory, overall issue, purpose.*

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

**Approach:**

- Single section and succinct.
- An outline of the job done is always written in past tense.
- Concentrate on shortening results—limit background information to a verdict or two.
- Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

**Introduction:**

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

*The following approach can create a valuable beginning:*

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

**Approach:**

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

**Procedures (methods and materials):**

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

**Materials:**

*Materials may be reported in part of a section or else they may be recognized along with your measures.*

**Methods:**

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

**Approach:**

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

**What to keep away from:**

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

**Results:**

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

**Content:**

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

**What to stay away from:**

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

**Approach:**

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

**Figures and tables:**

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

**Discussion:**

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- Give details of all of your remarks as much as possible, focusing on mechanisms.
- Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

**Approach:**

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

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|---|---|---|---|
| | **A-B** | **C-D** | **E-F** |
| ***Abstract*** | Clear and concise with appropriate content, Correct format. 200 words or below | Unclear summary and no specific data, Incorrect form<br><br>Above 200 words | No specific data with ambiguous information<br><br>Above 250 words |
| ***Introduction*** | Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited | Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter | Out of place depth and content, hazy format |
| ***Methods and Procedures*** | Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads | Difficult to comprehend with embarrassed text, too much explanation but completed | Incorrect and unorganized structure with hazy meaning |
| ***Result*** | Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake | Complete and embarrassed text, difficult to comprehend | Irregular format with wrong facts and figures |
| ***Discussion*** | Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited | Wordy, unclear conclusion, spurious | Conclusion is not cited, unorganized, difficult to comprehend |
| ***References*** | Complete and correct format, well organized | Beside the point, Incomplete | Wrong format and structuring |

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