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Highlights

- Minimum Values for Mass
- Quantum Mechanical Features
- Hybrid Quaternions of Pell
- Geometric Algebra Formalism

Discovering Thoughts, Inventing Future

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A Dimensionless Equation Linking the Maximum and Minimum Values for Mass (or Energy) and Time (or Length)

By Andre P. Steynberg

Abstract- The James Webb telescope can detect photons in the infrared wavelength from cosmic events with a known intensity and emission frequency. The redshift data from such measurements for photons from the early universe can be used to determine the dimensions of a finite spacetime manifold. Such measurements would differ from predictions using the assumption that space and time are infinite. If these measurements confirm that the spacetime manifold is finite, then there exists a dimensionless ratio between the maximum and minimum dimensions for space and time. Furthermore, the Schwarzschild equation can be used to calculate the total mass of the Universe to determine a dimensionless ratio for the maximum and minimum mass values. The resulting equation derivation is a tribute to the work of Albert Einstein, Alexander Friedmann, Karl Schwarzschild, and Max Planck. The derived dimensionless relationship is $M_u/m_P = t_m/\pi t_P = \ell_m/\pi \ell_P$.

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ADIMENSIONLESSEQUATIONLINKINGTHEMAXIMUMANDMINIMUMVALUESFORMASSORENERGYANDTIMEORLENGTH

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A Dimensionless Equation Linking the Maximum and Minimum Values for Mass (or Energy) and Time (or Length)

Andre P. Steynberg

Abstract- The James Webb telescope can detect photons in the infrared wavelength from cosmic events with a known intensity and emission frequency. The redshift data from such measurements for photons from the early universe can be used to determine the dimensions of a finite spacetime manifold. Such measurements would differ from predictions using the assumption that space and time are infinite. If these measurements confirm that the spacetime manifold is finite, then there exists a dimensionless ratio between the maximum and minimum dimensions for space and time. Furthermore, the Schwarzschild equation can be used to calculate the total mass of the Universe to determine a dimensionless ratio for the maximum and minimum mass values. The resulting equation derivation is a tribute to the work of Albert Einstein, Alexander Friedmann, Karl Schwarzschild, and Max Planck. The derived dimensionless relationship is $M_u/m_P = t_m/\pi t_P = \ell_m/\pi\ell_P$

I. Introduction

Alexander Friedmann worked out a solution to Einstein's field equations which applies to the entire Universe. A specific solution to the Friedmann equation applies to a finite, symmetrical Universe. For this solution, the entire mass of the Universe, M_u, can be related to the maximum radius of the Universe, R_m, by a version of the Schwarzschild equation, $R_m = 2GM_u/c^2$. Where c is the speed of light and G is the universal gravitational constant.

If the spacetime manifold has a constant finite structure, the maximum space dimension radius is reached when space ceases to expand. When expansion ceases, the Schwarzschild equation is then a solution to the Friedmann equation. A single parameter model has been proposed to predict photon redshift as a function of time, consistent with curved spacetime[1]. So, this theory for the structure of spacetime is falsifiable according to the scientific method. Data from the James Webb telescope, which can detect photons in the infrared wavelength range, is the type of equipment required to test this hypothesis.

Max Planck, a German physicist, is best known as the originator of the quantum theory of energy for which he was awarded the Nobel Prize in 1918. Today, the standard model of particle physics describes the smallest entities which exist. Although these entities are referred to as particles, it is better to think of them as packets of energy.

So, at both ends of the scale, from small to large, there is a finite metric. Nothing real is infinite. Everything with mass consists of finite packets of energy in a finite spacetime manifold.

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In particle physics and physical cosmology, Planck units are a set of units of measurement defined exclusively in terms of four universal physical constants, in such a manner that these physical constants take on the numerical value of 1 when expressed in terms of these units.

Originally proposed in 1899 by Max Planck, these units are a system of natural units because the origin of their definition comes only from properties of nature.

The four universal constants used to define the Planck units are:

- The speed of light in a vacuum, c
- The gravitational constant, G
- The reduced Planck constant, ħ (Originally h was used which differs from ħ by a factor of $(2\pi)^{½}$.)
- The Boltzmann constant, kB (which is used to give natural units for temperature)

The mass of a packet of energy with a wavelength equal to the Planck length is:

$$m_P = (\hbar c/G)^{½}.$$

The Planck length, denoted ℓ_P, is a unit of length defined as:

$$\ell_P = (\hbar G/c^3)^{½}$$

Planck time $t_P$is the time required for light to travel 1 Planck length in a vacuum,

$$t_P = (\hbar G/c^5)^{½} = Gm_P/c^3.$$

II. Equation Derivation

At $t = t_m$, when the space dimensions reach their maximum size, the time dimension and the space dimension are equal. (The referenced publication uses the symbol T for t_m). This assumes that the spacetime structure is symmetrical so that the curvature of the time dimension is the same as the curvature of the space dimensions. Applying some simple spherical geometry to this structure, $2\pi R_m = 4ct_m$ so that $R_m = 2ct_m/\pi$. When substituting t_m for R_m in the Schwarzschild equation, the result is $t_m = \pi GM_u/c^3$.

Dividing the maximum time, t_m, by Planck time, t_p, while the other side of the above equation is also divided by t_P using the relation $t_P = Gm_P/c^3$(which is obtained from the definitions of t_P and m_P, the Planck mass) gives the result:

$$t_m/t_P = \pi M_u/m_P$$

This can be rearranged to $M_u/m_P = t_m/\pi t_P = \ell_m/\pi\ell_P$

The length metric denoted by ℓ_m is the maximum possible distance that a photon can be separated from its source mass, and, like t_m, it is a quarter of the circumference of the spacetime manifold.

Note that $M_u/m_P = E_u/E_p$ where E_u is the total energy in the Universe contained in entities with mass and E_p is the Planck energy.

Any fundamental entity with mass has an associated Schwarzschild radius. This means that analogous equations apply if the mass of the Universe, M_u, is replaced by the mass of a black hole or any of the mass-containing fundamental entities in the standard model of particle physics. So, these fundamental entities are spherical packets of energy (in four dimensions)in a spacetime manifold. The distortion of spacetime by mass orthogonal to the manifold is observed as gravity. Therefore, gravity is the

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additive result from the summation of the distortions from the individual contributions by all the fundamental mass-containing entities.

III. Conclusion

If observations from the James Webb telescope do validate the single parameter model proposed to predict photon redshift as a function of time, then it can be concluded that there is a simple dimensionless representation relating mass (or energy) to time (or length) at the maximum possible scale. This relationship is derived using the definitions for the Planck units which are used to describe entities at the smallest possible scale. Nature would then be consistent for all entities with mass at any scale.

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Hybrid Quaternions of Pell and Jacobsthal

By M. C. Dos S. Mangueira, R. P. M. Vieira, F. R. V. Alves & P. M. M. C. Catarino

University of Trás-os-Montes

Abstract- Knowing that the Pell and Jacobsthal sequences are second-order linear recursive sequences and that they have similarities between them, this study aims to explore these sequences. Thus, an investigation will be carried out on the Pell and Jacobsthal numbers based on the hybrid numbers and their quaternions. In this way, it will be presented as a great among these hybrid themes of Pell and will be presented as a formula of hybrids of Pell Jacobsthal and will be presented, transforming function and its extension to the indices.

Keywords: pell sequence, jacobsthal sequence, pell and jacobsthal hybrid quaternions, hybrid numbers.

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HYBRIDQUATERNIONSOFPELLANDJACOBSTHAL

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Hybrid Quaternions of Pell and Jacobsthal

M. C. Dos S. Mangueira α, R. P. M. Vieira σ, F. R. V. Alves ρ & P. M. M. C. Catarino ω

Abstract- Knowing that the Pell and Jacobsthal sequences are second-order linear recursive sequences and that they have similarities between them, this study aims to explore these sequences. Thus, an investigation will be carried out on the Pell and Jacobsthal numbers based on the hybrid numbers and their quaternions. In this way, it will be presented as a great among these hybrid themes of Pell and will be presented as a formula of hybrids of Pell Jacobsthal and will be presented, transforming function and its extension to the indices.

Keywords: pell sequence, jacobsthal sequence, pell and jacobsthal hybrid quaternions, hybrid numbers.

I. Introduction

In this research we will address two second-order linear recursive sequences, they are: Pell and Jacobsthal. The Pell sequence came from the English mathematician John Pell (1611 - 1685), known for being one of the most enigmatic mathematicians of the century XVII. The Jacobsthal sequence is named after the mathematician Ernest Erich Jacobsthal (1882-1965), specialist in Number Theory.

These sequences have similarity in their recurrence, the Pell sequence is defined by:

$$Pe_{n+1} = 2Pe_n + Pe_{n-1}, n \geq 2, \tag{1.1}$$

being $Pe_0 = 0$ $Pe_1 = 1$ its initial conditions.

As for the Jacobsthal sequence, it is presented by the recurrence:

$$J_{n+1} = J_n + 2J_{n-1}, n \geq 2, \tag{1.2}$$

being $J_0 = 0$ $J_1 = 1$ its initial conditions.

On the other hand, we have the quaternions numbers, it is believed that the quaternions arose in an attempt to transform the complex number $z = a + bi$ in three dimensions [1]. Quaternions are presented as formal sums of scalars with usual vectors of three-dimensional space, existing four dimensions and is described by: $q = a+bi+cj+dk$, where a, b, c and d are real numbers and i, j, k the orthogonal part at the base $\mathbf{R}^3$. In the work of [3] the authors presents the quaternionic product as $i^2 = j^2 = k^2 = -1, ij = k = -ij, jk = i = -kj$ and $ki = j = -ik$.

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On the other hand, there are the hybrid numbers, presented initially by [4], which he studied and lists three sets of numbers, they are: the complex, hyperbolic and dual. A hybrid number is defined as:

$$\mathbf{K} = \{z = a + bi + c\varepsilon + dh : a, b, c, d \in \mathbf{R}, i^2 = -1, \varepsilon^2 = 0, h^2 = 1, ih = -hi = \varepsilon + i\}.$$

The hybridization of linear sequences consists of relating the hybrid numbers with linear sequences, researches around this hybridization are found in the mathematical literature, as we can find in [10, 11, 12, 13, 14, 15, 16]. Regarding the study on quaternions, we find work on the Padovan and Perrin quaternions in [5], Pell-Padovan in [6] and Fibonacci and Fibonacci Complexes in [2, 7, 8, 9].

Finally, in this research, we take as a basis the works of [17, 18] to present, in the following sections, the hybrid quaternions of Pell and Jacobsthal.

II. Pell Hybrid Quaternions Numbers

In this section, we will present Pell's hybrid quaternions based on the work of [17, 18].

Definition 2.1. Pell's hybrid number, denoted by HPe_n, is defined by:

$$HPe_n = Pe_n + Pe_{n+1}i + Pe_{n+2}\varepsilon + Pe_{n+3}h.$$

Definition 2.2. The Recurrence Relationship for Pell's Hybrids, $n \geq 2$, is defined by:

$$HPe_{n+1} = 2HPe_n + HPe_{n-1}, \tag{2.1}$$

with $HPe_0 = i + 2\varepsilon + 5h$ and $HPe_1 = 1 + 2i + 5\varepsilon + 12h$ its initial terms.

Definition 2.3. Pell's quaternion number, denoted by QPe_n, it is given by:

$$QPe_n = Pe_n + Pe_{n+1}i + Pe_{n+2}j + Pe_{n+3}k.$$

Definition 2.4. The recurrence relation for Pell's quaternions, $n \geq 2$, is defined by:

$$QPe_{n+1} = 2QPe_n + QPe_{n-1}, \tag{2.2}$$

with $QPe_0 = i + 2j + 5k$ and $QPe_1 = 1 + 2i + 5j + 12k$ its initial terms.

Now, from what was seen above, we will approach Pell's hybrid quaternions.

Definition 2.5. Pell's hybrid quaternion number is defined as:

$$\tilde{Pe}_n = HPe_n + HPe_{n+1}i + HPe_{n+2}j + HPe_{n+3}k.$$

where i, j, k are the units of the quaternions and HPe_n it's the n-th Pell hybrid number. Thus, Pell's hybrid quaternions can be rewritten by:

$$\begin{aligned}
\tilde{Pe}_n &= (Pe_n + Pe_{n+1}i + Pe_{n+2}\varepsilon + Pe_{n+3}h) + \\
&\quad (Pe_{n+1} + Pe_{n+2}i + Pe_{n+3}\varepsilon + Pe_{n+4}h)i + \\
&\quad (Pe_{n+2} + Pe_{n+3}i + Pe_{n+4}\varepsilon + Pe_{n+5}h)j + \\
&\quad (Pe_{n+3} + Pe_{n+4}i + Pe_{n+5}\varepsilon + Pe_{n+6}h)k \\
&= \widehat{HPe}_n + \widehat{HPe}_{n+1}i + \widehat{HPe}_{n+2}\varepsilon + \widehat{HPe}_{n+3}h.
\end{aligned}$$

where i, ε and h are the imaginary units of the hybrid numbers and $\widehat{HPe}_n = Pe_n + Pe_{n+1}i + Pe_{n+2}j + Pe_{n+3}k$.

Ref

6. D. Tasci, "Padovan and Pell-Padovan quaternions," *Journal of Science and Arts*, vol. 42, no. 1, pp. 125-132, 2018.

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Definition 2.6. The recurrence relationship for Pell's hybrid quaternions, $n \geq 2$, is defined by:

$$\tilde{Pe}_{n+1} = 2\tilde{Pe}_n + \tilde{Pe}_{n-1}, \tag{2.3}$$

with the following initial terms: $\tilde{Pe}_0 = HPe_0 + HPe_1 i + HPe_2 j + HPe_3 k$ and $\tilde{Pe}_1 = HPe_1 + HPe_2 i + HPe_3 j + HPe_4 k$.

And yet, extending these non-positive integer indices, we have:

Definition 2.7. The recurrence relation for the hybrid quaternions of non-positive Pell indices, $n \geq 0$, is defined by:

$$\tilde{Pe}_{-n} = \tilde{Pe}_{-n+2} - 2tildePe_{-n+1}.$$

Theorem 2.1. Pell's hybrid quaternion, $\tilde{Pe}_n$, satisfies the following recurrence:

$$\tilde{Pe}_{n+1} = 2\tilde{Pe}_n + \tilde{Pe}_{n-1}. \tag{2.4}$$

Proof.

$$\begin{aligned}
2\tilde{Pe}_n + \tilde{Pe}_{n-1} &= 2HPe_n + 2HPe_{n+1}i + 2HPe_{n+2}j + 2HPe_{n+3}k + \\
&\quad HPe_{n-1} + HPe_n i + HPe_{n+1}j + HPe_{n+2}k \\
&= (2HPe_n + HPe_{n-1}) + (2HPe_{n+1} + HPe_n)i + \\
&\quad (2HPe_{n+2} + HPe_{n+1})j + (2HPe_{n+3} + HPe_{n+2})k \\
&= HPe_{n+1} + HPe_{n+2}i + HPe_{n+3}j + HPe_{n+4}k \\
&= \tilde{Pe}_{n+1}.
\end{aligned}$$

According to the recurrence relationship, $\tilde{Pe}_{n+1} = 2\tilde{Pe}_n + \tilde{Pe}_{n-1}$, one can present its characteristic equation, defined by $x^2 - 2x - 1 = 0$ where is a quadratic equation having two real roots $x_1 = 1 + \sqrt{2}$ and $x_2 = 1 - \sqrt{2}$.

Definition 2.8. Pell's hybrid quaternion conjugate can be defined in three different types for $\tilde{Pe}_n = \widehat{HPe}_n + \widehat{HPe}_{n+1}i + \widehat{HPe}_{n+2}\varepsilon + \widehat{HPe}_{n+3}h$:

- *Quaternion conjugate, $\overline{\tilde{Pe}_n}$: $\overline{\tilde{Pe}_n} = \overline{\widehat{HPe}_n} + \overline{\widehat{HPe}_{n+1}}i + \overline{\widehat{HPe}_{n+2}}\varepsilon + \overline{\widehat{HPe}_{n+3}}h$;*
- *Hybrid conjugate, $(\tilde{Pe}_n)^C$: $(\tilde{Pe}_n)^C = \widehat{HPe}_n - \widehat{HPe}_{n+1}i - \widehat{HPe}_{n+2}\varepsilon - \widehat{HPe}_{n+3}h$;*
- *Total conjugate, $(\tilde{Pe}_n)^T$: $(\tilde{Pe}_n)^T = \overline{(\tilde{Pe}_n)^C} = \overline{\widehat{HPe}_n} - \overline{\widehat{HPe}_{n+1}}i - \overline{\widehat{HPe}_{n+2}}\varepsilon - \overline{\widehat{HPe}_n}^{+3}h$.*

Theorem 2.2. The generating function of Pell's hybrid quaternions, denoted by $G_{\tilde{Pe}_n}(x)$, with $n \in \mathbf{N}$, is presented by:

$$G_{\tilde{Pe}_n}(x) = \frac{\tilde{Pe}_0 + (\tilde{Pe}_1 - \tilde{Pe}_0)x}{(1 - 2x - x^2)}.$$

Proof. To define the generating function of Pell's hybrid quaternions we will write a sequence in which each term of the sequence corresponds to the coefficients.

$$G_{\tilde{Pe}_n}(x) = \sum_{n=0}^{\infty} \tilde{Pe}_n x^n.$$

Making algebraic manipulations due to the recurrence relation we can write this sequence as:

$$\begin{aligned}
\tilde{Pe}_n(x) &= \tilde{Pe}_0 + \tilde{Pe}_1 x + \tilde{Pe}_2 x^2 + \cdots + \tilde{Pe}_n x^n + \cdots \\
-2x\tilde{Pe}_n(x) &= -\tilde{Pe}_0 2x - \tilde{Pe}_1 2x^2 - \tilde{Pe}_2 2x^3 - \cdots - \tilde{Pe}_n 2x^{n+1} - \cdots \\
-x^2\tilde{Pe}_n(x) &= -\tilde{Pe}_0 x^2 - \tilde{Pe}_1 x^3 - \tilde{Pe}_2 x^4 - \cdots - \tilde{Pe}_n x^{n+2} - \cdots
\end{aligned}$$

Adding each member, we have:

$$\begin{aligned}
(1-2x-x^2)\tilde{Pe}_n(x) &= \tilde{Pe}_0 + (\tilde{Pe}_1 - \tilde{Pe}_0)x \\
\tilde{Pe}_n(x) &= \frac{\tilde{Pe}_0 + (\tilde{Pe}_1 - \tilde{Pe}_0)x}{1-2x-x^2}
\end{aligned}$$

Theorem 2.3. For $n \geq 0$, we have that the Binet formula for the Pell's hybrid quaternion is given by:

$$\tilde{Pe}_n = \frac{(\tilde{Pe}_1 - \tilde{Pe}_0 x_2)x_1^n - (\tilde{Pe}_1 - \tilde{Pe}_0 x_1)x_2^n}{x_1 - x_2},$$

where x_1 and x_2 are the real roots of the characteristic equation.

Proof. The Binet formula can be represented as follows:

$$\tilde{Pe}_n = Ax_1^n + Bx_2^n.$$

For $n = 0$, there is: $A + B = \tilde{Pe}_0$ a, for $n = 1$, we have $Ax_1 + Bx_2 = \tilde{Pe}_1$.
Thus, we have the following linear system:

$$\begin{cases} A + B = \tilde{Pe}_0 \\ Ax_1 + Bx_2 = \tilde{Pe}_1 \end{cases}$$

Solving the linear system, we have that the coefficients found were: $A = \frac{\tilde{Pe}_1 - \tilde{Pe}_0 x_2}{x_1 - x_2}$ and $B = \frac{\tilde{Pe}_0 x_1 - \tilde{Pe}_1}{x_1 - x_2}$.
Making the appropriate substitutions in the Binet formula, we have:

$$\tilde{Pe}_n = \frac{(\tilde{Pe}_1 - \tilde{Pe}_0 x_2)x_1^n - (\tilde{Pe}_1 - \tilde{Pe}_0 x_1)x_2^n}{x_1 - x_2}.$$

III. Jacobsthal Hybrid Quaternions Numbers

Jacobsthal's hybrid quaternion numbers are defined from below.

Definition 3.1. Jacobsthal's hybrid number, denoted by HJ_n, is defined by:

$$HJ_n = J_n + J_{n+1}i + J_{n+2}\varepsilon + J_{n+3}h.$$

Definition 3.2. The Recurrence Relationship for Jacobsthal's hybrid, $n \geq 2$, is defined by:

$$HJ_{n+1} = HJ_n + 2HJ_{n-1}, \tag{3.1}$$

with $HJ_0 = i + \varepsilon + 3h$ and $HJ_1 = 1 + i + 3\varepsilon + 5h$ its initial terms.

Definition 3.3. Jacobsthal's quaternion number, denoted by QJ_n, it is given by:

$$QJ_n = J_n + J_{n+1}i + J_{n+2}j + J_{n+3}k.$$

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Definition 3.4. The recurrence relation for Jacobsthal's quaternions, $n \geq 2$, is defined by:

$$QJ_{n+1} = QJ_n + 2QJ_{n-1}, \tag{3.2}$$

with $QJ_0 = i + j + 3k$ and $QJ_1 = 1 + i + 3j + 5k$ its initial terms.

Now, from what was seen above, we will approach Jacobsthal's hybrid quaternions.

Definition 3.5. Jacobsthal's hybrid quaternion number is defined as:

$$\tilde{J}_n = HJ_n + HJ_{n+1}i + HJ_{n+2}j + HJ_{n+3}k.$$

where i, j, k are the units of the quaternions and HJ_n it's the n-th Jacobsthal hybrid number. Thus, Jacobsthal's hybrid quaternions can be rewritten by:

$$\begin{aligned}
\tilde{J}_n &= (J_n + J_{n+1}i + J_{n+2}\varepsilon + J_{n+3}h) + \\
&\quad (J_{n+1} + J_{n+2}i + J_{n+3}\varepsilon + J_{n+4}h)i + \\
&\quad (J_{n+2} + J_{n+3}i + J_{n+4}\varepsilon + J_{n+5}h)j + \\
&\quad (J_{n+3} + J_{n+4}i + J_{n+5}\varepsilon + J_{n+6}h)k \\
&= \widehat{HJ}_n + \widehat{HJ}_{n+1}i + \widehat{HJ}_{n+2}\varepsilon + \widehat{HJ}_{n+3}h.
\end{aligned}$$

where i, ε and h are the imaginary units of the hybrid numbers and $\widehat{HJ}_n = J_n + J_{n+1}i + J_{n+2}j + J_{n+3}k$.

Definition 3.6. The recurrence relationship for Jacobsthal's hybrid quaternions, $n \geq 2$, is defined by:

$$\tilde{J}_{n+1} = \tilde{J}_n + 2\tilde{J}_{n-1}, \tag{3.3}$$

with the following initial terms: $\tilde{J}_0 = HJ_0 + HJ_1i + HJ_2j + HJ_3k$ and $\tilde{J}_1 = HJ_1 + HJ_2i + HJ_3j + HJ_4k$.

And yet, extending these non-positive integer indices, we have:

Definition 3.7. The recurrence relation for the hybrid quaternions of non-positive Jacobsthal indices, $n \geq 0$, is defined by:

$$\tilde{J}_{-n} = \frac{\tilde{J}_{-n+2} + \tilde{J}_{-n+1}}{2}.$$

Theorem 3.1. Jacobsthal's hybrid quaternion, $\tilde{J}_n$, satisfies the following recurrence:

$$\tilde{J}_{n+1} = \tilde{J}_n + 2\tilde{J}_{n-1}. \tag{3.4}$$

Proof.

$$\begin{aligned}
\tilde{J}_n + 2\tilde{J}_{n-1} &= HJ_n + HJ_{n+1}i + HJ_{n+2}j + HJ_{n+3}k + \\
&\quad 2HJ_{n-1} + 2HJ_ni + 2HJ_{n+1}j + 2HJ_{n+2}k \\
&= (HJ_n + 2HJ_{n-1}) + (HJ_{n+1} + 2HJ_n)i + \\
&\quad (HJ_{n+2} + 2HJ_{n+1})j + (HJ_{n+3} + 2HJ_{n+2})k \\
&= HJ_{n+1} + HJ_{n+2}i + HJ_{n+3}j + HJ_{n+4}k \\
&= \tilde{J}_{n+1}.
\end{aligned}$$

According to the recurrence relationship, $\tilde{J}_{n+1} = \tilde{J}_n + 2\tilde{J}_{n-1}$, one can present its characteristic equation, defined by $k^2 - k - 2 = 0$ where is a quadratic equation having two real roots $k_1 = 2$ and $k_2 = -1$.

Definition 3.8. Jacobsthal's hybrid quaternion conjugate can be defined in three different types for $\tilde{J}_n = \widehat{HJ}_n + \widehat{HJ}_{n+1}i + \widehat{HJ}_{n+2}\varepsilon + \widehat{HJ}_{n+3}h$:

- *Quaternion conjugate, $\overline{\tilde{J}_n}$: $\overline{\tilde{J}_n} = \overline{\widehat{HJ}_n} + \overline{\widehat{HJ}_{n+1}}i + \overline{\widehat{HJ}_{n+2}}\varepsilon + \overline{\widehat{HJ}_{n+3}}h$;*
- *Hybrid conjugate, $(\tilde{J}_n)^C$: $(\tilde{J}_n)^C = \widehat{HJ}_n - \widehat{HJ}_{n+1}i - \widehat{HJ}_{n+2}\varepsilon - \widehat{HJ}_{n+3}h$;*
- *Total conjugate, $(\tilde{J}_n)^T$: $(\tilde{J}_n)^T = \overline{(\tilde{J}_n)^C} = \overline{\widehat{HJ}_n} - \overline{\widehat{HJ}_{n+1}}i - \overline{\widehat{HJ}_{n+2}}\varepsilon - \overline{\widehat{HJ}_{n+3}}h$.*

Theorem 3.2. The generating function of Jacobsthal's hybrid quaternions, denoted by $G_{\tilde{J}_n}(x)$, with $n \in \mathbf{N}$, is presented by:

$$G_{\tilde{J}_n}(k) = \frac{\tilde{J}_0 + (\tilde{J}_1 - \tilde{J}_0)k}{1 - k - 2k^2}.$$

Proof. To define the generating function of Jacobsthal's hybrid quaternions we will write a sequence in which each term of the sequence corresponds to the coefficients.

$$G_{\tilde{J}_n}(k) = \sum_{n=0}^{\infty} \tilde{J}_n k^n.$$

Making algebraic manipulations due to the recurrence relation we can write this sequence as:

$$\begin{aligned}
G_{\tilde{J}_n}(k) &= \tilde{J}_0 + \tilde{J}_1 k + \tilde{J}_2 k^2 + \cdots + \tilde{J}_n k^n + \cdots \\
-kG_{\tilde{J}_n}(k) &= -k\tilde{J}_0 - k^2\tilde{J}_1 - k^3\tilde{J}_2 - \cdots - k^{n+1}\tilde{J}_n - \cdots \\
-2k^2 G_{\tilde{J}_n}(k) &= -2k^2\tilde{J}_0 - 2k^3\tilde{J}_1 - 2k^4\tilde{J}_2 - \cdots - 2k^{n+2}\tilde{J}_n - \cdots
\end{aligned}$$

Adding each member, we have:

$$\begin{aligned}
(1 - k - 2k^2)G_{\tilde{J}_n}(k) &= \tilde{J}_0 + (\tilde{J}_1 - \tilde{J}_0)k + (\tilde{J}_2 - \tilde{J}_1 - 2\tilde{J}_0)k^2 + \ldots \\
(1 - k - 2k^2)G_{\tilde{J}_n}(k) &= \tilde{J}_0 + (\tilde{J}_1 - \tilde{J}_0)k \\
G_{\tilde{J}_n}(k) &= \frac{\tilde{J}_0 + (\tilde{J}_1 - \tilde{J}_0)k}{(1 - k - k^2)}.
\end{aligned}$$

Theorem 3.3. For $n \geq 0$, we have that the Binet formula for the Jacobsthal's hybrid quaternion is given by:

$$\tilde{J}_n = \frac{\tilde{J}_0 - \tilde{J}_1}{3}k_1^n + \frac{2\tilde{J}_0 - \tilde{J}_1}{3}k_2^n,$$

where k_1 and k_2 are the real roots of the characteristic equation.

Proof. The Binet formula can be represented as follows:

$$\tilde{J}_n = Ax_1^n + Bx_2^n.$$

Notes

Notes

For $n=0$, there is: $A+B=\tilde{J}_0$ a, for $n=1$, we have $Ak_1+Bk_2=\tilde{J}_1$.
Thus, we have the following linear system:

$$\begin{cases} A+B=\tilde{J}_0 \\ Ak_1+Bk_2=\tilde{J}_1 \end{cases}$$

Solving the linear system, we have that the coefficients found were:

$$A = \frac{\tilde{J}_0-\tilde{J}_1}{3}$$

$$B = \frac{2\tilde{J}_0-\tilde{J}_1}{3}$$

Making the appropriate substitutions in the Binet formula, we have:

$$\tilde{J}_n = \frac{\tilde{J}_0-\tilde{J}_1}{3}k_1^n+\frac{2\tilde{J}_0-\tilde{J}_1}{3}k_2^n.$$

IV. Conclusion

This research was carried out around the numbers of Pell and Jacobsthal, which was made an investigation into the process of complexification and hybridization of these numbers, this research was based on the works of Dagdeviren and Kürüz (2020) and Mangueira, Alves and Catarino (2022). From the definition of the hybrid quaternions of Pell and Jacobsthal, it was possible to present their characteristic equations that have two real roots, their generating function and Binet's formula. Thus, it is expected that this research has contributed with important theorems for the studies of Pell and Jacobsthal sequences.

For future work, we can explore more properties around the Pell and Jacobsthal hybrid quaternions, as well as carry out investigation into the hybridization and complexification process in other numerical sequences.

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Resumo. Sabendo que as sequências de Pell e Jacobsthal são sequências lineares recursivas de segunda ordem e que apresentam similiariedades entre elas, neste estudo, tem-se o intuito de explorar essas sequências. Assim, será realizado uma investigação sobre os números de Pell e Jacobsthal com base nos números híbridos e os seus quaternions. Dessa forma, será realizado uma junção entre esses temas e será apresentado os quaternions híbridos de Pell e Jacobstahl, abordando sua equação característica, formula de Binet, função geradora e sua extensão para índices negativos.

Palavras-chave. Sequência de Pell, sequência de Jacobsthal, quaténions híbridos de Pell e Jacobsthal, números híbridos.

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Lehmann Type-2 Inverse Lomax Distribution and Applications to Reliability Data

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GJSFR-F Classification: *DDC Code: 536.73 LCC Code: QC318.E57*

LEHMANNTYPE2INVERSELOMAXDISTRIBUTIONANDAPPLICATIONSTORELIABILITYDATA

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Abstract- We developed a new three-parameter extended inverse Lomax distribution called the Lehmann Type-2 inverse Lomax distribution. We demonstrated its superiority over the inverse Lomax distribution through various theoretical and practical approaches. The derived properties include the quantiles, moments, incomplete moments, entropy (Renyi and Tsallis), and order statistics. Finally, we consider two life-time data sets to show how the practitioner can take advantage of the new Type-2 inverse Lomax model.

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I. Introduction

Over the last century, many significant distributions have been introduced to serve as models in applied sciences. The notable among them, generalized beta distribution developed by McDonald (1995), is at the top of the list in terms of usefulness. The main feature of the generalized beta distribution is that it introduces skewness and kurtosis into the baseline distribution that allows for modeling data of various forms of the shape of the hazard function which may be decreasing, increasing, decreasing-increasing, increasing decreasing, and inverted bathtub shapes. In this paper, we focus our attention on one of the most attractive of these distributions, known as the inverse Lomax (IL) distribution. Mathematically, it can be presented in the distribution form as $\mathrm{Y} = X^{-1}$, where X is a random variable following the famous Lomax distribution (see Lomax, 1954). Thus, the cumulative distribution function (CDF) of the IL distribution is given by

$$G(x;\lambda,\rho) = \left(1+\frac{\rho}{x}\right)^{-\lambda},\ x\ > 0 \tag{1}$$

And the corresponding PDF is given as

$$g(x;\lambda,\rho) = \lambda\rho x^{-2}\left(1+\frac{\rho}{x}\right)^{-\lambda-1}, x > 0 \tag{2}$$

where $\boldsymbol{\rho}$ is a positive scale parameter and $\boldsymbol{\lambda}$ is a positive shape parameter. The reasons forstudying the IL distribution are not limited to the following: It has proved itself as a statistical model in various applications, including economics and actuarial sciences (see Kleiber and Kotz, (2003)) and geophysics (see McKenzie et al. (2004)). Also, the mathematical and inferential aspects of the IL distribution have been studied. See, for example, Lorenz (2004), for the Lorenz ordering of order statistics, Rahman and Aslam

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(2013) studied the estimation of the parameters in a Bayesian setting, Yadav et al. (2016) examined the estimation of the parameters from hybrid censored samples, Singh et al. (2016) the study of the reliability estimator under type II censoring and Reyad, and Othman (2018) studied the Bayesian estimation of a two-component mixture of the IL distribution type I censoring. Despite an interesting compromise between simplicity and accuracy, the IL model suffers from a certain rigidity in the peak (punctual and roundness) and tail properties. This motivates the development of various parametric extensions, such as the inverse power Lomax distribution introduced by Hassan and Abd-Allah 92018), the Weibull IL distribution studied by Hassan, A.S.; Mohamed (2019) and, the Marshall-Olkin IL distribution developed by Maxwell et al. (2019). In this paper, we introduce and discuss a new extension of the IL distribution called Lehman Type-2 Inverse Lomax distribution which can model all forms of data exhibiting any shape of the hazard function because of its flexibility and tractability.

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II. Lehmann Type-2 Inverse Lomax Distribution

The CDF of the Lehmann type-2 family of distribution is given by

$$F(x) = 1 - \left(1 - G(x)\right)^{b} \tag{3}$$

And the associated PDF is given by

$$f(x) = bg(x)\left(1 - G(x)\right)^{b-1} \tag{4}$$

Putting (3) and (4) in (2), we have an expression for the Lehmann Type-2 inverse Lomax (LT-2IL) distribution given by

$$f(x; b, \lambda, \rho) = b\lambda\rho x^{-2}\left(1 - \frac{\rho}{x}\right)^{-\lambda-1}\left(1 - \left(1 + \frac{\rho}{x}\right)^{-\lambda}\right)^{b-1}, x > 0 \tag{5}$$

And the corresponding CDF is given by

$$F(x; b, \lambda, \rho) = 1 - \left(1 - \left(1 + \frac{\rho}{x}\right)^{-\lambda}\right)^{b}, \tag{6}$$

The survival and the hazard function are respectively given by

$$S(x; b, \lambda, \rho) = \left(1 - \left(1 + \frac{\rho}{x}\right)^{-\lambda}\right)^{b} \tag{7}$$

And

$$h(x; b, \lambda, \rho) = \frac{b\lambda\rho\left(1 - \frac{\rho}{x}\right)^{-\lambda-1}\left(1 - \left(1 + \frac{\rho}{x}\right)^{-\lambda}\right)^{b-1}}{x^{2}\left(1 - \left(1 + \frac{\rho}{x}\right)^{-\lambda}\right)^{b}}. \tag{8}$$

The graph of the density, hazard, and the survival function is given in figures 1, 2, and 3 for various values of the parameters.

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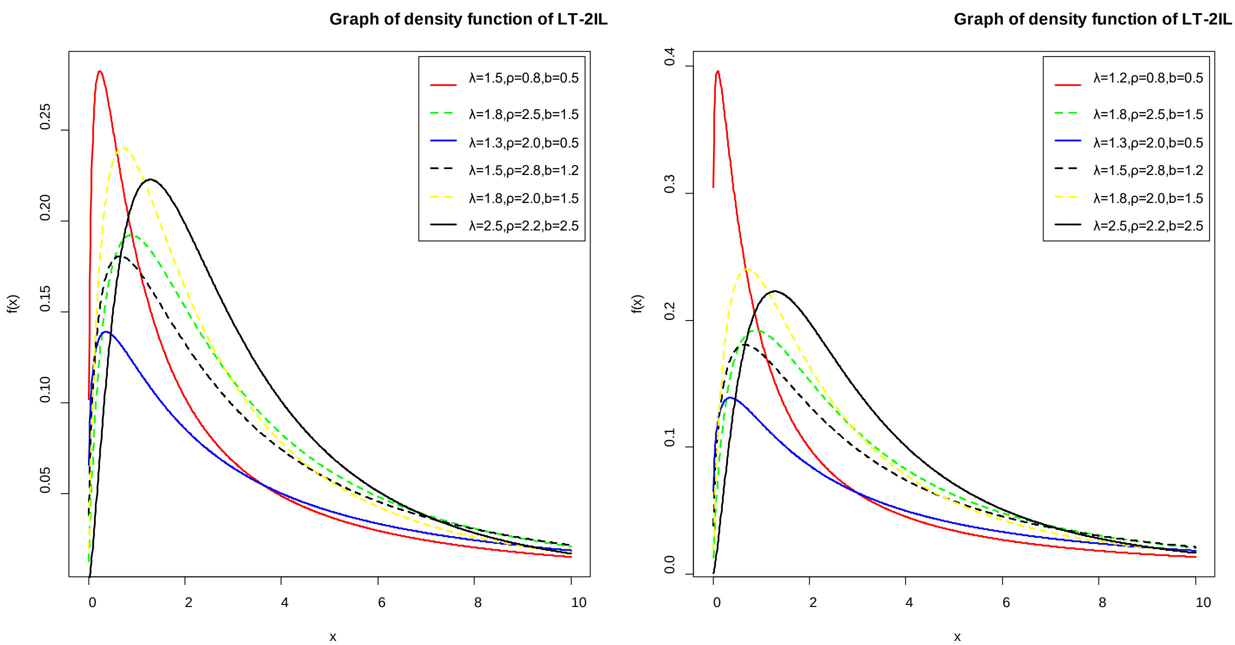

Figure 1: The graph of the density function of LT-2IL distribution

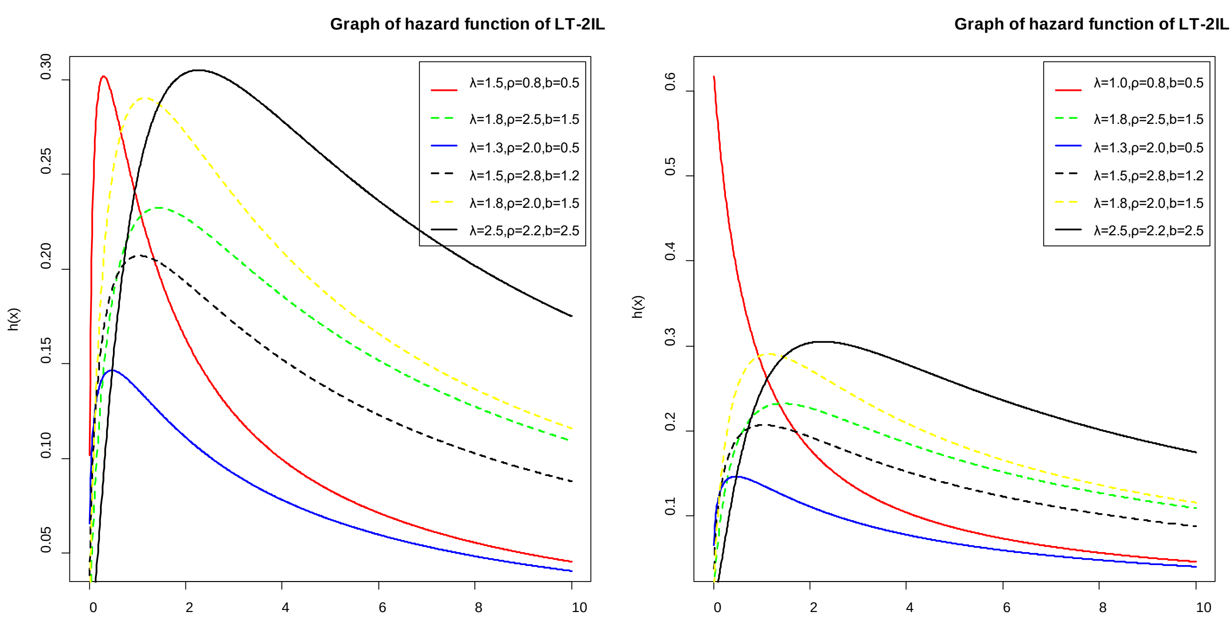

Figure 2: The graph of the hazard function of LT-2IL distribution

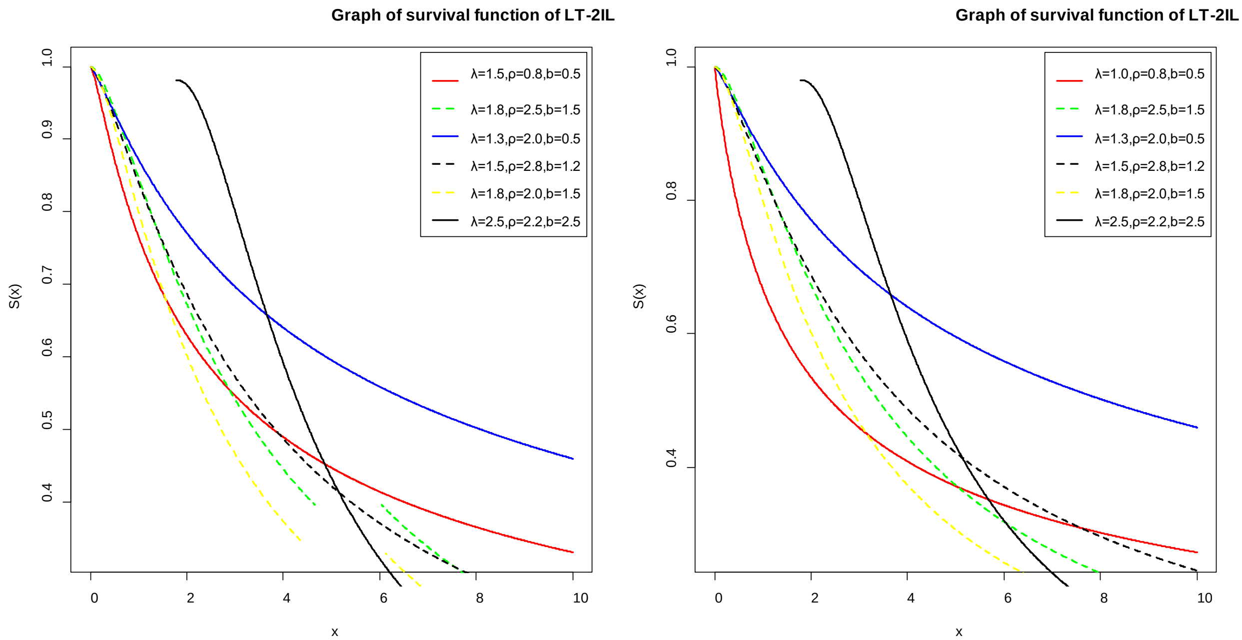

Figure 3: The graph of the survival function of $LT-2IL$ distribution

III. Useful Expansions

The binomial theorem, for $m > 0$and $|v| < 1$, can be expressed as

$$(1-v)^m = \sum_{i=0}^{\infty} (-1)^i \binom{m}{i} v^i \tag{9}$$

Then, applying the binomial series expansion given in (9) to (5), we have

$$\left(1-\left(1+\frac{\rho}{x}\right)^{-\lambda}\right)^{b-1} = \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} \left(1+\frac{\rho}{x}\right)^{-\lambda i}$$

Finally, we have

$$f(x) = b\lambda\rho x^{-2} \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} \left(1+\frac{\rho}{x}\right)^{-[\lambda(i+1)+1]} \tag{10}$$

Equation (10) represents the Exponentiated Inverse Lomax distribution, with shape parameter $[\lambda(i+1)+1]$ and scale parameter ρ. It then follows that the properties of Lehmann type-2 inverse Lomax distribution can be obtained from that of the Exponentiated inverse Lomax distribution.

a) *Quantile function*

The quantile function of the $LT-2IL$ distribution is defined by $Q(u;b,\lambda,\rho) = F^{-1}(u;b,\lambda,\rho)$, $u \in (0,1)$. After some mathematical manipulations, we obtain

$$Q(u;b,\lambda,\rho) = \rho\left\{\left[1-(1-u)^{1/b}\right]^{-1/\lambda} - 1\right\}, u \in (0,1) \tag{10.1}$$

Notes

From (10.1), we can obtain the lower quartile (q_1), middle quartile (q_2), also known as the median, and the upper quartiles (q_3)of the $LT-2IL$distribution by taking the values of u to be 0.25, 0.5, and 0.75 respectively. Then, we obtain an equation for the lower quartile, median, and the upper quartile of $LT-2IL$ distribution given respectively, by

Notes

$$Q(0.25;b,\lambda,\rho)=\rho\left\{\left[1-(0.75)^{1/b}\right]^{-1/\lambda}-1\right\}, \tag{10.2}$$

$$Q(0.5;b,\lambda,\rho)=\rho\left\{\left[1-(0.5)^{1/b}\right]^{-1/\lambda}-1\right\} \tag{10.3}$$

and

$$Q(0.75;b,\lambda,\rho)=\rho\left\{\left[1-(0.25)^{1/b}\right]^{-1/\lambda}-1\right\} \tag{10.4}$$

b) Moments of $LT-2IL$distribution

The k^{th} moment of the $LT-2IL$ distribution under certain regularity conditions; the k^{th} moment of TIIHLF distribution is obtained as

$$\mu_k'=\int_{-\infty}^{\infty}x^kf(x)dx=b\lambda\rho\boldsymbol{x}^{-2}\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}W(x) \tag{11}$$

where

$$W(x)=\int_{-\infty}^{\infty}x^{k-2}\left(\mathbf{1}+\frac{\boldsymbol{\rho}}{\boldsymbol{x}}\right)^{-[\boldsymbol{\lambda}(\boldsymbol{i}+\mathbf{1})+\mathbf{1}]}dx \tag{12}$$

Taking $z=\frac{\boldsymbol{\rho}}{\boldsymbol{x}}$ and putting it in (12), we have

$$W(x)=-\rho^{k-1}\int_{-\infty}^{\infty}z^{2-k}(1+z)^{-[\lambda(i+1)+1]}dz \tag{13}$$

Also, letting $y=\frac{v}{1-v}, dy=(1-v)^{-2}dv$ and putting it in (13), we have

$$W(x)=\rho^{k-1}\int_0^1 v^{2-r}(1-v)^{k+\boldsymbol{\lambda}(\boldsymbol{i}+\mathbf{1})-\mathbf{3}} \tag{14}$$

Then we have

$$W(x)=\rho^{k-1}B[1-k,k+\boldsymbol{\lambda}(\boldsymbol{i}+\mathbf{1})-\mathbf{2}]$$

Finally, we obtain an expression for the k^{th} moment of the $LT-2IL$ distribution given as

$$\mu_k'=b\lambda\sum_{i=0}^{\infty}(-1)^i\binom{b-1}{i}\rho^kB[1-k,k+\boldsymbol{\lambda}(\boldsymbol{i}+\mathbf{1})-\mathbf{2}] \tag{15}$$

Where $B(.,.)$ is a beta function

An expression for the first, second and third moments can be obtained by respectively taken the value of $k = 1,2$ and 3 as

$$\mu_1' = b\lambda \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} \rho B[\lambda(i+1) - 1], \tag{16}$$

$$\mu_2' = b\lambda \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} \rho^2 B[-1, \boldsymbol{\lambda}(\boldsymbol{i}+\mathbf{1})] \tag{17}$$

And

$$\mu_3' = b\lambda \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} \rho^3 B[-2, 1 + \lambda(i+1)] \tag{18}$$

c) Incomplete moments of LT-2IL distribution

The incomplete moment of LT-2IL distribution can be obtained using (13) as

$$\mu_k' = \int_{-\infty}^{\infty} x^k f(x) dx = b\lambda\rho \boldsymbol{x}^{-\mathbf{2}} \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} W(x) \tag{16}$$

where

$$W(x) = \int_0^t x^{k-2} \left(\mathbf{1} + \frac{\boldsymbol{\rho}}{\boldsymbol{x}}\right)^{-[\boldsymbol{\lambda}(\boldsymbol{i}+\mathbf{1})+\mathbf{1}]} dx \tag{17}$$

Taking $z = \frac{\boldsymbol{\rho}}{\boldsymbol{x}}$ and putting it in (17), we have

$$W(x) = -\rho^{k-1} \int_0^t z^{2-k} (\mathbf{1} + \boldsymbol{z})^{-[\boldsymbol{\lambda}(\boldsymbol{i}+\mathbf{1})+\mathbf{1}]} dz \tag{18}$$

Also, letting $y = \frac{v}{1-v}, dy = (1-v)^{-2} dv$ and putting it in (18), we have

$$W(x) = \rho^{k-1} \int_0^1 v^{2-r} (1-v)^{k+\boldsymbol{\lambda}(\boldsymbol{i}+\mathbf{1})-\mathbf{3}} \tag{19}$$

Then we have

$$W(x) = \rho^{k-1} B_{\rho/_{t+\rho}}[1-k, k+\lambda(i+1)-2] \tag{20}$$

Finally, we obtain an expression for the k^{th}incomplete moment of the $LT-2IL$ distribution given as

$$\mu_k' = b\lambda \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} \rho^k B_{\rho/_{t+\rho}}[1-k, k+\boldsymbol{\lambda}(\boldsymbol{i}+\mathbf{1})-\mathbf{2}] \tag{21}$$

$$\mu_1' = b\lambda \sum_{i=0}^{\infty} (-1)^i \binom{b-1}{i} \rho^k B_{\rho/_{t+\rho}}[1-k, k+\boldsymbol{\lambda}(\boldsymbol{i}+\mathbf{1})-\mathbf{2}] \tag{22}$$

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d) Renyi entropy

Renyi entropy was proposed by Renyi (1961). It can be obtained by

$$I_{\varphi}(x)=\frac{1}{1-\varphi}logM \tag{28}$$

Where

$$M=\int_{-\infty}^{\infty}f^{\varphi}(x)dx\,,\qquad \varphi>0,\varphi\neq 1 \tag{29}$$

Putting (5) in (28) followed by binomial expansion, we have

$$M=(b\lambda\rho)^{\varphi}\sum_{i=0}^{\infty}(-1)^{i}\binom{\varphi(b-1)}{i}\int_{-\infty}^{\infty}x^{2}\left(1+\frac{\rho}{x}\right)^{[\boldsymbol{\lambda}(\varphi-\mathbf{1})+\varphi]}dx \tag{30}$$

Taking $y=\frac{\rho}{x}, dx=-\rho y^{-2}dy$, putting it in (30), we have

$$M=(b\lambda)^{\varphi}\rho^{2+\varphi}\sum_{i=0}^{\infty}(-1)^{i}\binom{\varphi(b-1)}{i}\int_{0}^{\infty}y^{-2}(1+y)^{[\lambda(\varphi-1)+\varphi]}dy, \tag{31}$$

Furthermore, by letting $y=\frac{u}{1-u}, dw=(1-u)^{-2}du$ and putting it in (31) gives

$$M=(b\lambda)^{\varphi}\rho^{2+\varphi}\sum_{i=0}^{\infty}(-1)^{i}\binom{\varphi(b-1)}{i}\int_{0}^{\infty}u^{-2}(1-u)^{-[\boldsymbol{\lambda}(\varphi-\mathbf{1})+\varphi]}\,dy \tag{32}$$

$$M=(b\lambda)^{\varphi}\rho^{2+\varphi}\sum_{i=0}^{\infty}(-1)^{i}\binom{\varphi(b-1)}{i}B[-1,1-[\boldsymbol{\lambda}(\varphi-\mathbf{1})+\varphi]$$

Finally, we have an expression for the Renyi entropy of LT-2IL distribution as

$$I_{\varphi}(x)=\frac{1}{1-\varphi}log\left[(b\lambda)^{\varphi}\rho^{2+\varphi}\sum_{i=0}^{\infty}(-1)^{i}\binom{\varphi(b-1)}{i}B[-1,1-[\boldsymbol{\lambda}(\varphi-\mathbf{1})+\varphi]\right]$$

e) Tsallis Entropy

The Tsallis entropy, also known as β-entropy, was first discovered by Havrada and Charvat (1967) and later developed by Tsallis (1988). The Tsallis entropy of the LT-2IL distribution can be defined as

$$I_{T}^{(\varphi)}=\frac{1}{\varphi-1}\left[1-\int_{-\infty}^{\infty}f(x;b,\rho,\lambda)^{\varphi}\right],\quad \varphi>0,\varphi\neq 1 \tag{33}$$

Invariably, it may be written as

$$I_{T}^{(\varphi)}=\frac{1}{\varphi-1}[1-M^{\varphi}],\quad \varphi>0,\varphi\neq 1 \tag{34}$$

Putting (29) in (32), we obtain an expression for the Tsallis entropy given as

$$I_T^{(\varphi)} = \frac{1}{\varphi - 1}\left[1 - (b\lambda)^{\varphi}\rho^{2+\varphi}\sum_{i=0}^{\infty}(-1)^i\binom{\varphi(b-1)}{i}B[-1,1-[\boldsymbol{\lambda}(\varphi-\mathbf{1})+\varphi]\right] \quad (35)$$

f) Order statistics

Let X be a random variable from the $KGIL$ distribution and, given a random sample size n from X, say say $X_1,\ldots,X_n$, let $x_{i:n}$ be the i^{th} order statistic such that $x_{1:n} \leq x_{2:n} \leq x_{3:n} \leq \cdots \leq x_{n:n}$, given that $x_{h:n} \in \{X_1,\ldots,X_n\}$ for $h = 1,\ldots,n$. In particular, the study of order statistics is very important since naturally it appears in many applications, majorly those involving systems comprises of several components parts that can fail independently of each other. The density of $x_{i:n}$ is given by

$$f_{i:n}(x;) = \frac{n!}{(i-1)(n-i)}F(x;b,\rho,\lambda)^{i-1}R(x;b,\rho,\lambda)^{n-1}f(x;b,\rho,\lambda),\ \ x > 0 \quad (36)$$

Applying binomial theorem given in (36) to the expression above, it follows immediately that

$$f_{i:n}(x) = \frac{n!}{(i-1)(n-i)}\sum_{j=0}^{i-1}\binom{i-1}{j}(-1)^j R(x;b,\rho,\lambda)^{j+n-1}f(x;b,\rho,\lambda), x > 0 \quad (37)$$

Putting (5) and (7) in (37) gives us the following series expansion for the i^{th} order statistics for $LT-2IL$ distribution as

$$f_{i:n}(x) = \frac{n!\,b\boldsymbol{\lambda\rho x}^{-2}}{(i-1)(n-i)}\sum_{j=0}^{i-1}\binom{i-1}{j}(-1)^j\left[\left(1-\left(1+\frac{\rho}{x}\right)^{-\lambda}\right)^b\right]^{b+j+n-2}\left(1+\frac{\rho}{x}\right)^{-\lambda-1} \quad (38)$$

It should be noted that from (38) an expression for the smallest and the largest order statistics can be obtained.

IV. Maximum Likelihood Estimation (MLE) Method

Taking an observed sample $x_1,\ldots,x_n$ from the $LT-2IL$ distribution, the corresponding likelihood function can be represented as

$$L(b,\boldsymbol{\lambda},\boldsymbol{\rho}) = \prod_{i=1}^{n} f(x_i : b,\boldsymbol{\lambda},\boldsymbol{\rho}) \quad (39)$$

$$= \prod_{i=1}^{n} b\boldsymbol{\lambda\rho x}^{-2}\left(\mathbf{1}-\frac{\boldsymbol{\rho}}{\boldsymbol{x}}\right)^{-\boldsymbol{\lambda}-\mathbf{1}}\left(1-\left(\mathbf{1}+\frac{\boldsymbol{\rho}}{\boldsymbol{x}}\right)^{-\lambda}\right)^{b-1}$$

The MLEs of $p,q,\beta,$ and λ are denoted by $\hat{b},\hat{\boldsymbol{\rho}}$,and $\hat{\lambda}$, respectively. The log-likelihood function is given by

Notes

$$l = nlog(b\boldsymbol{\lambda\rho}) - (\boldsymbol{\lambda} + \mathbf{1})\sum_{i=1}^{n} log\left(\mathbf{1} + \frac{\boldsymbol{\rho}}{\boldsymbol{x_i}}\right) + (b-1)\sum_{i=1}^{n} log\left(1 - \left(\mathbf{1} + \frac{\boldsymbol{\rho}}{\boldsymbol{x_i}}\right)^{-\lambda}\right) \tag{40}$$

And the element of the score vector is given by

$$\frac{\partial l}{\partial b} = \frac{n}{b} + \sum_{i=1}^{n} log\left(1 - \left(\mathbf{1} + \frac{\boldsymbol{\rho}}{\boldsymbol{x_i}}\right)^{-\lambda}\right) \tag{41}$$

Notes

$$\frac{\partial l}{\partial \rho} = \frac{n}{\rho} - (\boldsymbol{\lambda} + \mathbf{1})\sum_{i=1}^{n}\left[\frac{\left(\mathbf{1} - \frac{\mathbf{1}}{x_i}\right)}{\left(\mathbf{1} - \frac{\rho}{x_i}\right)}\right] + \lambda(b-1)\sum_{i=1}^{n}\left[\frac{\left(\mathbf{1} + \frac{\mathbf{1}}{x_i}\right)^{-\boldsymbol{\lambda}-\mathbf{1}}}{\left(1 - \left(\mathbf{1} + \frac{\rho}{x_i}\right)^{-\lambda}\right)}\right] \tag{42}$$

$$\frac{\partial l}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} log\left(\mathbf{1} + \frac{\boldsymbol{\rho}}{\boldsymbol{x_i}}\right) + (b-1)\sum_{i=1}^{n}\left[\frac{\left(\mathbf{1} + \frac{\rho}{x_i}\right)^{-\lambda} \boldsymbol{log}\left(\left(\mathbf{1} + \frac{\rho}{x_i}\right)\right)}{\left(1 - \left(\mathbf{1} + \frac{\rho}{x_i}\right)^{-\lambda}\right)}\right] \tag{43}$$

V. Applications

In this section, two life-time data sets are provided to illustrate the importance of the $LT - 2IL$ distribution in modeling life-time data. We compare the $LT - 2IL$ model with other competitive models such as Kumaraswamy inverse Lomax (KIL), Kumaraswamy Frechet (KF), Exponentiated Lomax (EL), and the inverse Lomax (IL) distribution.

First, we consider the number of failures for the air conditioning system of jet airplanes. These data were reported by Cordeiro and Lemonte (2011) and Huang and Oluyede (2014): 194, 413, 90, 74, 55, 23, 97, 50, 359, 50, 130, 487, 57, 102, 15, 14, 10, 57, 320, 261, 51, 44, 9, 254, 493, 33, 18,209, 41, 58, 60, 48, 56, 87, 11, 102, 12, 5, 14, 14, 29, 37, 186, 29, 104, 7, 4, 72, 270, 283, 7, 61, 100, 61, 502, 220,120, 141, 22, 603, 35, 98, 54, 100, 11, 181, 65, 49, 12, 239, 14, 18, 39, 3, 12, 5, 32, 9, 438, 43, 134, 184, 20, 386, 182,71, 80, 188, 230, 152, 5, 36, 79, 59, 33, 246, 1, 79, 3, 27, 201, 84, 27, 156, 21, 16, 88, 130, 14, 118, 44, 15, 42, 106, 46,230, 26, 59, 153, 104, 20, 206, 5, 66, 34, 29, 26, 35, 5, 82, 31, 118, 326, 12, 54, 36, 34, 18, 25, 120, 31, 22, 18, 216, 139, 67, 310, 3, 46, 210, 57, 76, 14, 111, 97, 62, 39, 30, 7, 44, 11, 63, 23, 22, 23, 14, 18, 13, 34, 16, 18, 130, 90, 163, 208, 1,24, 70, 16, 101, 52, 208, 95, 62, 11, 191, 14, 71 .Some descriptive statistics for these data shows that the smallest and the largest values are 1 and 603, respectively. Further, the mean, median and variance are 92.07, 54.00 and 11645.93, respectively. The total time on test plot for the windshield data set is given in figure 4. The parameter estimates and the values for the goodness of fit test for the model is given is Table 1 and 2 respectively.

To check the adequacy of the fitted model in fitting the data considered, Akaike information criterion (AIC), consistent Akaike information criterion (CAIC),Bayesian information criterion (BIC), Kolmogorov-Smirnov(KS), Crammer-Von Misses (CM), Anderson Darling(AD) goodness of fit test and its p-value (PV) are obtained. In general, it is considered that the smaller the values of AIC, BIC, CAIC, HQIC and, K statistics and the larger the p-value, the better the fit of the model.

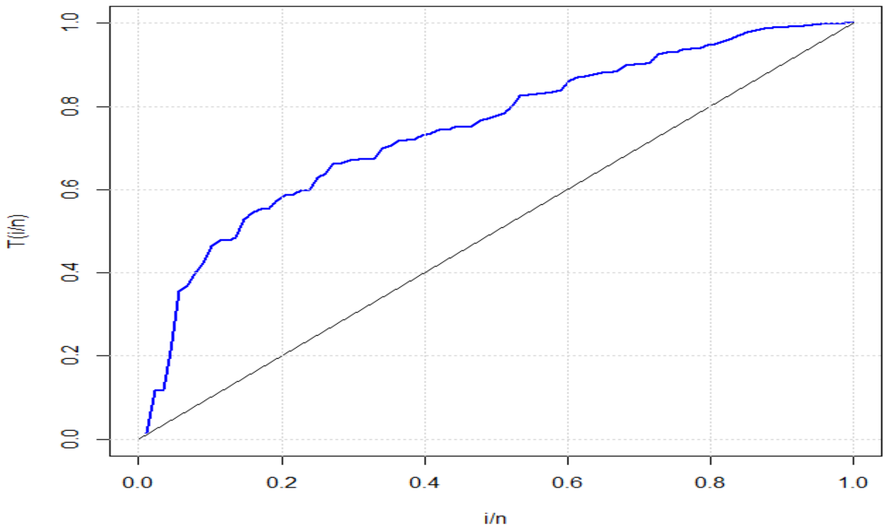

Figure 4: Graph of the total time on test plot for Jet air conditioning data

Table 1: MLEs and standard errors in braces for the first data set

Model	λ	ρ	b	a
$LT-2IL$	1.247(0.199)	125.111(77.185)	2.7769(0.961)	−(−)
EL	0.6547(0.161)	1.2489(0.368)	15.558(4.407)	−(−)
KIL	0.7724(18.669)	144.4729(101.187)	1.565(37.835)	3.0006(1.204)
KF	0.6137(0.110)	0.6451(0.1625)	8.8399(1.675)	5.001(1.353)
IL	2.0790(0.395)	18.822(4.938)	−(−)	−(−)

Table 2: AIC, BIC, CAIC, CM, AD KS statistic, and P- value (PV) for first data set

Model	−l	AIC	BIC	CAIC	CM	AD	KS	PV
$LT-2IL$	1034.77	2075.54	2085.25	2075.68	0.057	0.415	0.050	0.736
EL	1054.88	2115.77	2125.48	2115.90	0.357	2.393	0.089	0.099
KIL	1034.74	2077.69	2090.42	2077.69	0.061	0.433	0.049	0.741
KF	1039.52	2087.03	2099.98	2087.25	0.115	0.842	0.052	0.687
IL	1044.10	2092.20	2098.67	2092.26	0.173	1.175	0.068	0.343

The second data sets have been obtained from Murthy et al. (2004) is about the failure times of windshields and is given by 0.04, 0.3, 0.31, 0.557, 0.943, 1.07, 1.124, 1.248, 1.281, 1.281, 1.303, 1.432, 1.48,1.51, 1.51, 1.568, 1.615, 1.619, 1.652, 1.652, 1.757, 1.795, 1.866, 1.876, 1.899, 1.911, 1.912, 1.9141, 0.981, 2.010, 2.038, 2.085, 2.089, 2.097, 2.135, 2.154, 2.190, 2.194, 2.223, 2.224, 2.23, 2.3, 2.324, 2.349, 2.385, 2.481, 2.610, 2.625, 2.632, 2.646, 2.661, 2.688, 2.823, 2.89, 2.9, 2.934, 2.962, 2.964, 3, 3.1, 3.114, 3.117, 3.166, 3.344, 3.376, 3.385, 3.443, 3.467, 3.478, 3.578, 3.595, 3.699, 3.779, 3.924, 4.035, 4.121, 4.167, 4.240, 4.255, 4.278, 4.305, 4.376, 4.449, 4.485, 4.570, 4.602, 4.663, 4.694. Some descriptive statistics for these data shows that the smallest and the largest values are 0.04 and 4.694, respectively. Further, the mean, median and variance are 2.569, 2.367 and 1.286, respectively. The Total time on test plot for the windshield data set is given in figure 4. The parameter estimates and the values for the goodness of fit test for the model is given is Table 3 and 4 respectively.

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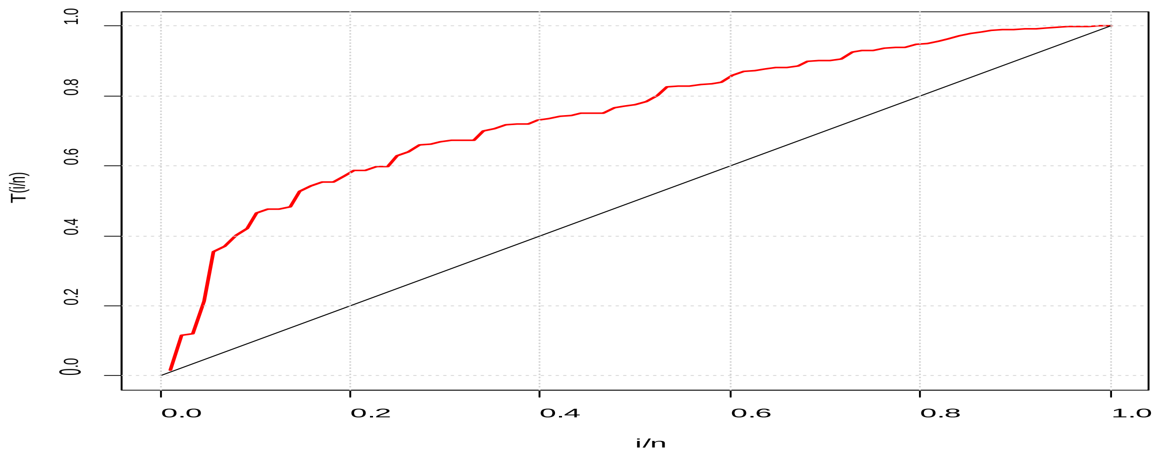

Figure 5: Graph of the total time on test plot for the second data

Table 3: MLEs and standard errors in braces for the second data set

Model	**λ**	**ρ**	**b**	**a**
$LT-2IL$	2.591(0.319)	16.442(8.468)	142.560(111.761)	$-(-)$
EL	1.584(0.351)	1.1683(0.348)	3.428(1.143)	$-(-)$
KIL	1.549(0.939)	4.3495(1.485)	1.939(1.176)	16.1458(5.434)
KF	0.7820(0.305)	1.0205(0.588)	7.163(3.946)	13.3993(8.685)
IL	4.123(1.321)	0.490(0.184)	$-(-)$	$-(-)$

Table 4: AIC, BIC, CAIC, CM, AD, KS statistic and P- value (PV) for the second data set

Model	**−l**	**AIC**	**BIC**	**CAIC**	**CM**	**AD**	**KS**	**PV**
$LT-2IL$		283.93	291.07	283.94	0.064	0.668	0.057	0.929
EL	166.92	339.84	347.27	340.13	0.705	4.571	0.169	0.013
KIL	143.95	295.89	305.80	296.38	0.137	1.203	0.094	0.408
KF	150.68	309.83	319.26	309.83	0.272	2.078	0.121	0.152
IL	187.02	378.05	383.0	378.19	0.825	5.233	0.336	$4.6e-09$

It could be observed from the results obtained from the two data sets considered that the Lehmann Type-2 Inverse Lomax model possesses the smallest AIC, BIC, CAIC, CM, AD, KS statistic and, the largest of a P-value. It could therefore be regarded as the best model in the class of the models considered based on the data used.

VI. Concluding Remarks

In this paper, we developed a study a novel three-parameter distribution called Lehmann-type-2 Inverse distribution. Some statistical properties of the new distribution are studied. The maximum likelihood estimation method is used to obtain the parameters. Two real data sets are presented to illustrate the applicability of the new model.

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Brazilian National Textbook Program: The Pedagogical Assessment of Mathematics Textbooks as Exam Technology

By José Wilson dos Santos
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Abstract- This objective of this paper is to analyze and describe the way in which the effects of disciplinary power that crosses the pedagogical evaluation of mathematics textbooks constitute a list of knowledge that feeds back this productive field. In this search, we take Michel Foucault's analysis of power as a parameter, more specifically the theories about disciplinary power. Inspired by the cartographic process, we produced the data from semi-structured interviews with former members of the National Textbook Program (PNLD), as well as with an editor who works in a large publishing group today. Data analysis allows us to describe the pedagogical evaluation of the PNLD as an important instrument for analysis, writing and validation of knowledge that is intended to be propagated, configuring itself in an examination of textbooks.

Keywords: evaluation of mathematics textbooks. mathematics curriculum. power relations.

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BRAZILIANNATIONALTEXTBOOKPROGRAMTHEPEDAGOGICALASSESSMENTOFMATHEMATICSTEXTBOOKSASEXAMTECHNOLOGY

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Brazilian National Textbook Program: The Pedagogical Assessment of Mathematics Textbooks as Exam Technology

Programa Nacional De Livros Didáticos: A Avaliação Pedagógica do Livro Didático de Matemática como Tecnologia de Exame

José Wilson dos Santos

Abstract- This objective of this paper is to analyze and describe the way in which the effects of disciplinary power that crosses the pedagogical evaluation of mathematics textbooks constitute a list of knowledge that feeds back this productive field. In this search, we take Michel Foucault's analysis of power as a parameter, more specifically the theories about disciplinary power. Inspired by the cartographic process, we produced the data from semi-structured interviews with former members of the National Textbook Program (PNLD), as well as with an editor who works in a large publishing group today. Data analysis allows us to describe the pedagogical evaluation of the PNLD as an important instrument for analysis, writing and validation of knowledge that is intended to be propagated, configuring itself in an examination of textbooks.

Keywords: evaluation of mathematics textbooks. mathematics curriculum. power relations.

Resumo- Este artigo tem por objetivo analisar e descrever o modo como efeitos do poder disciplinar que atravessa a avaliação pedagógica de livro didático de matemática, constituem um rol de saberes que retroalimentam esse campo produtivo. Nessa busca, tomamos como parâmetro a analítica do poder de Michel Foucault, mais especificamente as teorizações sobre o poder disciplinar. Com inspiração no processo cartográfico, produzimos os dados a partir de entrevistas semiestruturadas com ex-integrantes do Programa Nacional do Livro Didático (PNLD), bem como com um editor que atua em um grande grupo editorialda atualidade. A análise dos dados nos permite descrever a avaliação pedagógicado PNLD como importante instrumento de análise, escrita e validação de saberes que se deseja propaga, configurando se em um exame dos livros didáticos.

Palavras-chave: avaliação de livros de matemática. currículo de matemática. relações de poder.

I. Introdução

Embora tenha como foco o livro didático de matemática, o estudo ora proposto não tem por objetivo sua arqueologia, não visa percorrer as diferentes abordagens metodológicas ou conteúdos, nem mesmo analisar o impacto de ações e documentos oficiais sobre sua produção, como bem têm feito outros pesquisadores, como Schubring (2003), que apresenta uma história dos livros-textos de Matemática, destacando a interferência de fatores sociais em sua constituição, e Cassiano (2013) que relata como a articulação e influências recíprocas entre política e educação, revelam-se como

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estratégia de expansão de empresas espanholas do ramo livreiro em sua expansão para países da América latina e do caribe, de modo especial em solo brasileiro.

Diferentemente dos estudos citados, nossa pesquisa aproxima-se daquelas que analisam e descrevem o modo como o discurso pedagógico e econômico, sob a égide de neutralidade, vêm sendo endereçados nos livros didático de matemática. Interessa-nos a investigação dos valores morais que estes propagam, as relações de poder que se desenrolam no âmbito de sua produção, a condução das condutas dos sujeitose envolvidos eo modo como a governamentalidade neoliberal ocupa esses espaços e constitui sujeitos performáticos, empreendedores de si (Santos, 2019).

Por estas razões, mobilizamos neste artigo a seguinte questão: Quais efeitos de poder emergem da avaliação pedagógica do Programa Nacional de Livros didáticos (PNLD), sobre a produção dos livros de matemática? A partir desta questão, objetivamos analisar e descrever o modocomoefeitosdopoderdisciplinar que atravessa a avaliação pedagógica de livro didático de matemática, constitui um rol de saberes que retroalimentam esse campo produtivo.

Nesse contexto, tomamos o poder disciplinar como ferramentas para pensar a avaliação pedagógica do livro didático de Matemática, o modo como os efeitos de poder de dela derivam transita pelos fios e repercute em cada nó da rede produtiva, evidenciando efeitos de poder que não visam a retirada ou apropriação das forças do sujeito, mas, mais que isso: "(...) tem como função maior "adestrar"; ou, sem dúvida, adestrar para retirar e se apropriar ainda mais e melhor." (FOUCAULT, 2011, p. 164).

II. Perspectiva Teórico-Metodológica

Considerando que uma pesquisa não deve ser tomada como um experimento, mas como uma experimentação do pensamento, aberta às múltiplas possibilidades de compor no/com o processo (LARROSA, 2002), buscamos um afastamento de métodos catedráticos, onde cada etapa da pesquisa é previamente definida e aponta para o caminho certeiro, e optamos por praticar uma cartografia (KASTRUP, 2007), onde o caminho se faz ao caminhar.

Sob esta inspiração, a partir da realização de uma pesquisa mais ampla (SANTOS, 2019), selecionamos para este artigo enunciações produzidas a partir da realização de entrevistas semiestruturadas realizadas com três ex-integrantes do PNLD; João Bosco Pitombeira (Pitombeira), Marilena Bittar (Bittar) e José Luiz Magalhães (Magalhães), devido às longas trajetórias destes frente ao programa, ocupando cargos de coordenador de área, avaliadora/coordenadora adjunta, e avaliador respectivamente. Juntamos a estes, um editor que atua em um dos três grandes grupos editoriais que dominam o "[...] mercado editorial brasileiro na atualidade: Somos Educação, Santilhana e FTD, que juntos detêm 78% dasvendas ao MEC no período de 2005–2017" (SANTOS; SILVA, 2019, p. 252).

Os dados foram produzidos em sua grande maioria, no ambiente escolhidos pelos referidos sujeitos, na sede do grupo editorial localizada na cidade de São Paulo capital (no caso do editor), e nas residências dos entrevistados na cidade de Campo Grande -

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MS (no caso dos ex-integrantes do PNLD), com exceção da entrevista com Pitombeira que, diante da impossibilidadede agenda, ocorreu via Skype.

Cabe destacar que nas entrevistas, não buscamos acesso/resgate a uma memória esquecida. Não se trata de "(...) reencontrar uma fala primeira que aí estivesse enterrada, mas de inquietar as palavras que falamos." (FOUCAULT, 2010a, p. 412).

Uma vez concluídas e transcritas as entrevistas, realizamos a análise e descrição dos achados, apoiados no conceito de "fluxo do pensamento" (KASTRUP, 2007), onde a atenção do cartógrafo se compara ao voo de um pássaro em suas diferentes variações de velocidade, direção, altura, ângulos de visada, pousos e decolagens. O sobrevoo do cartógrafo é que definirá o foco de sua atenção. "Uma vez escolhido um lugar, o cartógrafo alterna pousos curtos ou longos, depois alça novos voos, visita outros lugares, aventura-se a diferentes encontros" (SANTOS, 2018, p.11).

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A partir desse sobrevoo sobre os dados produzidos, selecionamos a temática e os sujeitos que mobilizamos nesse artigo.

III. Mecanismo De Exame E A Avaliação De Livros Didáticos De Matemática

Ao d iscorrer sobre o poder disciplinar, Foucault (2011) apresenta-o como uma economia de poder, um sistema vigilante e disciplinador que substitui o desgastado poder soberano, outrora pautado no suplício, por outro mais econômico, em que a simples sensação de estar sendo vigiado doutrina o "delinquente", modifica seu comportamento e ajusta-o à prática desejada.

Tal poder é significativamente ampliado a partir dos mecanismos de exame, que permite não somente punir os comportamentos indesejáveis, como gratificar aqueles desejados, de modo a reforça-los, fazê-los perpetuar. Desta forma, "o sucesso do poder disciplinar se deve sem dúvida ao uso de instrumentos simples: o olhar hierárquico, a sanção normalizadora e sua combinação num procedimento que lhe é específico, o exame", definido por Foucault (2011, p.177) como:

> (...) um controle normalizante, uma vigilância que permite qualificar, classificar e punir. Estabelece sobre os indivíduos uma visibilidade através da qual eles são diferenciados e sancionados. É por isso que, em todos os dispositivos de disciplina, o exame é altamente ritualizado. Nele vêm-se reunir a cerimônia do poder e a forma da experiência, a demonstração da força e o estabelecimento da verdade.

Desta forma, o exame marca um jogo constante e minucioso de objetivação do sujeito, configurando uma microeconomia de diferenciação que determina o seu lugar e seu "valor". Para tanto, não basta apenas a observação pelo olhar incidente, mas o registro, a materialização dessa observação que torna possível o acesso e reavaliação minuciosa dos dados.

> É nesse aspecto que se faz imprescindível o exercício da escrita. Movimentos, gestos, práticas, comportamentos, discursos, tudo se torna objeto do olhar incisivo e especulativo do exame, transformando-os em dados, registros, fichas, tabelas, relatórios técnicos e boletins. Em outras palavras, o exame coloca em prática uma contabilidade penal que ritualiza a disciplina. No hospital,

> boletins médicos esquadrinham pacientes; nas fábricas, relatórios descrevem a "vida" dos funcionários; nas escolas, provas e boletins "medem" os alunos, tudo isso ao mesmo tempo que reserva ao médico, ao chefe ou ao mestre um saber novo (SANTOS, 2018, p. 17).

Cabe destacar que esse não é um processo espontâneo, mas, construído artesanalmente por discursos e relações de poder, onde cada lugar e sujeito é gradualmente transformado em uma peça dessa maquinaria, contexto onde movimentos muito bem calculados constroem as condições de possibilidade para o advento/expansão dos processos de avaliação que caracterizam/materializam toda uma escrita própria do exame, onde registros e estatísticas evidenciam o entrelaçamento entre saber-poder, de modo a subsidiar os arranjos que asseguram a perpetuação dos grandes grupos à frente da produção dos livros didáticos de Matemática.

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IV. Análise Dos Dados

Se considerarmos o contexto histórico da produção didática, veremos o modo como documentos oficiais (como o texto "Educação para Todos: caminho para a mudança", publicado em 1985, ou ainda "Recomendações para uma Política Pública de Livros Didáticos" de 2001) foram utilizados para criar/reforçar, oficializar uma verdade e, ao mesmo tempo, eliminar "[...] disposições contrárias à produção/expansão do livro didático como ativo econômico no mundo globalizado,evidenciando uma vontade de poder que abre espaço às grandes empresas multinacionais do ramo livreiro" (SANTOS; SILVA, 2018, p.18).

Contudo, nos pautaremos nessa parte do texto na análise e descrição das enunciações dos entrevistados, evidenciando o modo como a institucionalização ehierarquização dos saberes, a centralização de decisões e as normatizações dos processos de produção do livro didático de Matemática, caracterizam uma forma de exame.

Segundo Foucault (2011, p. 181) "[o] exame que coloca os indivíduos num campo de vigilância situa-osigualmente numa rede de anotações escritas; compromete-os em toda uma quantidade de documentos que os captam e os fixam." O filósofo exemplifica o modo como se dá o exame sobre os sujeitos no ambiente hospitalar, em que:

> (...) O ritual da visita é uma de suas formas mais evidentes (...). A inspeção de antigamente, descontínua e rápida, se transforma em uma observação regular que coloca o doente em situação de exame quase perpétuo. Com duas conseqüências: na hierarquia interna, o médico, elemento até então exterior, começa a suplantar o pessoal religioso (...); quanto ao próprio hospital, que era antes de tudo um local de assistência, vai tornar-se local de formação e aperfeiçoamento científico: viravolta das relações de poder e constituição de um saber. O hospital bem "disciplinado" constituirá o local adequado da "disciplina" médica; esta poderá então perder seu caráter textual e encontrar suasreferências menos na tradição dos autores decisivos que num campo de objetos perpetuamente oferecidos ao exame (FOUCAULT, 2011, p.178).

De modo análogo, entendemos que a avaliação pedagógica dos livros didáticos de matemática, reúne elementos que a caracterizam como forma de exame, não somente pela sua periodicidade ou rigor, mas pelo modo como dispõe, organiza e distribuem os sujeitos no campo, todos a postos com seus deveres específicos de observar e converter tal observação em uma rede de escritas que constituirá um campo de saber sobre o livro, que assume a posição de objeto a ser examinado.

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Ainda que não se trate diretamente do exame sobre o sujeito-autor, este não permanece ileso a tal avaliação, uma vez que o esquadrinhamento da obra que leva seu nome configura-se também em uma forma de qualifica-lo, determinando seu "valor", seu lugar nessa produção ou fora dela.

Nesse contexto, Bittar descreve o modo como o jogo de perguntas erespostas próprios do poder disciplinar, faz-se presente na avaliação de livros de matemática na atualidade: "(...) A ficha que a gente preenche, antes de preenche- la, ela já tem 17 páginas; então, é um processo (...). Na ficha vinha assim: quandoa gente está nos itens sobre atividades, pergunta: 'tem atividade de cálculo mental? ' (...) E aí a gente analisa se tem, se não tem" (Bittar em entrevista concedida ao autor).

Essa dinâmica de avaliação que segue a risca o rigor dos editais, ganha expressão no olhar minucioso e inquiridor do avaliador que é posteriormente convertido em relatórios e pareceres que classificam os livros, determinando aqueles que seguirão no jogo (aprovados) ou que serão excluídos do processo (reprovados). Desta forma, a avaliação pedagógica caracteriza-se como um exame dos livros didáticos, à medida que sobre eles incide um "(...) olhar vigilante,disciplinador e normalizante que o diferencia, coloca em prática o mecanismo sanção- gratificação, que credencia à competição de mercado aqueles considerados aptos e pune os inaptos com a sua exclusão do programa" (SANTOS, 2018, p.19).

Nesta dinâmica, observa-se nos relatos o modo como as observações sobre o livro são convertidas em escrita:

> (...) primeiro elas [as duplas de avaliadores] trabalham separadamente o material [livro], depois junta. O processo é todoassim, depois é feito um parecer de aprovação ou de exclusão. Oparecer de aprovação é o que tem lá, aprovação direta, ou aprovação com algumas coisas para corrigir (...). Aí esse parecerde exclusão ele vai para o MEC chancelar e vai para as editoras,que elas têm lá o prazo de não sei quantos dias eles têm que entrar com recurso! (Bittar em entrevista concedida ao autor).

Esta tecnologia de escrita que converte o olhar vigilante em uma gama de escritas, como relatórios e pareceressobre o livro de matemática, constitui-se como um importante mecanismo de produção de saberes que, posteriormente, serão colocados novamente em movimento, fornecendo subsídios para que autores, editores, *designers* e todo o campo editorial reconsiderem suasestratégias de produção, reorganizem o caminho traçado. Em outras palavras, (re)ativados, esses repertórios de saberes subsidiarão as decisões dos grupos editoriais em prol da aprova ção de futuras coleções, conforme explicita Pitombeira:

> (...) nós temos estatísticas completas, sabemos quais são os autores novos e as coleções apresentadas, e as coleções novas,algumas delas levam paulada da primeira vez, corrigem os problemas encontrados, e aí da vez seguinte conseguem ser aprovadas (...). É assim! E nos primeiros anos era muito pior, nasprimeiras avaliações. (Pitombeira em entrevista concedida ao autor).

Tal enunciação ressalta uma peculiaridade do exame no poder disciplinar, a capacidade de, não somente garantir a distribuição dos sujeitos no espaço a fim de promover a vigilância que dociliza os corpos, mas de fazer com que esse mesmo movimento constitua todo um rol de saberes que retroalimenta o campo onde circula.

Com efeito, é da presença dos avaliadores/examinadores neste estudo que vemos emergir outra particularidade própria do exame, a reversibilidade do olhar:

> O exame inverte a economia da visibilidade no exercício do poder: tradicionalmente, o poder é o que se vê, se mostra, se manifesta e, de maneira paradoxal, encontra o princípio de sua força no movimento com o qual a exibe. Aqueles sobre o qual ele é exercido podem ficar esquecidos; só recebem luz daquela parte do poder que lhes é concedida, ou do reflexo que mostram um instante. O poder disciplinar, ao contrário, se exerce tornando-se invisível: em compensação impõe aos que submete um princípio de visibilidade obrigatória. (FOUCAULT, 2011, p. 179).

Ora, não é esta mesma inversão que vemos na avaliação do livro didático de Matemática? São as obras avaliadas que estão sob os holofotes e não os avaliadores. Estes são sujeitos anônimos no processo. Poderiam ser quaisquer outros. Isso pouco significaria na dinâmica do poder. Importam as obras. É sobre elas que pairam os olhares dos que avaliam. É sobre elas que tratarão os editais de aprovação/exclusão, os recursos impetrados pelaseditoras contestando a avaliação (e não o avaliador), as propagandas das empresas, os divulgadores, professores, coordenadores, etc.

Dessa forma, o exame estabelece um ritual que reúne a relação saber- poder a uma economia de verdades. Ao inquirir, esmiuçar, individualizar e constituir um campo de saber em torno do livro de Matemática, o exame produz e faz circular discursos que instituem o verdadeiro sobre sua produção, evidenciando o dito por Foucault (2016, p. 22), de que não é possível o "(...) exercício do poder sem uma certa economia dos discursos de verdade que funcione dentro e a partir desta dupla exigência. Somos submetidos pelo poder à produção da verdade e só podemos exercê-lo através da produção da verdade".

Uma vez colocadas em jogo, essas verdades determinam o tipo de livro de matemática que é passível de ser produzido no contexto atual, conforme vemos nas enunciações: "(...) as editoras têm aquelas coleções que foram aprovadas muitas vezes e continuam sendo aprovadas; nessas que são aprovadas eles não mexem mais [...] eles não mexem mais, não vamos mexer para não dar zebra" (Pitombeira em entrevista concedida ao autor).

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O falta de mudanças significativas das obras, estacionando-se em um“modelo equilibrado” após sucessivas submissões ao processo de avaliação/exame é destacada por Pedro ao afirmar que:

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> Os livros de hoje são muito melhores do que os livros de ontem. O que pode ter ocorrido é que muito texto pode estar maispasteurizado, porque você tem a fórmula. Qual é a fórmula de aprovação? Você tem que ter isso, isso e isso! É isso que o MECquer. É isso que o Brasil quer para os seus brasileirinhos, então tem uma fórmula lá [no edital do PNLD], querendo ou não, tem lá! (Pedro em entrevista concedida ao autor).

Ao recorrer à enunciação de Pedro, não estamos com isso defendendo que seja o PNLD o lugar do poder, a fonte que determina os saberes a seremcolocados em prática na produção de livros de matemática, argumento já contestado por Santos e Silva (2019), mas apenas destacando o modo como, em nível maior ou menor, o exame realizado pelo programa produz saberes que são considerados por autores, editores e toda uma rede produtiva. Exemplo disso vê- se no relato do próprio Pedro, ao destacar as mudanças/ajustes nos livros de matemática ao longo do tempo, de modo a tornar os livros atuais, melhores que o de outrora. O mesmo acrescenta ainda: “[...] a visão de livro mudou, a visão deum livro mais belo, mais artístico, mais bem acabado [...], os professores batem o olho e se apaixonam, é a parte visual. Você vê, ‘nossa que bonito!’, mais as aberturas, essas ilustrações, ele se encanta” (Pedro em entrevista concedida ao autor).

Ainda nesse contexto, Magalhães sintetiza o processo onde os conhecimentos produzidos são reorganizados e dispostos novamente emcirculação na produção de novas obras: “(...) é assim, um olho no peixe o outro nogato. É um olho nos avaliadores e o outro na massa de professores que vão ter que trabalhar com aquele livro (...)” (Magalhães em entrevista concedida ao autor).

Magalhães complementa seus argumentos que, a nosso ver, reforçam oque estamos afirmando:

> [Editoras pensam assim]: olha, nós não podemos fazer um livro muito, muito avançado, muito inovador, que faz tanto o professor quanto o aluno pensar muito (...); enfim, que obriga tanto alunos quanto professores a estudarem muito, (...) não adianta fazer muitodesse livro, porque ele não vai vender”. E agradar bem aos avaliadores? É aquela história, ia ser sucesso de crítica e de avaliação, mas fracasso de venda! (Magalhães em entrevista concedida ao autor).

Nota-se desta forma que o exame sobre os livros de matemática induz o ajuste das obras ao contexto de um sistema vigilante, que articula diferentes discursos que atravessam o ambiente da produção didática. É o que podemos observar na fala de Pedro, ao afirmar: “(...) eu não tenho gosto quando eu edito, eu busco o melhor dentro daquilo que são as regras dojogo. Então, se o PNLD quer isso, normalmente é o ideal (...)” (Pedro em entrevista concedida ao autor).

Assim, o exame realizado pela avaliação atua como uma tecnologia de produção/ativação das relações saber-poder, por meio do qual editoras/editores, orientam e (re)ajustam suas obras, aproximando-as cada vez mais da normalidade, daquilo que está no âmbito do verdadeiro, criando um repertório de saberes que serão ampliados, (re)arranjados e disponibilizados novamente no campo livreiro a cada novo ciclo produtivo.

V. Considerações Finais

Ao considerar as relações de poder a partir de Michel Foucault, nos atentamos nesse artigo ao exame realizado no livro didático por meio da avaliação pedagógica do PNLD. Apoiados nas enunciações de editores e ex-avaliadores do PNLD, descrevemos nesse processo o modo como observações periódicas são convertidas em anotações que capturam e fixam o livro didático de matemática, tornando suas nuances e diferenciações mais acessíveis, facilmente legíveis. Assim, o olhar vigilante transforma observações em fichas, relatórios, pareceres, estatísticas e toda uma ordem de escrita.

Decorre desse processo a normalização do livro didático de Matemática que se ajusta ao discurso em vigor que determinando o que pode e o que não pode ser nele impresso, quem são os sujeitos autorizados a falar e quais são as regras válidas para sua produção. Enfim, reafirmam- se os efeitos do poder disciplinar sobre esta produção, processo em que ganha destaque o mecanismo sanção-gratificação explicitado por Foucault (2011), no qual se gratifica a conduta que se quer reforçar, por exemplo, a produção do livro "normal", aprovando-o no PNLD, e pune-se aquela que se afasta desta "normalidade", reprovando a obra.

Tal processo implica, portanto, um jogo de produção do verdadeiro e do falso que leva ao "sucesso" ou "fracasso" na avaliação/exame, visto que "[o]u a verdade fornece a força, ou a verdade desequilibra, acentua as dissimetrias e finalmente faz a vitória pender mais para um lado do que para o outro: a verdade é mais uma força, assim como ela só se manifesta nas relações de força." (FOUCAULT, 2010b, p. 45).

Cabe ainda ressaltar que ao considerar e (re)ativar em produções futuras os saberes produzidos no exame dos livros didáticos de matemática, gruposeditoriais asseguram não somente a capacidade de produzir um livro adequado às normas, logo, passível de aprovação, mas também alinhado aos anseios do mercado consumidor, bem como, de moldar/selecionar sujeitos mais aptos a essa produção.

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Explaining Some Weird Quantum Mechanical Features in Geometric Algebra Formalism

By Alexander Soiguine

Abstract- The Geometric Algebra formalism opens the door to developing a theory replacing conventional quantum mechanics. Generalizations, stemming from implementation of complex numbers as geometrically feasible objects in three dimensions, followed by unambiguous definition of states, observables, measurements, bring into reality clear explanations of some weird quantum mechanical features, particularly, the results of double-slit experiments where particles create diffraction patterns inherent to a wave, or modeling atoms as a kind of solar system.

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Explaining Some Weird Quantum Mechanical Features in Geometric Algebra Formalism

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Abstract- The Geometric Algebra formalism opens the door to developing a theory replacing conventional quantum mechanics. Generalizations, stemming from implementation of complex numbers as geometrically feasible objects in three dimensions, followed by unambiguous definition of states, observables, measurements, bring into reality clear explanations of some weird quantum mechanical features, particularly, the results of double-slit experiments where particles create diffraction patterns inherent to a wave, or modeling atoms as a kind of solar system.

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I. Introduction. States, Observables, Measurements

Complementarity principle in physics says that a complete knowledge of phenomena on atomic dimensions requires a description of both wave and particle properties. The principle was announced in 1928 by the Danish physicist Niels Bohr. His statement was that depending on the experimental arrangement, the behavior of such phenomena as light and electrons is sometimes wavelike and sometimes particle-like and that it is impossible to observe both the wave and particle aspects simultaneously.

In the following it will be shown that actual weirdness of all conventional quantum mechanics comes from logical inconsistence of what is meant in basic definitions and has nothing to do with the phenomena scale and the attached artificial complementarity principle.

It will be explained below that theory should speak not about complementarity but about perfect splitting of measurement process into the operator ("state" in confusing conventional terminology, though "wave function is a little better) and the operand (observable) components.

a) General definitions

Unambiguous definition of states and observables, does not matter are we in "classical" or "quantum" frame, should follow general paradigm, [1], [2], [3]:

- Measurement of observable $O(\mu)$ by state[2] $S(\lambda)$ is a map:

$$\big(S(\lambda), O(\mu)\big) \longrightarrow O(\nu),$$

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where $O(\mu)$ is an element of the set of observables. $S(\lambda)$ is element of, generally though not necessarily, another set, set of states.

- The result (value) of a measurement of observable $O(\mu)$ by state $S(\lambda)$ is a map sequence:

$$\big(S(\lambda), O(\mu)\big) \longrightarrow O(\nu) \longrightarrow V(B),$$

where V is a set of (Boolean) algebra subsets identifying possible results of measurements.

Thus, state and observable are different things. Evolution of state should be considered separately, and then action of modified state will be applied to observable in measurement.

b) *Classical kinematic illustration*

The importance of the above definitions becomes obvious even from trivial examples.

Take a point moving along straight line. The definitions are pictured as (see Fig.1.1):

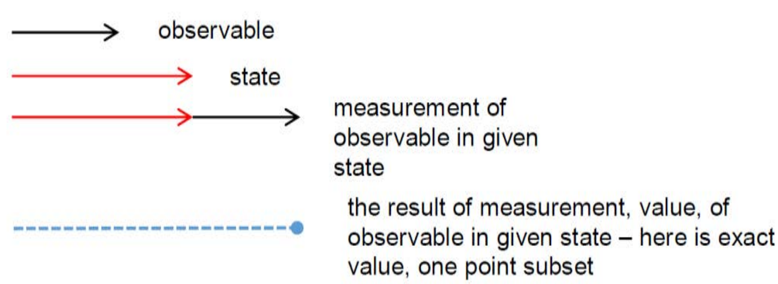

Fig. 1.1: States, observables, measurements on straight line

In this classical kinematic example, it does not formally matter do we consider evolution of "state" or of "measurement of observable by the state" or of "the result of measurement" because they differ only by an additive constant or the map of one-dimensional vector to its length.

The above one-dimensional situation radically changes if the process entities become belonging to a plane, that's dimensionality of physical process increases, though we continue watching results in one dimensional projection (see Fig.1.2):

[2] One should say "by a state". State is operator acting on observable.

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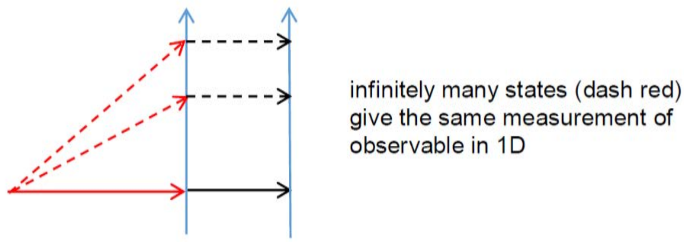

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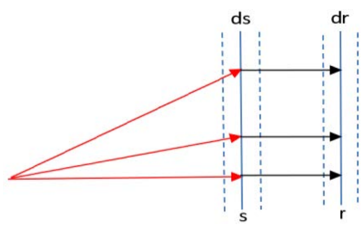
Fig. 1.2: States, observables, measurements on plane, projected on straight line

In a not deterministic evolution, the central point of randomness of observed values is the fact that their probabilities are associated with partition of the space of states. Each partition element is fiber (level set)[3] of each of the observable value under the action of the state on observable. Probabilities are (relative) measures of those fibers (see Fig.1.3):



Probability to get result of measurement in interval **dr** around **r** (making no sense to say "find system in state **r**" as in conventional quantum mechanics) is the integral of probability density of states over the strip **ds**.

Fig. 1.3: Probabilities as measures of partition elements

The option to expand, to lift the space where physical processes are considered, may have critical consequence to a theory. A kind of expanding is the core of the suggested formulation aimed at the theory deeper than conventional quantum mechanics.

II. Working with G-Qubits Instead of Qubits

A theory that is an alternative to conventional quantum mechanics has been under development for a while, see, [1], [2], [4], [5].

Its novel features are:

- Replacing complex numbers by elements of even subalgebra of geometric algebra in three dimensions, that's by elements of the form "scalar plus bivector".
- Elementary physical objects follow the structure: position in space plus explicitly defined object as the G_3, geometric algebra in three dimensions, elements.
- Operators acting on those objects are identified as direct sums of position translation and points on the three-sphere $\mathbb{S}^3$ defining rotations. Those points are connected, due to hedgehog theorem, by parallel (Clifford) translations.
- Evolution of the $\mathbb{S}^3$ part of operators by Clifford translations is governed by generalization of the Schrodinger equation with unit bivectors in three dimensions instead of formal imaginary unit.

[3] Recall that fiber of a point y in Y under a function $f : X \to Y$ is the inverse image of $\{y\}$ under f : $f^{-1}(\{y\}) = \{x \in X : f(x) = y\}$

In the following the $\mathbb{S}^3$ part of the operators will only be considered.

Qubits, identifying states in conventional quantum mechanics, mathematically are elements of the two-dimensional complex spaces:

$\binom{x_1+iy_1}{x_2+iy_2}$, conditioned by $\|x_1+iy_1\|^2+\|x_2+iy_2\|^2=1$, that is unit value elements of C^2.

Imaginary unit i is used formally with the property $i^2=-1$. In another accepted notations a qubit is:

$$C^2 \ni \binom{z_1}{z_2} = z_1\binom{1}{0} + z_2\binom{0}{1} = z_1|0\rangle + z_2|1\rangle$$

In the suggested formalism complex numbers $x+iy$ are replaced with elements of even subalgebra of G_3 – geometric algebra in three dimensions.

Even subalgebra G_3^+ is subalgebra of elements of the form $M_3=\alpha+I_S\beta$, where α and β are (real)[4] scalars and I_S is some unit bivector arbitrary placed in three-dimensional space. Elements of G_3^+ can be depict as in Fig. 2.1.

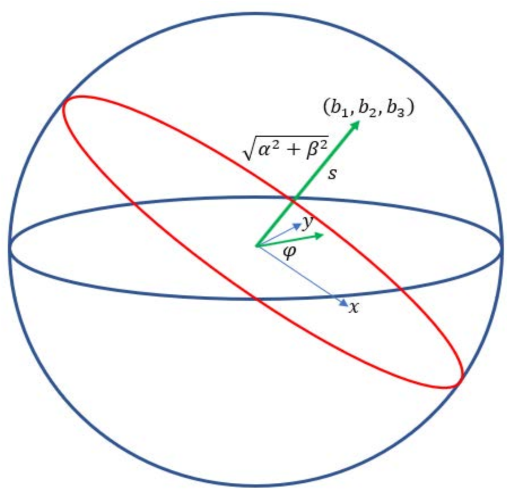


Fig. 2.1: An element of G_3^+

Unit value elements of G_3^+, when $\alpha^2+\beta^2=1$, will be called *g-qubits.* The wave functions (states in the suggested approach) implemented as g-qubits store much more information than qubits, see Fig 2.2.

[4] In the current formalism scalars can only be real numbers. “Complex” scalars make no sense anymore, see, for example, [2], [5].

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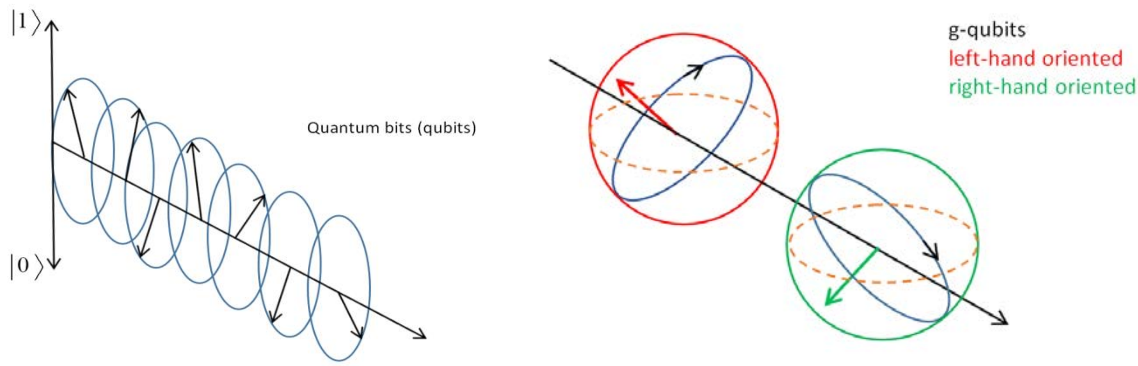

Fig. 2.2: Geomectrically picted qubits and g-qubits

III. Lift of Qubits to G-Qubits

a) Lift of quantum mechanical qubit states to g-qubits

Take right-hand screw oriented basis $\{B_1, B_2, B_3\}$ of unit value bivectors, with the multiplication rules $B_1B_2 = -B_3$, $B_1B_3 = B_2$, $B_2B_3 = -B_1$, $I_3B_1I_3B_2I_3B_3 = I_3$ (or equivalently $B_1B_2B_3 = 1$), where I_3 is oriented unit value volume, pseudoscalar, in three dimensions, see Fig.3.1.

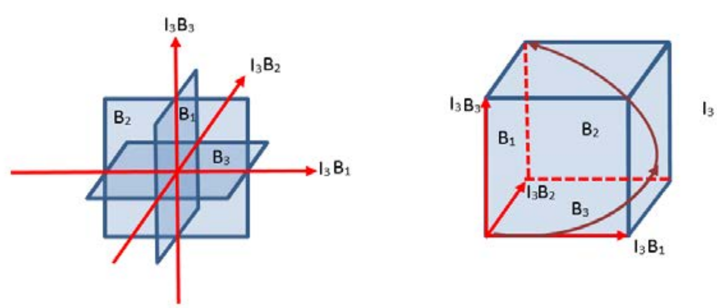

Fig. 3.1: Basis of bivectors, dual vectors and unit value pseudoscalar

The quantum mechanical qubit state, $|\psi\rangle = z_1|0\rangle + z_2|1\rangle$, is linear combination of two basis states $|0\rangle$ and $|1\rangle$. In the G_3^+ terms these two states correspond to the following classes of equivalence in G_3^+, depending particularly on which basis bivector is selected as complex plane:

- If B_1 is taken as complex plane, then

- State $|0\rangle$ has fiber (level set) of the G_3^+ elements $so(\alpha, \beta, S)_{|0\rangle}$ (0-type G_3^+ states):

$$\alpha + \beta_1 B_1,\ \alpha^2 + \beta_1^2 = 1$$

- State $|1\rangle$ has fiber of the G_3^+ elements $so(\alpha, \beta, S)_{|1\rangle}$ (1-type G_3^+ states):

$$\beta_3\ B_3 + \beta_2\ B_2 = (\beta_3 + \beta_2 B_1)B_3,\ \beta_3^2 + \beta_2^2 = 1$$

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- If B_1 is taken as complex plane, then
- State $|0\rangle$ has fiber (level set) of the G_3^+ elements $so(\alpha,\beta,S)_{|0\rangle}$ (0-type G_3^+ states):

$$\alpha + \beta_2 B_2,\ \alpha^2 + \beta_2^2 = 1$$

- State $|1\rangle$ has fiber of the G_3^+ elements $so(\alpha,\beta,S)_{|1\rangle}$ (1-type G_3^+ states):

$$\beta_1\ B_1 + \beta_3\ B_3 = (\beta_1 + \beta_3 B_2)B_1,\ \beta_1^2 + \beta_3^2 = 1$$

- If B_3 is taken as complex plane, then
- State $|0\rangle$ has fiber (level set) of the G_3^+ elements $so(\alpha,\beta,S)_{|0\rangle}$ (0-type G_3^+ states):

$$\alpha + \beta_3 B_3,\ \alpha^2 + \beta_3^2 = 1$$

- State $|1\rangle$ has fiber of the G_3^+ elements $so(\alpha,\beta,S)_{|1\rangle}$ (1-type G_3^+ states):

$$\beta_1\ B_1 + \beta_2\ B_2 = (\beta_2 + \beta_1 B_3)B_2,\ \beta_2^2 + \beta_1^2 = 1$$

b) Implementation of definitions 1.1 in the g-qubit state case

General definition of measurement in the suggested approach is based on:

- the set of observables, particularly elements of G_3^+,

- the set of states, normalized elements of G_3^+, g-qubits,

- special case of measurement of a G_3^+ observable $C = C_0 + C_1B_1 + C_2B_2 + C_3B_3$ by g-qubit (wave function) $\alpha + I_S\beta = \alpha + \beta_1B_1 + \beta_2B_2 + \beta_3B_3$ is defined as

$$(\alpha - I_S\beta)C(\alpha + I_S\beta)$$

with the result:

$$C_0 + C_1B_1 + C_2B_2 + C_3B_3 \xrightarrow{\alpha+\beta_1B_1+\beta_2B_2+\beta_3B_3} C_0 +$$

$$\big(C_1[(\alpha^2+\beta_1^2) - (\beta_2^2+\beta_3^2)] + 2C_2(\beta_1\beta_2 - \alpha\beta_3) + 2C_3(\alpha\beta_2 + \beta_1\beta_3)\big)B_1 +$$

$$\big(2C_1(\alpha\beta_3 + \beta_1\beta_2) + C_2[(\alpha^2+\beta_2^2) - (\beta_1^2+\beta_3^2)] + 2C_3(\beta_2\beta_3 - \alpha\beta_1)\big)B_2 +$$

$$(2C_1(\beta_1\beta_3 - \alpha\beta_2) + 2C_2(\alpha\beta_1 + \beta_2\beta_3) + C_3[(\alpha^2+\beta_3^2) - (\beta_1^2+\beta_2^2)])B_3 \qquad (3.1)$$

Since g-qubit (state, wave function) is normalized, the measurement can be written in exponential form:

$$e^{-I_S\varphi}Ce^{I_S\varphi}$$

where $\varphi = \cos^{-1}\alpha$.

The lift from C^2 to G_3^+ needs a $\{B_1, B_2, B_3\}$ reference frame of unit value bivectors. This frame, as a solid, can be arbitrary rotated in three dimensions. In that sense we have principal fiber bundle $G_3^+ \to C^2$ with the standard fiber as group of rotations which is also effectively identified by elements of G_3^+.

Notes

Suppose we are interested in the probability of the result of measurement in which the observable component $C_1 B_1$ does not change. This is relative measure of states

$\sqrt{\alpha^2+\beta_1^2}\left(\frac{\alpha}{\sqrt{\alpha^2+\beta_1^2}}+\frac{\beta_1}{\sqrt{\alpha^2+\beta_1^2}}B_1\right)$ in the measurements:

$$\sqrt{\alpha^2+\beta_1^2}\left(\frac{\alpha}{\sqrt{\alpha^2+\beta_1^2}}-\frac{\beta_1}{\sqrt{\alpha^2+\beta_1^2}}B_1\right)C\sqrt{\alpha^2+\beta_1^2}\left(\frac{\alpha}{\sqrt{\alpha^2+\beta_1^2}}+\frac{\beta_1}{\sqrt{\alpha^2+\beta_1^2}}B_1\right)$$

Notes

That measure is equal to $\alpha^2+\beta_1^2$, that is equal to z_1^2 in the down mapping from G_3^+ to $z_1|0\rangle + z_2\,|1\rangle$. Thus, we have clear explanation of common quantum mechanics wisdom on "probability of finding system in state $|0\rangle$".

Similar calculations explain correspondence of $\beta_3^2+\beta_2^2$ to z_2^2 in the qubit $z_1|0\rangle +$ $z_2 1\rangle$ when the component $C_1 B_1$ in measurement just got flipped.

Any arbitrary G_3^+ state $so(\alpha,\beta,S)=\alpha+\beta_1 B_1+\beta_2 B_2+\beta_3 B_3$ can be rewritten either as 0-type state or 1-type state:

$$\alpha+\beta_1 B_1+\beta_2 B_2+\beta_3 B_3=\alpha+I_{S(\beta_1,\beta_2,\beta_3)}\sqrt{\beta_1^2+\beta_2^2+\beta_3^2},$$

where $I_{S(\beta_1,\beta_2,\beta_3)}=\frac{\beta_1 B_1+\beta_2 B_2+\beta_3 B_3}{\sqrt{\beta_1^2+\beta_2^2+\beta_3^2}}$, 0-type,

or

$$\alpha+\beta_1 B_1+\beta_2 B_2+\beta_3 B_3=(\beta_3+\beta_2 B_1-\beta_1 B_2-\alpha B_3)B_3=\Big(\beta_3+$$

$$I_{S(\beta_2,-\beta_1,-\alpha)}\sqrt{\alpha^2+\beta_1^2+\beta_2^2}\Big)B_3,$$

where $I_{S(\beta_2,-\beta_1,-\alpha)}=\frac{\beta_2 B_1-\beta_1 B_2-\alpha B_3}{\sqrt{\alpha^2+\beta_1^2+\beta_2^2}}$, 1-type.

All that means that any G_3^+ state $\alpha+\beta_1 B_1+\beta_2 B_2+\beta_3 B_3$ measuring observable $C_1 B_1 + C_2 B_2 + C_3 B_3$ does not change the observable projection onto plane of $I_{S(\beta_1,\beta_2,\beta_3)}=\frac{\beta_1 B_1+\beta_2 B_2+\beta_3 B_3}{\sqrt{\beta_1^2+\beta_2^2+\beta_3^2}}$ and just flips the observable projection onto plane $I_{S(\beta_2,-\beta_1,-\alpha)}=\frac{\beta_2 B_1-\beta_1 B_2-\alpha B_3}{\sqrt{\alpha^2+\beta_1^2+\beta_2^2}}$.

IV. Evolution of G-Qubit States

Measurement of observable C by a state $e^{I_S\varphi}$ is defined as $e^{-I_S\varphi}Ce^{I_S\varphi}$. Evolution of a state is its movement on surface of $\mathbb{S}^3$.

Consider necessary formalism.

Multiplication of two geometric algebra exponents reads, see Sec.1.2 of [5]:

$$e^{I_{S_1}\alpha}e^{I_{S_2}\beta} = (\cos\alpha + I_{S_1}\sin\alpha)(\cos\beta + I_{S_2}\sin\beta)$$
$$= \cos\alpha\cos\beta + I_{S_1}\sin\alpha\cos\beta + I_{S_2}\cos\alpha\sin\beta + I_{S_1}I_{S_2}\sin\alpha\sin\beta$$

It follows from the formula for bivector multiplication:

$$g_1g_2 = \alpha_1\alpha_2 - (s_1 \cdot s_2)\beta_1\beta_2 + I_{S_1}\alpha_2\beta_1 + I_{S_2}\alpha_1\beta_2 - I_3(s_1 \times s_2)\beta_1\beta_2$$

with vectors to which the unit bivectors I_{S_1} and I_{S_2} are duals: $s_1 = -I_3I_{S_1}$, $s_2 = -I_3I_{S_2}$.

In the current case

$$\alpha_1 = \cos\alpha,\ \ \alpha_2 = \cos\beta,\ \ \beta_1 = \sin\alpha,\ \ \beta_2 = \sin\beta,$$

and we get above formula for $e^{I_{S_1}\alpha}e^{I_{S_2}\beta}$.

The product of two exponents is again an exponent, because generally $|g_1g_2| = |g_1||g_2|$ and $\left|e^{I_{S_1}\alpha}e^{I_{S_2}\beta}\right| = \left|e^{I_{S_1}\alpha}\right|\left|e^{I_{S_2}\beta}\right| = 1$, see Sec.1.3 of [5].

Multiplication of an exponent by another exponent is often called *Clifford translation.* Using the term *translation* follows from the fact that Clifford translation does not change distances between the exponents it acts upon when we identify exponents as points on unit sphere $\mathbb{S}^3$*:*

$$\cos\alpha + I_S\sin\alpha = \cos\alpha + b_1\sin\alpha\, B_1 + b_2\sin\alpha\, B_2 + b_3\sin\alpha\, B_3$$
$$\Leftrightarrow \{\cos\alpha\,, b_1\sin\alpha\,, b_2\sin\alpha\,, b_3\sin\alpha\}$$

$$(\cos\alpha)^2 + (b_1\sin\alpha)^2 + (b_2\sin\alpha)^2 + (b_3\sin\alpha)^2 = 1$$

This result follows again from $|g_1g_2| = |g_1||g_2|$:

$$|e^{I_S\alpha}(g_1 - g_2)| = |e^{I_S\alpha}||g_1 - g_2| = |g_1 - g_2|$$

Assume the angle α in Clifford translation is a variable one. Then in the case $I_{S_1} = const$:

$$\frac{\partial}{\partial\alpha}e^{I_{S_1}\alpha} = I_{S_1}e^{I_{S_1}\alpha}$$

If I_{S_1} is dual to some unit vector H, $I_{S_1} = -I_3H$ (this is the case of the matrix Hamiltonian map to G_3^+, see [3]), then $e^{I_{S_1}\alpha} = e^{-I_3H\alpha} \equiv \psi(H,\alpha)$ and

$$\frac{\partial}{\partial\alpha}\psi(H,\alpha) = -I_3H\psi(H,\alpha)$$

that is obviously Geometric Algebra generalization of the Schrodinger equation.

If vector H varies in time we get, assuming $\alpha \equiv t$:

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$$\frac{\partial}{\partial t}\psi(H(t),t) = I_3\left(-H(t) - t\frac{\partial}{\partial t}H(t)\right)\psi(H(t),t)$$

with, generally, $\psi(H(t),t) = e^{-I_3\left(\frac{H(t)}{|H(t)|}\right)|H(t)|t}$.

Assume again constant H and its unit length, $|H| = 1$. We see that displacement with $\alpha = \Delta t$ along big circle, intersection of the unit sphere $\mathbb{S}^3$ by plane $-I_3H$, rotates $\psi(H,t)$ lying on $\mathbb{S}^3$ by angle Δt in that plane.

Let us take two planes orthogonal to the plane of $-I_3H$ and comprising right-hand screw with it: $-I_3H_1$ and $-I_3H_2$. Right-handedness means:

$$(-I_3H)(-I_3H_1) = I_3H_2,$$

$$(-I_3H)(-I_3H_2) = -I_3H_1 \text{ and}$$

$$(-I_3H_1)(-I_3H_2) = -I_3H$$

(See the earlier definition of the right-hand oriented triple of basis bivectors.) Then the three above formulas mean that the planes $-I_3H_1$ and $-I_3H_2$ rotate synchronically with $-I_3H$, correspondingly in planes $-I_3H_2$ and $-I_3H_1$. Thus, the triple of planes ro tates as solid while moving along big circle on $\mathbb{S}^3$.

V. Double-Slit Experiment

Taking the set of g-qubits and projection of them onto C^2: $\pi: G_3^+ \to C^2$, we get fiber bundle. The projection depends on which basis bivector plane is selected as corresponding to formal imaginary unit plane. If we take, for example B_3, the projection is:

$$\pi: so(\alpha,\beta,S) = \alpha + \beta_1B_1 + \beta_2B_2 + \beta_3B_3 \to \begin{pmatrix}\alpha+i\beta_3\\ \beta_2+i\beta_1\end{pmatrix}$$

Then for any $z = \begin{pmatrix}x_1+iy_1\\ x_2+iy_2\end{pmatrix} \in C^2$ the fiber in G_3^+ consists of all elements $F_z = x_1 + y_2B_1 + x_2B_2 + y_1B_3$ with an arbitrary triple of orthonormal bivectors $\{B_1, B_2, B_3\}$ satisfying multiplication rules. That particularly means that the standard fiber is group of rotations of basis bivectors in the standard fiber F_z. Thus, the fiber bundle is principal fiber bundle.

Let one first slit is only open, and the fiber, wave function, is some $F^1 = x_1^1 + y_2^1B_1 + x_2^1B_2 + y_1^1B_3$. For the only open second slit the fiber is different: $F^2 = x_1^2 + y_2^2B_1 + x_2^2B_2 + y_1^2B_3$. When both slits are open the corresponding fiber is defined by connection, parallel transport anywhere between fibers F^1 and F^2.

Let we have a smooth curve $\gamma(t, P_1, P_2), 0 \le t \le 1$, connecting points $P_1 = (x_1^1, y_2^1, x_2^1, y_1^1)$ and $P_2 = (x_1^2, y_2^2, x_2^2, y_1^2)$, on three-dimensional sphere $\mathbb{S}^3$ such that $\gamma(0, P_1, P_2) = P_1$ and $\gamma(1, P_1, P_2) = P_2$. The easiest way to define parallel transport is $\gamma(t, P_1, P_2) = (1-t)P_1 + tP_2$.

For convenience purposes let us write F^1 and F^2 as exponents:

$$F^1 = x_1^1 + y_2^1 B_1 + x_2^1 B_2 + y_1^1 B_3 = x_1^1 + \sqrt{(y_2^1)^2 + (x_2^1)^2 + (y_1^1)^2}\left(\frac{y_2^1}{\sqrt{\left(y_2^1\right)^2+\left(x_2^1\right)^2+\left(y_1^1\right)^2}} B_1 + \frac{x_2^1}{\sqrt{\left(y_2^1\right)^2+\left(x_2^1\right)^2+\left(y_1^1\right)^2}} B_2 + \frac{y_1^1}{\sqrt{\left(y_2^1\right)^2+\left(x_2^1\right)^2+\left(y_1^1\right)^2}} B_3\right) = e^{I_{S_1}\varphi_1},$$

where $\varphi_1 = cos^{-1} x_1^1$,

$$I_{S_1} = \frac{y_2^1}{\sqrt{\left(y_2^1\right)^2+\left(x_2^1\right)^2+\left(y_1^1\right)^2}} B_1 + \frac{x_2^1}{\sqrt{\left(y_2^1\right)^2+\left(x_2^1\right)^2+\left(y_1^1\right)^2}} B_2 + \frac{y_1^1}{\sqrt{\left(y_2^1\right)^2+\left(x_2^1\right)^2+\left(y_1^1\right)^2}} B_3.$$

Angle φ_1 is not uniquely defined since it can be any of $\cos^{-1} x_1^1 \pm 2\pi k_1$, $k_1 = 0,1,2,\ldots$, where $\cos^{-1} x_1^1$ is, by definition, taken from interval $[0, \pi]$. The angle $\cos^{-1} x_1^1$ will be denoted as $\varphi_1(0)$.

$$F^2 = x_1^2 + y_2^2 B_1 + x_2^2 B_2 + y_1^2 B_3 = x_1^2 + \sqrt{(y_2^2)^2 + (x_2^2)^2 + (y_1^2)^2}\left(\frac{y_2^2}{\sqrt{\left(y_2^2\right)^2+\left(x_2^2\right)^2+\left(y_1^2\right)^2}} B_1 + \frac{x_2^2}{\sqrt{\left(y_2^2\right)^2+\left(x_2^2\right)^2+\left(y_1^2\right)^2}} B_2 + \frac{y_1^2}{\sqrt{\left(y_2^2\right)^2+\left(x_2^2\right)^2+\left(y_1^2\right)^2}} B_3\right) = e^{I_{S_2}\varphi_2},$$

where $\varphi_2 = cos^{-1} x_1^2$,

$$I_{S_2} = \frac{y_2^2}{\sqrt{\left(y_2^2\right)^2+\left(x_2^2\right)^2+\left(y_1^2\right)^2}} B_1 + \frac{x_2^2}{\sqrt{\left(y_2^2\right)^2+\left(x_2^2\right)^2+\left(y_1^2\right)^2}} B_2 + \frac{y_1^2}{\sqrt{\left(y_2^2\right)^2+\left(x_2^2\right)^2+\left(y_1^2\right)^2}} B_3.$$

As above, $\varphi_2 = \cos^{-1} x_1^2 \pm 2\pi\, k_2$, $k_2 = 0,1,2,\ldots$ The angle $\cos^{-1} x_1^2$ will be denoted as $\varphi_2(0)$.

Measurement of an observable

$$C = C_0 + C_1 B_1 + C_2 B_2 + C_3 B_3 = |C|\left(\frac{C_0}{|C|} + \frac{C_1}{|C|} B_1 + \frac{C_2}{|C|} B_2 + \frac{C_3}{|C|} B_3\right) =$$

$$|C|\left(\frac{C_0}{|C|} + \sqrt{1 - \frac{C_0^2}{|C|^2}}\left(\frac{C_1}{|C|\sqrt{1-\frac{C_0^2}{|C|^2}}} B_1 + \frac{C_2}{|C|\sqrt{1-\frac{C_0^2}{|C|^2}}} B_2 + \frac{C_3}{|C|\sqrt{1-\frac{C_0^2}{|C|^2}}} B_3\right)\right) = |C| e^{I_S \varphi},$$

where $|C| = \sqrt{C_0^2 + C_1^2 + C_2^2 + C_3^2}$, $\varphi = \cos^{-1}\frac{C_0}{|C|}$, $I_S = \frac{C_1}{|C|\sqrt{1-\frac{C_0^2}{|C|^2}}} B_1 + \frac{C_2}{|C|\sqrt{1-\frac{C_0^2}{|C|^2}}} B_2 + \frac{C_3}{|C|\sqrt{1-\frac{C_0^2}{|C|^2}}} B_3$,

Notes

Notes

by the wave function $e^{Is_1\varphi_1}$ is:

$$M_1 = e^{-Is_1\varphi_1}\,|C|e^{Is\varphi}\;e^{Is_1\varphi_1}$$

Measurement by $e^{Is_2\varphi_2}$ is:

$$M_2 = e^{-Is_2\varphi_2}\,|C|e^{Is\varphi}e^{Is_2\varphi_2}$$

Measurement by any intermediate parallel transport wave function $(1-t)e^{Is_1\varphi_1} + te^{Is_2\varphi_2}$ then reads:

$$(1-t)^2 e^{-Is_1\varphi_1}|C|e^{Is\varphi}\;e^{Is_1\varphi_1} + t^2 e^{-Is_2\varphi_2}|C|e^{Is\varphi}e^{Is_2\varphi_2} +$$

$$|C|t(1-t)\left(e^{-Is_1\varphi_1}e^{Is\varphi}e^{Is_2\varphi_2} + e^{-Is_2\varphi_2}e^{Is\varphi}e^{Is_1\varphi_1}\right) =$$

$$(1-t)^2 e^{-Is_1\varphi_1}|C|e^{Is\varphi}e^{Is_1\varphi_1} + t^2 e^{-Is_2\varphi_2}|C|e^{Is\varphi}e^{Is_2\varphi_2} +$$

$$t(1-t)\left(e^{-Is_1\varphi_1}e^{Is_2\varphi_2}e^{-Is_2\varphi_2}|C|e^{Is\varphi}e^{Is_2\varphi_2} + e^{-Is_2\varphi_2}e^{Is_1\varphi_1}e^{-Is_1\varphi_1}|C|e^{Is\varphi}e^{Is_1\varphi_1}\right) =$$

$$(1-t)^2 M_1 + t^2 M_2 + t(1-t)\left(e^{-Is_2\varphi_2}e^{Is_1\varphi_1}M_1 + e^{-Is_1\varphi_1}e^{Is_2\varphi_2}M_2\right)$$

Let us make natural for double slit experiment assumption $S_1 = S_2 = S_0$ (that is the two wave functions, measuring states, are of 0-type with identical bivector planes.) Then we get the measurement result by the intermediate parallel transport wave function:

$$(1-t)^2 M_1 + t^2 M_2 + t(1-t)\left(e^{-Is_2\varphi_2}e^{Is_1\varphi_1}M_1 + e^{-Is_1\varphi_1}e^{Is_2\varphi_2}M_2\right) =$$

$$(1-t)^2 M_1 + t^2 M_2 + t(1-t)\left(e^{Is_0(\varphi_1-\varphi_2)}M_1 + e^{Is_0(\varphi_2-\varphi_1)}M_2\right)$$

It is easily seen that the result of measurement is M_1 when $t = 0$ and M_2 when $t = 1$.

Consider the following simplified scenario.

Assume we are only interested in the projections of M_1 and M_2 onto the plane of their rotations, S_0, $M_1(S_0)$ and $M_2(S_0)$. Then from the general formula

$$e^{Is_2\varphi_2}e^{Is_1\varphi_1} = \cos\varphi_1\cos\varphi_2 - (s_1\cdot s_2)\sin\varphi_1\sin\varphi_2 + I_3 s_2\cos\varphi_1\sin\varphi_2 + I_3 s_1\cos\varphi_2\sin\varphi_1 - I_3(s_2\times s_1)\sin\varphi_1\sin\varphi_2$$

we get that up to some factors $e^{Is_0(\varphi_1-\varphi_2)}M_1(S_0)$ is $M_1(S_0)$ rotated in S_0 by angle $\varphi_1-\varphi_2$ and $e^{Is_0(\varphi_2-\varphi_1)}M_2(S_0)$ is $M_2(S_0)$ rotated in S_0 by angle $\varphi_2-\varphi_1$.

Without loss of generality suppose that the angles $\varphi_1(0)$ and $\varphi_2(0)$ are equal by values but opposite in sign:

$$\varphi_1(0) = -\varphi_0,\;\; \varphi_2(0) = \varphi_0,$$

$$\varphi_1(0) - \varphi_2(0) = -2\varphi_0$$

$$\varphi_2(0) - \varphi_1(0) = 2\varphi_0$$

Then it follows that in Clifford translations the projection $M_1(S_0)$ rotates in S_0 additionally by $-2\left(\varphi_0 \pm \pi(k_1-k_2)\right)$, $k_1 = 0,1,2,\dots$, $k_2 = 0,1,2,\dots$, and projection $M_2(S_0)$ rotates in S_0 additionally by $2\left(\varphi_0 \pm \pi(k_1-k_2)\right)$, $k_1 = 0,1,2,\dots$, $k_2 = 0,1,2,\dots$.

Thus, in addition to $(1-t)^2 M_1(S_0)$ and $t^2 M_2(S_0)$, we get infinite number of copies of $M_1(S_0)$ and $M_2(S_0)$ multiplied every time by $t(1-t)$ and separated by $\pm 2\pi$ along the big circle of intersection of plane S_0 with the sphere $\mathbb{S}^3$, see Fig. 5.1.

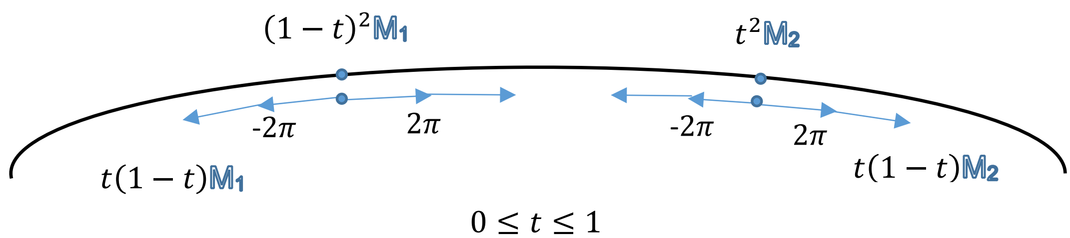

Fig. 5.1: Multiple results of measurements when both slits get opened

VI. Model of Hydrogen Atom

Let the state has the Hamiltonian type form:

$$\psi(H(t),t) = e^{-I_3\left(\frac{H(t)}{|H(t)|}\right)|H(t)|t} \tag{6.1}$$

where $H(t)$ is vector in three dimensions. An observable it will act upon is something of a torsion kind, $|r|e^{I_S\omega t}$. Thus, at instant of time t we have the following result of action of state (6.1):

$$e^{I_3\left(\frac{H(t)}{|H(t)|}\right)|H(t)|t}|r|e^{I_S\omega t}e^{-I_3\left(\frac{H(t)}{|H(t)|}\right)|H(t)|t} \tag{6.2}$$

The Hamiltonian type wave function (6.1) bears its origin from proton, while the observable $|r|e^{I_S\omega t}$ represents electron.

The geometric algebra existence of the hydrogen atom can only follow from stable sequence of measurement results (6.2) with appropriate combination(s) of $H(t)$ and ω.

Let

$$H(t) = -h_1(t)I_3B_1 - h_2(t)I_3B_2 - h_3(t)I_3B_3$$

Then $|H(t)| = \sqrt{h_1^2(t)+h_2^2(t)+h_3^2(t)}$, bivector part of (6.1) is $\frac{\sin(|H(t)|t)}{|H(t)|}(h_1(t)B_1 + h_2(t)B_2 + h_3(t)B_3)$ and the scalar part of the wave function (6.1) is $\cos(|H(t)|t)$.

If initial bivector plane of observable is $c_1B_1 + c_2B_2 + c_3B_3$, $c_1^2+c_2^2+c_3^2 = 1$, scalar part then is $|r|\cos\omega t$, thus $|r|e^{I_S\omega t} = |r|\cos\omega t + |r|\sin\omega t(c_1(t)B_1 + c_2(t)B_2 + c_3(t)B_3)$.

Notes

Let us denote the plane $-I_3\left(\frac{H(t)}{|H(t)|}\right) \equiv I_H$. Then the sequence of transformations (6.2) reads:

$$e^{-I_H|H|\Delta t}\left(\dots\left(e^{-I_H|H|\Delta t}\left(e^{-I_H|H|t}|r|e^{I_S\omega t}e^{I_H|H|t}\right)e^{I_H|H|\Delta t}\right)\dots\right)e^{I_H|H|\Delta t}$$

Notes

If $I_S = I_H$ and assuming that $H(t)$ does not depend on time. we get:

$$|r|e^{-I_H|H|t}e^{I_H\omega t}e^{I_H|H|t}$$

Angular velocity ω should be synchronized with Hamiltonian rotation by $2|H|$[5], though it can be integer times greater than $2|H|$.

Now assume that $I_S \neq I_H$. Thus, the result of (6.2) is:

$$|r|e^{-I_H|H|t}e^{I_S\omega t}e^{I_H|H|t}$$

The vector of length $|r|$ rotates in plane I_S with angular velocity ω while element $|r|e^{I_S\omega t}$ rotates in plane I_H. Again, for stability, angular velocity ω should be integer times greater than $2|H|$.

Take the general formula (3.1) and substitute $C_0 = |r|\cos\omega t$, $C_1 = |r|c_1\sin\omega t$, $C_2 = |r|c_2\sin\omega t$, $C_3 = |r|c_3\sin\omega t$, where c_i are components of I_S in the basis $\{B_1, B_2, B_3\}$, and $\alpha = \cos(|H|t)$, $\beta_i = h_i\sin(|H|t)$, h_i are components of I_H in the basis $\{B_1, B_2, B_3\}$: $I_H = h_1B_1 + h_2B_2 + h_3B_3$. The result of measurement after multiple transformations reads:

$$\frac{|r|\sin 2|H|t}{2|H|^2}\{\left[c_1\left((1+cos\,2\,|H|t)|H|^2 + (1-cos\,2\,|H|t)(2h_1^2-|H|^2)\right) + 2c_2\left((1-cos\,2\,|H|t)h_1h_2 - |H|h_3sin2|H|t\right) + 2c_3\left((1-cos\,2\,|H|t)h_1h_3 + |H|\,sin2|H|t\,h_2\right)\right]B_1 + \left[2c_1\left((1-cos\,2\,|H|t)h_1h_2 - |H|h_3sin2|H|t\right) + c_2\left((1+cos\,2\,|H|t)|H|^2 + (1-cos\,2\,|H|t)(2h_2^2-|H|^2)\right) + 2c_3\left((1-cos\,2\,|H|t)h_2h_3 - |H|\,sin2|H|t\,h_1\right)\right]B_2 + \left[2c_1\left((1-cos\,2\,|H|t)h_1h_3 - |H|\,sin2|H|t\,h_2\right) + 2c_2\left((1-cos\,2\,|H|t)h_2h_3 - |H|\,sin2|H|t\,h_1\right) + c_3\left((1+cos\,2\,|H|t)|H|^2 + (1-cos\,2\,|H|t)(2h_3^2-|H|^2)\right)\right]B_3\} \quad (6.3)$$

Formula (6.3) gives stable rotation of observable $|r|\cos\omega t + |r|\sin\omega t(c_1(t)B_1 + c_2(t)B_2 + c_3(t)B_3)$ (electron) due to action of the state $\psi(H(t),t) = e^{-I_3\left(\frac{H(t)}{|H(t)|}\right)|H(t)|t}$ (proton.)

[5] Rotation by the double of the exponential is known from rotational rules in three-dimensional geometric algebra, see, for example [3].

VII. Conclusions

It was demonstrated that the geometric algebra formalism along with generalization of complex numbers and subsequent lift of the two-dimensional Hilbert space valued qubits to geometrically feasible elements of even subalgebra of geometric algebra in three dimensions allows, particularly, to resolve the double-slit experiment results with diffraction patterns inherent to wave diffraction. This weirdness of the double - slit experiment is milestone of all further difficulties in interpretation of conventional quantum mechanics. The approach also allows elimination of the Bohr's planetary model of the hydrogen atom.

Notes

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Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

14. Arrangement of information: Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. Multitasking in research is not good: Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. Never copy others' work: Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. Go to seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. Refresh your mind after intervals: Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

20. Think technically: Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

21. Adding unnecessary information: Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

22. Report concluded results: Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

The introduction: This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear: Adhere to recommended page limits.

Mistakes to avoid:

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

Title page:

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

- o Single section and succinct.
- o An outline of the job done is always written in past tense.
- o Concentrate on shortening results—limit background information to a verdict or two.
- o Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

The following approach can create a valuable beginning:

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

Materials may be reported in part of a section or else they may be recognized along with your measures.

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- o You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- o Give details of all of your remarks as much as possible, focusing on mechanisms.
- o Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- o One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- o Recommendations for detailed papers will offer supplementary suggestions.

Approach:

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

The Administration Rules

Administration Rules to Be Strictly Followed before Submitting Your Research Paper to Global Journals Inc.

Please read the following rules and regulations carefully before submitting your research paper to Global Journals Inc. to avoid rejection.

Segment draft and final research paper: You have to strictly follow the template of a research paper, failing which your paper may get rejected. You are expected to write each part of the paper wholly on your own. The peer reviewers need to identify your own perspective of the concepts in your own terms. Please do not extract straight from any other source, and do not rephrase someone else's analysis. Do not allow anyone else to proofread your manuscript.

Written material: You may discuss this with your guides and key sources. Do not copy anyone else's paper, even if this is only imitation, otherwise it will be rejected on the grounds of plagiarism, which is illegal. Various methods to avoid plagiarism are strictly applied by us to every paper, and, if found guilty, you may be blacklisted, which could affect your career adversely. To guard yourself and others from possible illegal use, please do not permit anyone to use or even read your paper and file.

Criterion for Grading a Research Paper (Compilation) by Global Journals

Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals.

Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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