Global Journal of Science Frontier Research: F Mathematics & Decision Sciences

Volume 22 Issue 4 (Ver. 1.0)

Open Association of Research Society
Global Journals Inc.

Publisher’s Headquarters office

Global Journals® Headquarters
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Framingham Massachusetts Pin: 01701,
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Boosting Human Insight by Cooperative AI: Foundations of Shannon-Neumann Logic

By Edouard Siregar

Abstract- We present the logical foundation of an artificial intelligence (AI) capable of dealing with complex dynamic challenges, that would be very hard to handled using traditional approaches (e.g. predicate logic and deep learning). The AI is based on a cooperative questioning game, to boost insight. Insight gains are measured by information, probability, uncertainty (Shannon), as well as utility (von Neumann).

The framework is a two-person cooperative iterated Q&A game, in which both players (human, AI agent) benefit (positive-sum): the human player gains insight and the AI player learns to improve its suggestions. Generally speaking, valuable insight is typically gained by asking 'good' questions about the 'right' topic, at the 'appropriate' time and place: by posing insightful questions. In this study, we propose a logical and mathematical framework, for the meanings of 'good, right, appropriate', within clearly-defined classes of human intentions.

Keywords: artificial general intelligence, complexity, cooperative learning games, frame drift problem, information entropy, insight problems, predicate logic, renormalization, utility, value-alignment problem.

GJSFR-F Classification: DDC Code: 006.3 LCC Code: Q335

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AI based on this Shannon-Neumann Logic, combines symbolic AI with cooperative learning. It is transparent (no hidden layers), explainable (no unjustifiable moves), and remains human-aligned (no AI vs human contradictions) because of continuous cooperation (positive-sum game). In this paper, we focus uniquely on logical validity, and leave the complex topic scientific soundness for future research.

Keywords: artificial general intelligence, complexity, cooperative learning games, frame drift problem, information entropy, insight problems, predicate logic, renormalization, utility, value-alignment problem.

I. Introduction & Motivation

Purely algorithmic AI, from Predicate Logic [1] to Deep Learning neural nets [2–4], have proven highly effective for static, well-defined, narrow problems [5]. For dynamic, complex challenges, traditional AI becomes too ‘brittle’ (fails due to inappropriate application), and human insight is necessary to guarantee sound, human-aligned solutions. Solutions built on insufficient insight, can have deep long-lasting, human and economic consequences (e.g. conflict avoidance, war on drugs, pandemics or climate ill-preparedness).

Insight is usually gained (besides randomness and serendipity), by knowing when/where to pose which types of questions, about what topic: that is, by posing ‘insightful questions’. This ability thus requires a precise logical and mathematical meaning for the variables {when, where, what, which}, within well-defined contexts C, of human cognitive mindsets.

In this paper, the task of generating insightful questions, uses a framework we call Shannon-Neumann or SN-Logic, to cope with the fundamental concepts in

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insight-gains (see paper I [8]): built by combining information, probability, uncertainty [6] and utility [7]. This paper is structured as follows:

- In section 1, we discussed algorithmic vs human intelligence, and the purpose of SN-Logic.
- In section 2, we present the two-person (human $H$, AI agent $A_{SN}$) cooperative Iterated Questioning (IQ) game’s role, from both $H$’s and $A_{SN}$’s perspectives
- In section 2.3, we discuss the dynamic drift problem: coping with the changing human understanding of a given complex challenge, using a dynamic optimization process. It’s impossible to clearly define a single problem, in complex challenges (e.g. war on drugs) so that they can last for decades
- In sections 3.1-3.2, we discuss SN-Logic’s requirements to cope with insight (which involves causality, information, logic, probability, uncertainty and utility) and the spaces over which SN-Logic operates
- In sections 3.3-3.4, we introduce SN-Logic’s grammar: semantics + syntax. The syntax is used by question generators, to build millions of possible questions
- In section 3.5, we present SN-Logic predicates of two classes: problem difficulty-minimizing, and solution quality-maximizing, used in all inferences
- In section 3.6, we discuss the complexity and scope of SN-Logic, and section 3.7 highlights the distinction between knowledge acquisition (symbolic AI) and cooperative (machine) learning, both present in our AI
- In section 3.8, we introduce the normal form for making SN-inferences, about a question’s insightfulness
- In section 4, we introduce the Insight Gain Tensor $\mu$(when, where, what, which) to select sound inferences, from the many valid normal-form inferences, and measures of insight gains associated to these questions
- In section 5, we illustrate the use of SN-Logic, and we perform a validation test, to show how SN-Logic/IQ-game helps finding a solution path, to a component of a hard real-world solved case (quantum field theory research topic)

II. Two-Person Cooperative IQ-Game

a) IQ-game: Human player perspective

The Iterated Questioning or IQ game, is described in paper I. During a game session, the AI-agent, $A_{SN}$, poses the human player $H$, a question $q \in Q$, it thinks is most insightful, given $H$’s current cognitive mindset $C(t)$. $H$ then explores it, and reports if it was insightful. These are the game’s cooperative policies, both players agree to adopt for each Q&A episode. The game serves several purposes which benefits both players (positive-sum game) [7, 9]

For the human player, $H$, the IQ-game has the following main roles:

- The IQ-game is a Q&A process that reduces uncertainty and increases information about a specific problem, via a sequence of Q&As. It provides an effective tool, to gain insight on the many aspects of a complex challenge.
- The IQ-game drives a sequential (mostly left-hemispheric) conscious reasoning for solving well-defined (narrow) tasks. This process is mirrored by algorithmic AI. For complex tasks, this process alone fails to deliver full solutions. Conceptual solutions to such problems require the next process: insight-gaining.

Ref

• The IQ-game drives a parallel (mostly right-hemispheric) non-conscious process, for gaining insights leading to an 'aha' moment. Largely non-conscious processing can be used, where the first process proves too slow or impossible (task is too broad, ill-defined and complex).

• The IQ-game is driven by dual goals: minimizing obstacles and maximizing solution qualities. The minimizing questions guide \( H \) to eliminate or reduce difficulties in the problem, when possible. The maximizing questions guide \( H \) to boost specific solution qualities, when constraints allow it. It is a dynamic optimization (changes with \( H \)'s understanding). We discuss this process in section 3.4.

• The IQ-game provides a non-brittle reasoning framework, which continuously adapts to the human player \( H \)'s cognitive intentions \( C \). This mindset \( C \) evolves as \( H \)'s understanding of the challenge progresses. The IQ-game copes with the framework drift problem (section 2.3).

b) IQ-game: AI player perspective

For the AI-agent, \( A_{SN} \), the IQ-game has these roles:

• The IQ game produces game session episodes, from which the agent \( A_{SN} \) can learn via cooperative learning.

• The IQ game ensures the agent remains human-aligned [10], because of the continuous human judgments. What is useful, informative, insightful for a human player \( H \), does not necessarily mean the same for \( A_{SN} \), even if it starts that way. In the learning process, these values can drift apart, due to many factors. In the IQ game, human valuation is the ultimate arbiter, for the insight value of a question (since any AI short of a full AGI super-intelligence, will fail miserably at this task), while SN-Logic estimates the insight values, given \( C(t) \).

• The IQ game taps into a most valuable human resource: our collective evidence-based knowledge, undeniably our greatest accomplishment (culture, science, technology).

Note that our collective belief-based human selections are often poor (e.g. who we put in power as our leader). The forces here are complex and evolutionary: desire for control, cognitive biases and herd mentality from the fear of social isolation (e.g. [11]).

These factors are absent in the IQ procedure, since decisions are individual, and based directly on one’s own experience of a question’s insight, within a very specific cognitive context \( C(t) \). It uses direct evidence-based judgment, where \( H \)'s main incentive is to make life easier for herself. There are, of course individual variations in the experienced insightfulness of questions, but only stable patterns (across many individuals) are retained in cooperative learning (not presented in this paper).

c) Framework drift problem

A complex challenge is typically time-evolving, multi-objective, multi-solution, multi-discipline, multi-level and open-ended, making it hard from the start, to clearly define a single problem, even when it is urgent (e.g. a crisis) or critical (e.g. sustainability), or both (e.g. a pandemic)

Instead, there is a drift in the framing of problem and its solutions, as we accumulate new insights about a challenge: a framework drift problem. The drift cannot be handled with a static AI/ML system, focused on a given narrow problem.

The IQ-game, copes with the framework drift, by using an adaptive reasoning framework, and an adaptive cognitive intention \( C = \{ \text{framework, where, when, what} \} \)
Boosting Human Insight by Cooperative AI: Foundations of Shannon-Neumann Logic

which tracks the human player $H$’s current understanding of the conceptual framework. It follows $H$’s evolving understanding of the challenge, helping the SN-logic suggest the insightful questions, within each context $C$. The IQ-game doesn’t define a problem from the start, but instead, let’s $H$ describe the

III. Predicate SN-Logic

(a) SN-Logic requirements

Standard Logic Programming (predicate logic) is very effective when making strict deductions, but it cannot cope with the cooperative 2-person IQ-game. The purpose of SN-Logic is to provide an inference engine with the following requirements: it has to be ...

- precise (ambiguity-free) semantics axioms
- consistent (contradiction-free) framework within which, all SN-inferences can be made (normal-form inferencing)
- transparent (natural language, no hidden layers)
- explainable (no unjustifiable moves)
- human-aligned (no conflicts of with human cognitive intentions)
- non-brittle able to cope with fundamental concepts related to human-insight: causality (causes of insight), time-dependence (evolving understanding), information, probability, uncertainty (Shannon), utility (von Neumann), and insight (paper I). Britteness is a common cause of AI failures.

To satisfy these requirements, we need a consistent set of SN-Logic definitions, axioms and rules, to which we now turn.

(b) SN-Logic Spaces

To reason using a predicate logic (such as SN-Logic), the variables $x$ need spaces $X$, to scope the quantification: $\forall x \in X, \exists x \in X$. SN-Logic’s concepts are partitioned in six compact concept spaces, over which we can perform inferences (see appendices A-F):

Five vector spaces $\{T, S_D, S_C, S_G, S_S\}$, are used to describe the human player $H$’s changing cognitive mindset $C(t)$, during the IQ-game. The AI agent, $A_{SN}$, needs to know $C(t)$, because the insightfulness of a question, depends on $H$’s increasing understanding of the challenge and its possible solutions, as insight is accumulated.

The (tensor product) space $S_A$, of possible conceptual actions (operation $\times$ object) provide the raw material to build conceptual solutions.
• Vector space $T$ of exploration stages: vector variable $[\text{when} \in T]$ describes the current stage $\text{when}$ of the exploration cycle. The vector $[\text{when}]$ rotates in $T$ over time (appendix A).

• Vector space $S_D$ of mental obstacles: vector variable $[\text{where} \in S_D]$ describes where the human player’s $H$ difficulties reside. The vector $[\text{where}]$ rotates in $S_D$ over time while exploring the challenge (appendix B).

• Vector space $S_C$ of difficulty causes: vector variable $[\text{what} \in S_C]$ describes what in the reasoning’s framework, is causing $H$ difficulty. The vector $[\text{what}]$ rotates in $S_C$ over time while exploring the challenge (appendix C).

• Vector space $S_G$ of mental goals: vector variable $[\text{where} \in S_G]$ describes the solution quality, $H$ intends to improve. The vector $[\text{where}]$ rotates in $S_G$ over time while exploring the challenge (appendix D).

• Vector space $S_S$ of solution elements: vector variable $[\text{what} \in S_S]$ describes what aspect of the solution, $H$ intends to improve. The vector $[\text{what}]$ rotates in $S_S$ over time while exploring the challenge (appendix E).

• Tensor space of conceptual actions $S_A = O_p \times O_b$: action variable $[\text{which} \equiv \text{action} \in S_A]$ is composed of a mental operation (verb $\in O_p$) attached to a target object (noun $\in O_b$). Space $S_A$ provides the building-blocks of conceptual solutions. (appendix F).

c) SN-Grammar: Axioms of Semantics

SN-Logic’s role, is to provide guidance for insight-building via a Q&A process: suggesting when/where to pose which questions about what topic. To be used in inferences, the meanings of the parts of speech (variables $\{\text{when}, \text{where}, \text{what}, \text{which}\}$), and the sentence structure (questions $\text{which} \equiv q \in Q$), have to be both consistent and precise.

$A_{SN}$ needs a basic grammar (syntax, semantics, vocabulary) to communicate effectively with the human player $H$, in a consistent and precise manner. SN-Logic is based on four consistent (contradiction-free) axioms, to define its semantics precisely (ambiguity-free).

Let the human-player $H$’s cognitive mindset $C(\text{framework}, p)$ be defined by the current reasoning $\text{framework}$ (next section), and three (intention) parameters: $p = \{\text{when} = p_1, \text{where} = p_2, \text{what} = p_3\}$, then:

(Sem 1) **Shannon-informative questions**: a question (which) $q(p, \text{action})$, that reduces uncertainty (Shannon entropy) for $H$, who’s mindset is $C(\text{framework}, p)$

(Sem 2) **Neumann-useful questions**: a question (which) $q(p, \text{action})$, that has a human-aligned (via the 2-person IQ-game) utility, within a mindset $C(\text{framework}, p)$. It helps $H$ make progress towards a solution.

(Sem 3) **SN-insightful questions**: question (which) $q(p, \text{action})$ satisfying (Sem 1, Sem 2) is SN-insightful, within a mindset $C(\text{framework}, p)$, otherwise it is SN-insightless.

(Sem 4) **SN-Valid inferences**: an inference is SN-valid, if and only if it has the SN normal form (section 3.6)
d) **SN-Grammar: Syntax for Dual-Optimization**

The cooperative IQ-game is driven by dual-objectives: to minimize the problem’s causes of difficulty, and to maximize the solution’s quality. The optimization must continuously adapt to \( H \)’s understanding of the challenge, over an IQ-game session.

The SN-grammar has a simple syntax, specified for each question class \( Q \). All questions \( q \in Q \) will fall into two classes \( Q = \{ Q_{min}, Q_{max} \} \), from two complementary (dual) perspectives: (a) causes of cognitive difficulty (to minimize), (b) qualities of insight (to maximize). Each question class generates many of specific questions, aimed at making insight-gains.

The purpose of SN-Logic is to incrementally boost our insight about solutions, by suggesting when/where to pose which types of questions about what topic, while adapting to a moving target: our current understanding the obstacles in a challenge.

The question generator, or \( Q-gen \), of difficulty-minimizing questions, uses a specific syntax for an evolving cognitive mindset \( C_{min}(frame, topic, p_1, p_2, p_3) \). There is a lot of freedom in which questions to pose, even at a specific place and time, within a well-defined framework. We select a set of six commonly useful problem-solving questions, to illustrate the procedure.

**Q-Gen Syntax: difficulty-minimizing questions**

\[ q_{min1}: \text{at what exploration stage are we in now? (specifies when = } p_1 \in T) \]
\[ q_{min2}: \text{what reasoning frame are we operating in, now? (specifies [frame])} \]
\[ q_{min3}: \text{what topic in [frame] are we focusing on, now? (specifies [topic])} \]
\[ q_{min4}: \text{where does the main difficulty reside? (specifies where = } p_2 \in S_D) \]
\[ q_{min5}: \text{what, more specifically, causes this difficulty? (specifies what = } p_3 \in S_C) \]
\[ q_{min6}: \text{can you reduce the difficulty (where) and avoid its causes (what), by using these actions? (specifies action } \in S_A \text{ and which } = q_{min6} \in Q_{min}) \]

The variable \( [action] \in S_A \equiv O_p \times O_b \), is a product \( [verb \ operation] \in O_p \) x \( [noun \ object] \in O_b \) (appendices and section 5).

The [frame] variable, labels the reasoning framework currently being used (e.g. a discipline, a subject, a specialty, a model, a system, a theory, a technology etc.). This framework can change from one exploration stage to the next. It is a moving target, which mirrors our current understanding of a complex challenge.

The [topic] variable, labels a set of items we’re focusing on, within [frame] (e.g. agents, assumptions, bounds, properties, qualities, relations, statements, strategies, tactics, techniques etc.). Typically, [topic] is a tool we use within [frame], to make progress. For a concrete example, see section 5.

Questions \( q \in Q_{min} \) are SN-insightful, only if they are SN-informative (axiom Sem 1): they attempt to reduce a maximum possible amount of uncertainty (alternatives, ignorance, options, possibilities), within the context \( C_{min} \).

The generator of quality-maximizing questions, uses a specific syntax for an evolving cognitive mindset \( C_{max}(frame, topic, p_1, p_2, p_3) \):
Questions in $Q_{\text{max}}$ are SN-insightful, only if they are SN-informative (axiom Sem 1): they attempt to reduce a maximum amount of uncertainty (alternatives, ignorance, options, possibilities), within the context $C_{\text{max}}$. They are specificity-boosting questions which reduce uncertainty (Shannon entropy) to increase the solution’s quality.

e) SN-Logic predicates $q(x)$

The SN concept of insight involves notions in information, logic, probability, uncertainty and utility (see paper I). To cope with these, we need a logic with quantifiers for scoping the variables $x$ to specific spaces $X$. In standard predicate logic, a predicate is a function $p$ of a variable $x$, which maps a variable $x \in X$, into the predicate’s truth values $\{T, F\}$ [12].

$$X \rightarrow \{T, F\} \text{ and } x \in X \rightarrow p(x) = T \text{ or } F$$

In SN-Logic, an SN-predicate is a function $q$ of a variable $x$, which maps a variable $x \in X$, into the predicate’s insight values $\{\text{insightful } I^+, \text{insightless } I^0\}$.

$$X \rightarrow \{I^+, I^0\} \text{ and } x \in X \rightarrow q(x) = I^+ \text{ or } I^0$$

In SN-Logic we define the two classes (minimizing, maximizing) of predicates $q(x)$, the mindset parameter $p \in P \equiv \{\text{when, where, what}\}$ and the predicate variable ‘cognitive action’:

- SN-predicate questions $q(p, \text{action}) \in Q_{\text{min}}$, where $p \in P$, $\text{action} \in S_A$
- SN-predicate questions $q(p, \text{action}) \in Q_{\text{max}}$, where $p \in P$, $\text{action} \in S_A$

The parameter $p \in P$ is in the space $P$ of cognitive mindsets $C_{\text{min}}(\text{framework}, p)$: the set of $H$’s intentions, during the IQ-game. The AI needs to know this intent, to make useful cooperative suggestions. The mindset parameter $p$, encodes the type of insight, $H$ wants to boost, at any given time.

f) SN-Logic Complexity & Scope

SN-Logic only requires concept spaces $\{T, S_D, S_C, S_G, S_Q, O_p, O_b\}$ of very small size $N = \text{Card}(\text{Space}) \approx 10^2$ (see appendices).

- Number of distinct cognitive mindsets: $N_{\text{cogn}} = O(\text{Card}(P)) = O(\text{Card}(T) \times \text{Card}(S_D) \times \text{Card}(S_C)) = 10 \times 10 \times 10 = 10^3$
- Number of possible conceptual actions: $N_{\text{acts}} = O(\text{Card}(S_A)) = O(\text{Card}(O_p) \times \text{Card}(O_b)) = 10^2 \times 10^2 = 10^4$
- Number of possible distinct questions: $N_{\text{ques}} = \text{Card}(Q) = N_{\text{cogn}} \times N_{\text{acts}} = 10^7$ minimizing questions, posed by the $Q_{\text{min}}$-generator (same for maximizing questions).
These numbers already compare favorably to a typical human problem-solver $H$, working by herself. But the real power of SN-Logic (its scope of applications), comes from the combinatorial possibilities: the possible combinations and permutations of insight-boosting questions, needed to solve each class of challenges:

- Number of combinations: $N_{comb} = 2^{N_{ques}}$
- Number of permutations: $N_{perm} = N_{ques}!$

Thus, the number of distinct classes of challenges SN-Logic can cope with, is effectively infinite ($N = 10^{7!}$), yet, based on a few small, compact concept spaces (cardinality $\approx 10^2$). In this sense, SN-Logic is economical (Occam’s razor).

**g) Symbolic AI (knowledge acquisition) vs Learning**

The computed complexity of SN-Logic is a theoretical upper bound, to determine the scope of SN-Logic. In practice the computational cost will be much lower, due to universal constraints (common to all challenge classes), because they are imposed by (mostly) challenge-independent forces:

- causality: universal root causes of cognitive difficulties (e.g. confusion due to ambiguity, indecision due to missing information) and solution quality (e.g. accuracy, adaptability)
- logic: valid inferences with sound semantics
- planning: logically necessary chronology of solution steps
- problem-solving: universal tactics to minimize obstacles (to avoid/reduce), and maximize solution quality (to target/increase/maximize) (e.g. divide-and-conquer, minimize ambiguity, maximize order, simplify)
- information: a question is only informative, if it reduces uncertainty by eliminating alternatives, options, outcomes, possibilities, within a cognitive mindset (intention) $C$, restricting the insightful questions to a manageable subset: $q \in Q^*(C) \subset Q$, with $Card(Q^*(C)) << Card(Q)$
- utility: a question is only useful, if it helps $H$, overcome obstacles, given a cognitive intention $C$, restricting the insightful questions to a manageable subset: $q \in Q^*(C) \subset Q$, with $Card(Q^*(C)) << Card(Q)$

These rules impose a lot of structure on the SN-agent’s insight grain tensor $\mu(frame, topic, when, where, what, which)$, which is, in its fully general form, a high-dimensional rank-6 tensor, but in practice, very sparse and decomposable into simpler tensors and convolution kernels.

The structure imposed by the universal (challenge class-independent) constraints, is sufficient to construct factored (‘vanilla’) tensors $\mu^*$ of much lower dimensions and lower rank: knowledge acquisition. A ‘flavor’ is then learned to fine-tune the tensors to each class of challenge, via cooperative learning (not described in this paper). Given the complexity upper-bounds of SN-Logic, the fine-tuning possibilities are vast.

**h) SN-Logic Normal Form**

$A_{SN}$’s fundamental problem, is to use the IQ-game, to guide a human player $H$, in *when* and *where*, to pose *which* types of questions about *what* topic, to gain a maximum amount of insight into a complex challenge.

A standard normal form inferencing (analogous to conjunctive and disjunctive normal forms, in digital and predicate logic), is necessary for the AI to cope with the computational complexity of SN-Logic. The AI can efficiently search for predicate variables $action \in S_A$, used as building-blocks for conceptual solutions. Given an evolving inferencing framework ($frame, topic$), SN-normal forms are the following:
SN normal-form for minimizing inferences

Given a minimizing mindset \( C_{\text{min}}(\text{frame}, \text{topic}, p) \), where \( p \in P = \{\text{when}, \text{where}, \text{what}\} \):

if \( \exists \) action \( \in S_A \), such that \( \mu_{\text{min}}(\text{frame}, \text{topic}, p, \text{action}) > \mu_{\text{crit}} \),

then

\( q(p, \text{action}) \in Q^*_{\text{min}}(C_{\text{min}}) \subset Q_{\text{min}} \), and

\( q(p, \text{action}) \) is SN-insightful, within \( C_{\text{min}} \)

SN normal-form for maximizing inferences

Given a maximizing mindset \( C_{\text{max}}(\text{frame}, \text{topic}, p) \), where \( p \in P = \{\text{when}, \text{where}, \text{what}\} \):

if \( \exists \) action \( \in S_A \), such that \( \mu_{\text{max}}(\text{frame}, \text{topic}, p, \text{action}) > \mu_{\text{crit}} \),

then

\( q(p, \text{action}) \in Q^*_{\text{max}}(C_{\text{max}}) \subset Q_{\text{max}} \), and

\( q(p, \text{action}) \) is SN-insightful, within \( C_{\text{max}} \)

The sets \( Q^*(C) \), are maximum-insight subsets of \( Q_{\text{min}} \) or \( Q_{\text{max}} \), and \( \mu(\text{frame}, \text{topic}, p, \text{action}) \) is an insight-gain tensor (discussed shortly) whose insight gains are above a minimum critical cutoff \( \mu_{\text{crit}} \). The purpose of an insight-gain cutoff scale is intuitive, but its mathematical justification is outside the scope of this paper, which focuses only on logical validity, and ignores scientific soundness. The cutoff is related to a scale-invariance due to a conformal symmetry, under the renormalization of probabilities (unitarity). Scale-separation is used in quantum field theories [13], but justified by the conformal symmetry [14] of a renormalization group [15].

To perform successful inferences autonomously, the AI agent needs to possess the means of deciding whether a predicate variable \( \text{action} \in S_A \), leads to insight gains above a minimum lower bound (that is, \( \text{action} \in S_A(C) \subset S_A \)). The insight-gain tensor provides the SN-agent, the ability to select sound inferences, from a vast number of merely, valid ones (that is, of SN normal-form).

IV. Insight Gain Tensors \( \mu \)

a) Need for Insight-Gain Tensors

The AI performs SN normal-form inferences, to suggest insightful questions to explore, given human-targeted insight gains \( C(p) \). These 'most insightful' questions, lie in a restricted subspace \( Q^*(C) = \{Q^*_{\text{min}}(C_{\text{min}}), Q^*_{\text{max}}(C_{\text{max}})\} \), within a large space \( Q \), of possible questions (\( \text{Card}(Q) = 10^7 \)). Given a current mindset \( C(p) \), \( A_{SN} \) must find a subspace of questions \( Q^*(C) \). This is where an insight-gain measure \( \mu(p, \text{action}) \) (convolution tensors and their kernels, used to restrict searches to optimal sub-spaces) are essential, to make sound inferences (real-world accurate), rather than merely valid ones (SN normal-form inferences). This will be presented elsewhere. For now, we simply discuss general constraints imposed by SN-Logic, on the tensor elements.
b) Constraints on Insight-Gain Tensors \( \mu \)

The AI’s capacity to generate \( SN\)-insightful \( I^+ \) questions, from a vast possibility of \( SN\)-insightless \( I^0 \) ones (with \( actions \in S_A \)), resides in the structure a high-dimensional \( insight-gain \) tensor \( \mu(when, where, what, which) \equiv \mu(p, action) \), for each challenge \( class \) and reasoning \( frame \). So the full rank-7 tensor is actually \( \mu(class, frame, topic, p_1, p_2, p_3, action) \). This function outputs the value \( g \) of insight gain associated to exploring a question \( which \equiv q(p, action) \in Q \), where \( p \in P \) encodes \( H\)’s targeted insight gains. To be useful, the tensor \( \mu \) is required to satisfy the following properties:

- \( \mu : Cl \times Fr \times P \times S_A \rightarrow [0,1] \), where \( Cl = \) set of challenge classes, \( Fr = \) set of reasoning frameworks (frame+topic), \( P = T \times S_1 \times S_2 \), \( S_A = O_p \times O_b \), \( S_1 = S_D \) or \( S_G \), and \( S_2 = S_C \) or \( S_Q \)
- it is a measure of insight gain \( \mu(class, frame, topic, p, action) = g \in [0,1] \) (normalized)
- probability of all possible \( actions \) with a mindset \( p \), must sum to one (unitarity)
- \( \mu_{crit} \in ]0,1[ \) (minimum critical insight-gain value \( \mu > \mu_{crit} \))
- \( g = 0 \) when \( q(p, action) \) is \( SN\)-insightless \( I^0 \), given the mindset \( p \)
- \( g = 1 \) when \( q(p, action) \) is maximally \( SN\)-insightful \( I^+ \), given the mindset \( p \)
- \( \mu \) is initialized by satisfying heuristics from causality, information, logic, planning, problem solving and utility. These constraints provide the initial (challenge class-independent) approximation for \( \mu \)
- \( \mu \) gets optimized (fine-tuned) for specific classes of challenges, by \textit{cooperative learning}, using the IQ-game’s session episodes

V. Validation Test: Post-Doc Researcher’s Dilemma

We can now illustrate how \( SN\)-Logic is used, on a real challenge. In the IQ-game, both players (human: \( H \), \( A_{SN} \)) agree to use simple \textit{cooperative strategies}, given \( H\)’s current mindset \( C \):

1. \( A_{SN} \) suggests its guess at a most insightful question \( (q \in Q^*(C)) \)
2. \( H \) reports questions \( q \) she actually finds insightful

The game’s Q&A session, cycles over each obstacle, encountered within a challenge. Hundreds of such sub-problems may be encountered, to solve a challenge. Usually, the number and nature of these obstacles is unknown ahead of time, in real-world challenges.

For clarity, we use a single, static, not so complex, yet most difficult challenge. The scenario is: a young post-doctoral researcher, \( H \), is trying to find a good quantum field topic, to spend her next ten years on. The first few moves (Q&As) of the two-person IQ-game, could proceed as follows:

\( Q \) from \( A_{SN} \): ’Greetings! What \textit{class} of challenge are we exploring today (sample which depends on what \( SN\)-Logic is being used for):
To compose something e.g. music, arts, literature, programming
To compute something e.g. any domain
To construct something e.g. any domain
To design something e.g. engineering, technology
To discover something e.g. science, mathematics
To govern something e.g. crisis mitigation, leadership
To invent something e.g. engineering, science, technology
To manage something e.g. corporate, government
To optimize something e.g. any domain
To predict something e.g. any domain
To reconstruct something e.g. intelligence, inverse problems, sensing
To solve something e.g. any domain

A by H: I want to improve on standard quantum field theory, its a discover class of challenge ([class] = discover).

1. Q from A_{SN}: Which exploration stage are we in, now:
   (AI is using q_{min1} in Q-gen)
   - to specify a current obstacle
   - to minimize the obstacle
   - to explore solution ideas
   - to question a solution idea
   - to verify a solution idea

   A by H: 'I want to identify the current obstacle'

2. Q from A_{SN}: 'What is our current reasoning framework?
   (AI is using q_{min2} in Q-generator)

   The framework is composed of a topic and a frame

The topic can be any useful tool we select, for overcoming the obstacle (select the closest match):
- actions e.g. activities or behaviors
- agents e.g. catalysts or inhibitors
- limits e.g. lower, upper, extremes
- computations e.g. algorithms
- equations e.g. model or representation
- laws e.g. laws of quantum physics
- procedures e.g. protocols or decision
- processes e.g. interactions or communications
- properties e.g. pattern or symmetry
- qualities e.g. strengths or weaknesses
- relationships e.g. hierarchy or priorities
- restrictions e.g. constraints or conditions
- rules e.g. allowed or forbidden
- statements e.g. assumptions, conditions or theorems
- states e.g. equilibrium or criticality
- strategies e.g. divide-and-conquer
- structures e.g. classes, partitions, sets
- tactics e.g. explore special cases
- techniques e.g. calculation or construction
...
The reasoning **frame** is the clearly-defined context, within which **topic** is being used (select the closest match):

- **discipline** e.g. molecular biology
- **subject** e.g. protein folding
- **context** e.g. social revolution
- **environment** e.g. location and time
- **event** e.g. activity or pandemic crisis
- **model** e.g. just-in-time supply-chains
- **principle** e.g. quantum computing
- **method** e.g. optimization
- **network** e.g. communication or transport
- **theory** e.g. general relativity
- **specialty** e.g. programming
- **system** e.g. quantum communications
- **technology** e.g. fresh water extractor

... 

Note SN-logic’s non-brittleness: at any given time, the reasoning **frame** can adapt to any required abstraction level and scope. Such frame changes are typically **unpredictable** at the start of a real-world challenge.

A by **H**: For my research direction, I want to identify a weakness in quantum field theory (QFT) (so here, [frame] \(\equiv\) theory, and [topic] \(\equiv\) qualities).

3. Q from **ASN**: 'What’s your main difficulty with the [frame], right now? (select the closest match)
   (AI is using \(q_{\text{min}4}\) from Q-generator + an insight-gain tensor/kernel)

   - inability to apply [topic] in/of [frame]
   - inability to compute [topic] in/of [frame]
   - inability to construct [topic] in/of [frame]
   - inability to decide [topic] in/of [frame]
   - inability to evaluate [topic] in/of [frame]
   - inability to exploit [topic] in/of [frame]
   - inability to identify [topic] in/of [frame]
   - inability to select [topic] in/of [frame]
   - inability to simplify [topic] in/of [frame]
   - inability to solve [topic] in/of [frame]
   - inability to understand [topic] in/of [frame]

A by **H**: I can’t evaluate the weaknesses [topic = qualities] of quantum field theory [frame = theory]

4. Q from **ASN**: 'More specifically, why can’t you evaluate the [topic] in [frame]?
   (select the closest root cause of the difficulty)

   (AI is using \(q_{\text{min}5}\) from Q-generator + an insight-gain tensor/kernel)

   - missing comparison for [topic] in/of [frame]
   - missing constraint on [topic] in/of [frame]
   - missing criterion for [topic] in/of [frame]
   - missing direction in [topic] in/of [frame]
   - missing information about [topic] in/of [frame]
   - missing intuition for [topic] in/of [frame]
   - missing knowledge of [topic] in/of [frame]
   - missing metric for [topic] in/of [frame]
   - missing ranking of [topic] in/of [frame]
   - missing standard for [topic] in/of [frame]
   - missing value of [topic] in/of [frame]

A by **H**: 'I lack an intuition for the weaknesses of QFT'
5. Q from $A_{SN}$: awesome, so our current obstacle is, our missing intuition for the weaknesses of QFT. Let’s try to eliminate this obstacle.

A by $H$: ‘Okay, I’m all ears!’

6. Q from $A_{SN}$: can we gain intuition to evaluate QFT’s weakness, by...
(explore any question you think is promising, or move-on)
(AI is using $q_{min}$ in Q-generator + SN normal-form inferences + insight-gain
tensors/kernels)
by exploring:

- idealized cases of the theory (QFT)
- solved cases of the theory (QFT)
- simple cases of the theory (QFT)

by outlining:

- consequences (causal) of the theory (QFT)
- implications (logical) of the theory (QFT)
- predictions (temporal) of the theory (QFT)
- tests (experimental) of the theory (QFT)

by identifying:

- inconsistent aspects of the theory (QFT)
- limitations of the theory (QFT)
- problematic aspects of the theory (QFT)
- uncertain aspects of the theory (QFT)
- unjustified aspects of the theory (QFT)
- untested aspects of the theory (QFT)

by looking for:

- ambiguities (imprecision)
- contradictions (logical, evidence)
- counter-examples (exceptions)
- discrepancies (differences)
- dogma (cognitive traps)
- errors (math, procedures)
- falsehoods (logical)
- flaws (procedure, reasoning)
- gaps (missing pieces)
- implicit assumptions (reasoning)
- impossibilities (logical, physical)
- inaccuracies (scientific, technical)
- incompatibilities (between two items)
- inconsistencies (logical)
- limitations (scope of applicability)
- unexplained items (no explanation)
- unjustified items (lack justification)
- unsupported items (lack evidence)
- violations (law-breaking)
- weaknesses (logical)

A from $H$: ‘I find some questions quite insightful, because
(click on each insightful one, and note the reasons for your record):

I outlined the implications of QFT (e.g. including matrix unitarity), and
QFT’s experimental tests (e.g. including neutron decay experiments). I
found reported incompatibilities (e.g. known violations of the CKM matrix’s
unitarity [16], in neutron decay experiments [17]). That seems like an interesting research area of quantum field theory, for me.
7. $Q$ from $A_{SN}$: 'Do you want to identify a new obstacle, now? ...
Note: for a complex challenge, limitless combinations of obstacles can be explored in this manner.

This scenario shows how suggested questions from $A_{SN}$, can replicate real-world solutions to obstacles, via a cooperative $QE$A dialog. The researchers do something similar between themselves, early-on, to decide what to work on. But AI’s complementary strength, is to cover many exploration paths, which are very often overlooked, yet may be key to quality solutions. This dynamic ‘human-AI’ interaction would be even more fruitful, in a group brainstorming session, where each member of the team, can select directions to explore and possible answers.

VI. Discussion

a) Tensor Construction & Cooperative learning

We mentioned (section 3.7), that insight-gain convolution tensors and kernels, form the bridge between the SN normal form inferencing (SN-validity), and measures of insight (SN-soundness); the bridge between logic (validity) and science (soundness). Initially, the tensors $\mu$ are the AI’s ‘vanilla’ core, then, learned flavors are added to it, via machine learning to optimize the core AI, to distinct challenge classes.

The AI’s core will be initialized by heuristics from causality, information, logic, planning, problem-solving, and utility. These apply to all types of challenges. The tensors’ added flavor, needs to be learned using cooperative learning via a renormalization procedure, from the IQ-game’s episodes. The construction of the insight-gain tensors and cooperative learning will be described in future work.

b) Conclusion

We presented the foundations of SN-Logic, designed to boost human insight, to help overcome challenges that are hard to deal with, using traditional AI (mainly, predicate logic and deep learning neural nets). This required a logic, capable of coping with the concepts necessary to measure insight-gains: causality (causes of insight gains), dynamics (adaptive reasoning frameworks), information, probability, uncertainty (Shannon) and utility (von Neumann).

In this paper, we presented the following:

- The two-person ($H, A_{SN}$) cooperative IQ-game’s role from both $H$’s and $A_{SN}$’s perspectives
- The frame drift problem: coping with the changing understanding of a challenge, using a (non-brittle) logic and optimization process, which continuously adapt to the current human understanding and intention
- SN-Logic’s requirements to compute insightfulness (which involves causality, information, logic, probability, uncertainty and utility) and the concept spaces over which SN-Logic operates (to scope the quantifiers)
- SN-Logic’s grammar: semantics + syntax for posing questions $q \in Q$ from a vast space of potential questions. The syntax is used by a dual question generator ($q \in Q_{min}, q \in Q_{max}$), from which all questions are built ($N_{ques} = O(10^7)$)
- SN-Logic predicates of two question classes: problem difficulty-minimizing, and solution quality-maximizing, used in all inferences
- The complexity of SN-Logic, and show it’s broad scope and capability of coping with a large number of distinct challenge classes.
- The SN normal-form for making valid inferences, about a question’s insightfulness, efficiently within a vast space of possibilities

Ref

• *Insight Gain Tensors* $\mu(\text{when}, \text{where}, \text{what}, \text{which})$ are necessary to select *sound* inferences (real-world accurate), from a vast (effectively infinite) number of *valid* ones (those with SN normal-form). $\mu$ measures the human insight gains, associated to questions posed, within their cognitive mindsets ($C_{\text{min}}, C_{\text{max}}$)

• A validation test, to show that SN-Logic can replicate the solution steps, to a real-world solved case (discovery in quantum field theory)

This paper focused solely on logic and validity of SN-inferences. It has not dealt with the equally important issue of scientific soundness and accuracy. We will present the construction of the insight-gain convolution tensors and kernels, and the learned structure (cooperative learning), in future papers.

### VII. Appendices

A: Vector Space of Exploration Steps $T$ (sample)

<table>
<thead>
<tr>
<th>Time basis vector: $\text{when} \equiv p_1 \in T$</th>
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<tbody>
<tr>
<td>to identify an obstacle</td>
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<td>to minimize the obstacle</td>
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<tr>
<td>to explore solution ideas</td>
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<td>to question a solution idea</td>
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<td>to verify a solution idea</td>
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B: Vector Space of Cognitive Difficulties $S_D$ (sample)

<table>
<thead>
<tr>
<th>Basis vectors of cognitive obstacles: $\text{where} \equiv p_2 \in S_D$</th>
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<tbody>
<tr>
<td>inability to classify</td>
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<td>inability to compute</td>
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<td>inability to predict</td>
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<tr>
<td>inability to rank</td>
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<td>inability to relate</td>
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<tr>
<td>inability to select</td>
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<td>inability to simplify</td>
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<td>inability to solve</td>
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<tr>
<td>inability to transform</td>
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<tr>
<td>inability to verify</td>
</tr>
</tbody>
</table>

etc.
C: Vector Space of Difficulty Causes $S_C$ (sample)

<table>
<thead>
<tr>
<th>Basis vectors of causes: $p_3 \in S_C$</th>
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<tbody>
<tr>
<td>level of abstraction of item</td>
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<tr>
<td>level of ambiguity of item</td>
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<tr>
<td>level of complexity of item</td>
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<td>level of dependencies in item</td>
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<td>level of flaws in item</td>
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<td>level of fragmentation of item</td>
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<td>level of implicitness in item</td>
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<td>level of impracticality of item</td>
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<td>level of imprecision of item</td>
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<td>level of incompleteness of item</td>
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<td>level of inconsistency in item</td>
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<tr>
<td>level of indecision about item</td>
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<td>level of indetermination in item</td>
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<td>level of inefficiency of item</td>
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<td>level of insufficiency of item</td>
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<td>level of impracticality of item</td>
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<td>level of inefficiency of item</td>
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<td>level of insufficiency of item</td>
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<tr>
<td>level of impracticality of item</td>
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<tr>
<td>level of imprecision of item</td>
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<tr>
<td>etc.</td>
</tr>
<tr>
<td>missing assumption about item</td>
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<tr>
<td>missing bounds on item</td>
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<tr>
<td>missing capacity for item</td>
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<tr>
<td>missing classification of item</td>
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<tr>
<td>missing confidence in item</td>
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<tr>
<td>missing connections in item</td>
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<tr>
<td>missing constraints on item</td>
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<tr>
<td>missing evidence for item</td>
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<tr>
<td>missing explanation for item</td>
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<td>missing freedom to item</td>
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<tr>
<td>missing information about item</td>
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<td>missing intuition for item</td>
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<td>missing justification for item</td>
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<td>missing restriction on item</td>
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<tr>
<td>missing scales in item</td>
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<tr>
<td>missing statements in item</td>
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<td>missing tools for item</td>
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<td>missing verification of item</td>
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<tr>
<td>etc.</td>
</tr>
</tbody>
</table>
### D: Vector Space of Mental Goals $S_G$ (sample)

<table>
<thead>
<tr>
<th>Basis vectors of cognitive goals: $\text{where} \equiv p_2 \in S_G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>clarity about the</td>
</tr>
<tr>
<td>confidence in the</td>
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<tr>
<td>construction of the</td>
</tr>
<tr>
<td>criticism of the</td>
</tr>
<tr>
<td>exploitation of the</td>
</tr>
<tr>
<td>imagination for the</td>
</tr>
<tr>
<td>intuition for the</td>
</tr>
<tr>
<td>understanding of the</td>
</tr>
<tr>
<td>etc.</td>
</tr>
</tbody>
</table>

Note: mental goals [where] are intentions one tries to maximize, under constraints. The vector $\text{where} \in S_G$ rotates in $S_G$, with the mindset $C$ about the challenge.

### E: Vector Space of Solution Elements $S_S$ (sample)

<table>
<thead>
<tr>
<th>Basis vectors of solution elements: $\text{what} \equiv p_3 \in S_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>solution’s agents</td>
</tr>
<tr>
<td>solution’s cases</td>
</tr>
<tr>
<td>solution’s components</td>
</tr>
<tr>
<td>solution’s consequences</td>
</tr>
<tr>
<td>solution’s constraints</td>
</tr>
<tr>
<td>solution’s dimensions</td>
</tr>
<tr>
<td>solution’s economy</td>
</tr>
<tr>
<td>solution’s efficiency</td>
</tr>
<tr>
<td>solution’s effectiveness</td>
</tr>
<tr>
<td>solution’s ethics</td>
</tr>
<tr>
<td>solution’s form</td>
</tr>
<tr>
<td>solution’s framework</td>
</tr>
<tr>
<td>solution’s information</td>
</tr>
<tr>
<td>solution’s justification</td>
</tr>
<tr>
<td>solution’s methods</td>
</tr>
<tr>
<td>solution’s plan</td>
</tr>
<tr>
<td>solution’s properties</td>
</tr>
<tr>
<td>solution’s qualities</td>
</tr>
<tr>
<td>solution’s relationships</td>
</tr>
<tr>
<td>solution’s requirements</td>
</tr>
<tr>
<td>solution’s resources</td>
</tr>
<tr>
<td>solution’s restrictions</td>
</tr>
<tr>
<td>solution’s space</td>
</tr>
<tr>
<td>solution’s statements</td>
</tr>
<tr>
<td>solution’s sustainability</td>
</tr>
<tr>
<td>solution’s utility</td>
</tr>
<tr>
<td>solution’s value</td>
</tr>
<tr>
<td>etc.</td>
</tr>
</tbody>
</table>
F: Space of Actions \( S_A = O_p \times O_b \) (tiny sample)

<table>
<thead>
<tr>
<th>Conceptual Action Space: operation ( \in O_p \times ) object ( \in O_b )</th>
</tr>
</thead>
</table>

Actions to minimize indecision:
avoiding, comparing, demanding, imposing, evaluating, excluding, justifying, maximizing, minimizing, optimizing, prioritizing, ranking, requiring, selecting, weighing items etc.

Actions to minimize incomprehension:
classifying, collecting, defining, explaining, exploring, exploiting, decomposing, grouping, imposing, interpreting, isolating, reconstructing, relating, removing, separating items etc.

Actions to minimize inexperience:
exploring cases, exploring examples, exploring idealisations, exploring simplifications etc.

Actions to minimize skepticism:
comparing, demanding, excluding, explaining, gathering, imposing, justifying, reasoning, refuting, rejecting, requiring, searching for, testing, verifying items etc.

Actions to minimize unfamiliarity:
building an analogy, building a model, defining concepts, looking for items, outlining facts

Actions to maximize ability:
training to abstract, training to eliminate, training to exploit, training to organize, training to perform, training to relate, training to select, training to simplify, training to solve, training to transform etc.

Actions to maximize clarity:
classifying, connecting, defining, idealizing, ordering, organizing, outlining, reducing, relating, removing, separating, simplifying, summarizing items etc.

Actions to maximize criticism:
questioning an assumption, questioning a premise, questioning the framework, questioning a representation, questioning the necessity, questioning the sufficiency, questioning a method, questioning a path, questioning a solution, questioning the value etc.

Actions to maximize exploitation:
using an assumption, using a fact, using a given, using a constraint, using a property, using a relationship, using a restriction, using a statement, using a theorem etc.

Actions to maximize imagination:
weakening an assumption, weakening a bound, weakening a condition, weakening a constraint, weakening a requirement, weakening a restriction, weakening a rule, weakening a statement etc.

Actions to maximize intuition:
exploring an analogy, exploring a case, exploring an example, exploring a diagram, exploring a metaphor, exploring a model, exploring a story, exploring a simplification etc.
References Références Referencias

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Computational Thinking and the Curriculum of Mathematics in Portugal

By Sandra dos Anjos Canário Custódio Ribeiro
Researcher at Le@d at Universidade Aberta

Abstract- The emphasis on the importance of programming and computational thinking has been a constant in recent pedagogical trends (Wing, 2006, 2010; NRC, 2011).

In the same perspective Pollock et al. (2019) characterize computational thinking as decomposition, algorithms, data and abstraction.

According to Selby & Woolard (2013) and Tabesh (2017), computational thinking, in addition to being associated with decomposition, pattern recognition, algorithms and abstraction, identifies the importance of debugging, that is, the ability to test and evaluate the effectiveness of the solution, correct errors and seek to refine and optimize the solution.

Keywords: teaching and learning of mathematics, computational thinking, problem solving.

GJSFR-F Classification: DDC Code: 005.1 LCC Code: QA76.6
Abstract: The emphasis on the importance of programming and computational thinking has been a constant in recent pedagogical trends (Wing, 2006, 2010; NRC, 2011).

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According to Selby & Woolard (2013) and Tabesh (2017), computational thinking, in addition to being associated with decomposition, pattern recognition, algorithms and abstraction, identifies the importance of debugging, that is, the ability to test and evaluate the effectiveness of the solution, correct errors and seek to refine and optimize the solution.

This was the framework that was taken into account for the start of the MatemaTIC pilot project, promoted by the Directorate-General for Education (DGE), with the joint organization of the Association of Mathematics Teachers (APM), University of Coimbra (UC) and of the CCTIC of the University of Évora (CCTIC UE).

This project, started in 2019, involved teachers of the 1st Cycle of Basic Education from 30 Groups of Schools in Portugal and its main objective was to create resources and training contexts for teachers of this level of education, to support the development of their skills, professional skills in the fields of mathematics and ICT, so that they are able to work on issues of computational thinking, algorithms and computing, in the classroom, with students.

The final considerations point to the importance of the theme in the awareness of the learning that they intend to consolidate in the students; the importance of the contents to be developed; in the process of supporting students with gradually more complex tasks, helping to build reasoning and develop mathematical language; the importance of generalizing and transferring the problem-solving process to a wide variety of similar tasks; the importance of collaborative work; in the active role of the student in the construction of knowledge; and in the importance of involving the student in the evaluation process in the sense of self-assessment and self-correction.

Keywords: teaching and learning of mathematics, computational thinking, problem solving.
Foi este o enquadramento que foi tido em conta para o início do projeto piloto MatemaTIC, promovido pela Direção-Geral da Educação (DGE), com a organização conjunta da Associação de Professores de Matemática (APM), da Universidade de Coimbra (UC) e do CCTIC da Universidade de Évora (CCTIC UE).

Este projeto, iniciado em 2019, envolveu professores do 1.º Ciclo do Ensino Básico de 30 Agrupamentos de Escolas de Portugal continental e teve como objetivo principal criar recursos e contextos de formação para professores deste nível de ensino, para dar suporte ao desenvolvimento das suas competências profissionais nos domínios da matemática e das TIC, para que fiquem habilitados a trabalhar as questões do pensamento computacional, da programação e da computação, em sala de aula, com os alunos.

As considerações finais apontam para a importância da temática na tomada de consciência das aprendizagens que pretendem consolidar nos alunos; na importância dos conteúdos a desenvolver; no processo de apoio aos alunos com tarefas gradualmente mais complexas, ajudando a construir raciocínios e a desenvolver linguagem matemática; na importância da generalização e transferência do processo de resolução de problemas para uma ampla variedade de tarefas semelhantes; na importância do trabalho colaborativo; no papel ativo do aluno na construção do conhecimento; e na importância de envolver o aluno no processo de avaliação no sentido de se autoavaliar e de se autocorrigir.

Palavras-Chave: ensino e aprendizagem da matemática, pensamento computacional; resolução de problemas.

**Artigo**

Em contexto educativo, numa sociedade marcada por rápidas mudanças sociais, decorrentes da evolução científica e tecnológica, torna-se premente que os alunos passem de meros consumidores tecnológicos a produtores de conteúdos. Os alunos de hoje têm de ser preparados para profissões que não existem, pelo que, mais do que conhecimentos, necessitam de adquirir competências para o século XXI, que lhes vão permitir reconstruir o conhecimento à medida das necessidades das novas profissões. O Pensamento Computacional surge, assim, como uma das competências-chave dentro do quadro das 21ª Century Skills, de acordo com o referido no DigiCompEdu - Quadro Europeu de Competência Digital para Educadores.

A ênfase na importância da programação e pensamento computacional tem sido uma constante nas correntes pedagógicas recentes (Wing, 2006, 2010; NRC, 2011). Nas áreas das Ciências, Tecnologias, Engenharias, Artes, Matemática (STEAM), a programação e a robótica revestem-se de particular importância, sendo reconhecidas como indispensáveis no desenvolvimento de competências, tais como a resolução de problemas e o aumento da eficiência através da automação (Wing, 2010).

Desta forma, afigura-se necessário o desenvolvimento do pensamento computacional porque envolve a resolução de problemas, o que enfatiza a ideia de Polya (2004), que definiu a abstração (definida como a combinação de analogia, generalização e especialização) e a decomposição de problemas como cruciais para o sucesso na resolução de problemas.

Na mesma óptica Pollock et al. (2019) caracterizam o pensamento computacional como a decomposição (divide um problema em sub-problemas), algoritmos (cria uma série de etapas ordenadas para resolver um problema ou alcançar uma meta), dados (analisa um conjunto de dados para garantir que facilita a descoberta de padrões e relações) e abstração (reduz a complexidade para criar uma representação geral de um
processo ou grupo de objetos para que não seja apenas apropriado para o objeto imediato, mas também para que possa ser usado em diferentes contextos).

Selby & Woolard (2013) e Tabesh (2017) mencionam que o pensamento computacional para além de estar associado à decomposição, reconhecimento de padrões, algoritmia e abstracção, identificam a importância da depuração, ou seja, a capacidade para testar e avaliar a eficácia da solução, corrigir erros e procurar refinar e otimizar a solução.

Desta forma, há uma convergência de várias áreas que promovem o desenvolvimento da literacia matemática, ou seja, a capacidade de utilizar conhecimentos matemáticos na resolução de problemas da vida quotidiana (Ponte, 2003).

Foi este o enquadramento que foi tido em conta para o início do projeto piloto MatemaTIC, promovido pela Direção-Geral da Educação (DGE), com a organização conjunta da Associação de Professores de Matemática (APM), da Universidade de Coimbra (UC) e do CCTIC da Universidade de Évora (CCTIC UE).

Para além das referências anteriormente mencionadas tivemos em consideração os documentos curriculares, atualmente em vigor, nomeadamente: o Perfil dos Alunos à Saída da Escolaridade Obrigatória, as Orientações curriculares para as TIC no 1.º CEB e as Aprendizagens Essenciais da Matemática.

Estes documentos curriculares têm um carácter central, enquanto conteúdos de aprendizagem na área curricular de Matemática, tanto nas transversais, como conhecimentos matemáticos, assim como nas atitudes face à matemática.

Este projeto surgiu tendo como referência os autores de renome na área do pensamento computacional e por outro lado como uma necessidade de dar resposta às necessidades dos alunos do século XXI. Salientamos que o pensamento computacional já fazia parte do currículo de 11 países, tais como (Euforia, República Checa, Dinamarca, Finlândia, França, Grécia, Hungria, Itália, Lituânia, Suíça e Turquia.

Este projeto, que contribuiu para a reflexão do Grupo Trabalho de Matemática, criado no âmbito do Despacho n.º 12530/2018, alterado pelo Despacho n.º 7269/2019, que reconheceu como necessidade futura relativa à educação matemática o pensamento computacional e as diferentes áreas a ele associadas.

Este projeto, iniciado em 2019, envolveu professores do 1.º Ciclo do Ensino Básico de 30 Agrupamentos de Escolas de Portugal continental e teve como objetivo principal a criação de recursos e contextos de formação para professores deste nível de ensino, para dar suporte ao desenvolvimento das suas competências profissionais, nos domínios da matemática e das TIC, para os habilitar para trabalhar as questões do pensamento computacional, da algoritmia e da computação, em sala de aula, com os alunos.
Figura 1: A literacia matemática como ponto de convergência

Os objetivos gerais deste projeto foram: (1) Sensibilizar para a importância destes conteúdos na formação dos alunos do século XXI; (2) Preparar e realizar uma formação para professores em que se mobilizem conceitos do currículo da matemática, de algoritmia, pensamento computacional e programação e se preparam atividades práticas para a sala de aula; (3) Identificar e avaliar as potencialidades do recurso a tarefas que relacionem a Matemática com as TIC; (4) Criar um conjunto de recursos educativos que sirvam de suporte ao trabalho de outros docentes deste nível de ensino.

Integradas na fase de implementação do projeto MatemaTIC, as formação de formadores e de professores decorreram ao longo do ano letivo 2020/21 em regime de e-learning. No processo de formação participaram 11 formadores e 107 professores, distribuídos por 9 turmas de norte a sul do país.

Estas formações, na modalidade de curso, realizaram-se de forma articulada, permitindo a existência de tempo, por um lado, para que os formadores discutissem conceitos e dinâmicas de formação e, por outro, para que os professores se apropriassem dos conceitos trabalhados nas tarefas propostas e, deste modo, tivessem condições para as implementar com os seus alunos. Foram, ainda, criadas condições para que os professores pudessem desenhar e/ou adaptar tarefas matemáticas com a intencionalidade de integração e desenvolvimento de práticas do pensamento computacional.
Figura 2: Esquema com organização da formação de formadores e com a formação de professores

No âmbito da formação, foram trabalhadas tarefas matemáticas, que contemplam o uso de recursos muito diversos (desde o papel e lápis, às linguagens de programação por blocos) e as respetivas práticas de sala de aula, tendo sido planificadas com a intencionalidade de desenvolver o pensamento computacional, através da explicitação de práticas como a abstracção, a decomposição, o reconhecimento de padrões e a depuração. Os recursos educativos trabalhados, discutidos e implementados, serão tornados públicos, sob a forma de e-portfolio, para serem utilizados por outros professores deste nível de ensino.

Ao longo do projeto, foram organizadas reuniões de acompanhamento, para as quais foram envolvidos elementos da DGE, da APM, bem como da Universidade de Coimbra e do CCTIC da Universidade de Évora. Paralelamente, foi criada uma comunidade de prática, que serviu de apoio e suporte à formação, facilitando o processo de comunicação entre os formadores e professores, através da interação em fóruns de diálogo e partilha de ideias/materiais.

Deste projeto será elaborado um relatório, pela equipa da Universidade de Coimbra, que será tornado público, com dados relativos a:

- Metodologias de Ensino;
- Processos e métodos de implementação de práticas inovadoras;
- Impacto dessas práticas no desenvolvimento profissional dos professores;
- Outros dados considerados pertinentes.

Ainda no âmbito do projeto, decorreu, nos dias 7 e 8 de maio de 2021, o Evento Nacional MatemaTIC, com o título: "O Pensamento Computacional e o Currículo da Matemática em Portugal".

Na 1.ª parte do evento, concretizou-se uma discussão em torno dos fundamentos de diversas áreas das Ciências da Computação, em articulação com o currículo da Matemática e, na 2.ª parte, foram apresentadas algumas das atividades práticas desenvolvidas com professores e alunos. Participaram 1330 professores, formadores, especialistas e público em geral.
Deste projeto resultou a reflexão de que as ações de formação tiveram um impacto muito positivo quer nos formandos, quer nos formadores, porque, para além do desenvolvimento de conhecimentos, capacidades e atitudes, se promoveu a ligação entre especialistas e professores e foram sugeridas boas práticas que se assumiram como modelos para a replicação da formação.

Como resultados desta formação sublinhamos uma evolução dos formandos no domínio e na aplicação de práticas do pensamento computacional e foram desenvolvidas aprendizagens e metodologias e estratégias de ensino ativas e motivadoras que levaram os alunos e os professores a desenvolverem competências básicas ao nível da matemática e da prática do pensamento computacional. É ainda de salientar a reflexão que os formandos fazem da formaçãoporque evidenciaram:

- A tomada de consciência das aprendizagens que pretendem consolidar nos alunos;
- A importância dos conteúdos a desenvolver;
- O processo de apoio aos alunos com tarefas gradualmente mais complexas, ajudando a construir raciocínios e a desenvolver linguagem matemática;
- O importância da generalização e transferência do processo de resolução de problemas para uma ampla variedade de tarefas semelhantes;
- A Importância do trabalho colaborativo;
- O papel ativo do aluno na construção do conhecimento; e
- A importância de envolver o aluno no processo de avaliação no sentido de se autoavaliar e de se autocorrigir;

Neste processo destacamos como dificuldades a adaptação da formação na modalidade de oficina, com regime presencial para a modalidade de curso em regime de e-learning uma vez que a manipulação de materiais concretos constituía uma mais-valia da formação. Porém, apesar dos constrangimentos verificou-se uma melhoria gradual da capacidade de explorar e discutir matemática e consequente melhoria das segundas versões das tarefas realizadas nas sessões assíncronas.

Em jeito de síntese, podemos destacar a ênfase crescente nas práticas de pensamento computacional na abordagem das tarefas matemáticas e também é de salientar a importância da discussão de conceitos matemáticos e práticas de sala de aula e do desenvolvimento do pensamento computacional de forma integrada.

Neste contexto, apresentamos como sugestão o desenvolvimento e alargamento das ações a criação de redes de escolas e de formadores que possam replicar a formação, permitindo a construção de uma comunidade de apoio ao trabalho a ser produzido, criando uma estrutura de apoio intermédia.

**References Références Referencias**


*Legislação referida*


Despacho n.º 7269/2019, DR n.º 156/2019, Série II de 16 de agosto.
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An Improved Hungarian Algorithm for a Special Case of Unbalanced Assignment Problems

By Mohammad Shyfur Rahman Chowdhury

International Islamic University Chittagong

Abstract- The current Hungarian approach to solving unbalanced assignment issues is based on the notion that some tasks should be delegated to fictitious or covert components, and those studies should be left unperformed. In real-world scenarios, it may be desirable to carry out all of the tasks on fundamental details. To do this, multiple tasks may be distributed to a single machine. The current research's enhanced Hungarian method for addressing unbalanced assignment challenges results in the ideal work assignment policy. An example using numbers shows how well the suggested strategy works and how effective it is. The acquired result is then likened to other current approaches to demonstrate our algorithm's superiority.

GJSFR-F Classification: DDC Code: 005.1 LCC Code: QA76.6

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Mohammad Shyfur Rahman Chowdhury

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I. Introduction

One of the most significant applications of optimization theory is the Assignment Problem, where various tasks need to be distributed across components to be completed, such as spreading personnel to offices and drivers to buses, among other things. There have been numerous ways of exploring the best policy for assigning jobs to components. The Hungarian approach is the most frequently used method for determining the best assignment policy. The Hungarian Method was named by Kuhn (Kuhn, 1995) and was based in significant part on earlier work by two Hungarian mathematicians, Egervary and D. Konig. When the Hungarian approach is used to address an uneven assignment problem, the technique assigns the jobs to fake components that do not perform them (if the number of jobs is greater than the number of components). It seems impossible to leave jobs unfinished in real-world situations. As a result, rather than assigning extra work to dummy components, it is recommended to do each and every job that may be done by assigning multiple jobs to a single component.

To tackle assignment problems, the Hungarian algorithm (Chen., 2011) (W. B. Lee, 1997) and specific heuristic algorithms (e.g., simulated annealing method (B. Li, 2002) ant colony algorithm (Y. Liang, 2005) (R. K. Yin, 2008) particle swarm algorithm (W. F. Tan, 2007) and genetic algorithm (S. Q. Tao, 2004). Heuristic approaches are frequently employed to solve problems with assignments of high complexity. However, because the search is conducted at random, it cannot guarantee that the optimal result will be obtained. The Hungarian algorithm is an algorithm with a mathematical foundation. The Hungarian algorithm is commonly used to tackle assignment problems because of its simplicity and ability to find the best solution without requiring validation.

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In (J. L. Du, 2010), an enhanced Hungarian algorithm, the "add zero row approach," was presented to handle the incomplete assignment problem based on a study of the standard Hungarian algorithm. The Hungarian method was first introduced by (T. M. Chang, 2004) and it was used to solve a common assignment problem, such as a marriage assignment. (Ma., 2014) suggested a new method, the "difference method," for solving the non-standard assignment problem: "tasks more than the number of people." This method is more straightforward than the standard algorithm because it does not require using a new matrix to replace the original coefficient matrix at the beginning and instead solves the problems directly on the old coefficient matrix. (Qiu., 2013) suggested an enhanced Hungarian algorithm for studying multiple maintenance scheduling problems in hostile environments. A quick order reduction optimization approach based on the classical Hungarian algorithm was proposed (J. X. Ren, 2014) to increase the efficiency of the distribution of cloud computing activities. The order of the matrix is quickly lowered and, the computing efficiency is increased by deleting the matrix elements that are determined. Reference (R, 2014) used the Hungarian technique to investigate the dynamic power allocation of weapon-targets by changing it into an assignment issue. Furthermore, the traditional Hungarian algorithm has been used to tackle business and technical challenges in a variety of disciplines (P. Hahn, 1998) (Kuhn., 2012) (E. M. Loiola, 2007) (T. Tassa, 2008) (S. Promparamote, 2006) (M. H. Paul, 2013).

According to many authors, the unbalanced assignment problem has many solutions, all of which assume that all jobs are finished. Kumar (Kumar, 2006) came up with a fresh approach to address the problem of uneven assignments. The decision-maker can allocate several tasks to a single component using his methodology. The Lexi Search Approach, developed by Haragopal and Yadaiah (V. Yadaiah, 2016), is a more effective technique for dealing with imbalanced assignment problems that yield the same outcomes as Kumar (Kumar, 2006). In the same year, Kumar’s (Kumar, 2006), Haragopal’s and Yadaiah’s (V. Yadaiah, 2016) methods were surpassed by an approach provided by Betts and Vasko (Vasko, 2016).

II. Mathematical Formulation

Consider the processing cost of the jth job on the ith component be Cij, where i = 1, 2..., m; j = 1, 2..., n. The challenge is to create an ideal work assignment method that ensures that every task is finished while keeping the overall cost of doing so as low as possible.

Mathematical model of an unbalanced assignment problem can be expressed as,

Minimize: \[ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}X_{ij} \]

Subject to constraints

\[ \sum_{i=1}^{n} X_{ij} \geq 1, i = 1,2,\ldots, m \]  \hspace{1cm} (1)

\[ \sum_{j=1}^{n} X_{ij} \geq 1, j = 1,2,\ldots, n \]  \hspace{1cm} (2)

\[ X_{ij} = 0 \text{ or } 1 \]
III. Proposed Algorithm

Think about the issue of distributing a group of "n" jobs. $J = J_1, J_2, ..., J_n$ to an execution set with "m" components. $C = (C_1, C_2, ..., C_m)$. $X_{ij}, i = 1, 2, ..., m; j = 1, 2, ..., n$ (n > m), indicating that there are more tasks than components. In this case, n stands for columns and m for rows.

**Step-01:** Enter the following values: m, n

**Step-02:** Each column’s lowest cost should be subtracted from the column it belongs. This process results in each column reducing the cost column by having at least a single zero.

**Step-03:** Determine each row’s lowest cost and then deduct it from the associated row.

**Step-04:** Analyze the feasibility of doing an ideal task so that the smallest number of lines needed to cover each zero is calculated. If the number of lines and rows equals one, proceed to Step 7; if not, proceed to Step 5.

Choose the minimum uncovered cost if the number of lines exceeds the number of rows.

1. Take the least exposed cost in the table and subtract it from each exposed cost.
2. The cost at each intersection point is added by those minimum cost.

**Step 06:** If the case (the total number of lines and rows is equal) fails, then continue steps 04 and 05.

To assign the work, look for a row with only one zero. Choose that zero and block the other zeros in the relevant columns (the same component can be performed on more than one job, but the same job cannot be assigned more than one component).

**Step-08:** Assign the value with the lowest cost in the initial problem if there is a tie, that is, if any rows have two or more zeros.

**Step-09:** Repeat steps 7 and 8 until all positions have been filled, that is, all jobs have been assigned to one or more processing components.

**Step-10:** End

IV. Parallelism between Proposed Algorithm and Hungarian Algorithm

<table>
<thead>
<tr>
<th>Proposed</th>
<th>Hungarian</th>
</tr>
</thead>
<tbody>
<tr>
<td>• This tactic is employed to address issues with unbalanced assignments.</td>
<td>• This tactic is employed to address issues with unbalanced assignments.</td>
</tr>
<tr>
<td>• If the number of jobs exceeds the number of processing components, all jobs must be completed using the available components.</td>
<td>• If the number of jobs exceeds the number of components, the remaining jobs are performed by dummy components.</td>
</tr>
<tr>
<td>• There are no unfinished projects.</td>
<td>• Some jobs aren’t being completed.</td>
</tr>
<tr>
<td>• Related to at least one job that can be performed by a single component.</td>
<td>• A single component can only do one thing.</td>
</tr>
<tr>
<td>• A single component can only be assigned to a single job.</td>
<td>• Only one component can be allocated to a single job.</td>
</tr>
</tbody>
</table>
V. Mathematical Analysis

Let us take a problem of 8 jobs and 5 processing components with associated execution costs as given in Table 2.

**Table 2**

<table>
<thead>
<tr>
<th></th>
<th>J₁</th>
<th>J₂</th>
<th>J₃</th>
<th>J₄</th>
<th>J₅</th>
<th>J₆</th>
<th>J₇</th>
<th>J₈</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>270</td>
<td>260</td>
<td>220</td>
<td>190</td>
<td>300</td>
<td>320</td>
<td>180</td>
<td>250</td>
</tr>
<tr>
<td>C₂</td>
<td>210</td>
<td>190</td>
<td>300</td>
<td>200</td>
<td>290</td>
<td>180</td>
<td>190</td>
<td>310</td>
</tr>
<tr>
<td>C₃</td>
<td>190</td>
<td>260</td>
<td>230</td>
<td>220</td>
<td>280</td>
<td>190</td>
<td>300</td>
<td>290</td>
</tr>
<tr>
<td>C₄</td>
<td>250</td>
<td>210</td>
<td>180</td>
<td>190</td>
<td>290</td>
<td>240</td>
<td>190</td>
<td>300</td>
</tr>
<tr>
<td>C₅</td>
<td>160</td>
<td>180</td>
<td>160</td>
<td>140</td>
<td>210</td>
<td>170</td>
<td>180</td>
<td>200</td>
</tr>
</tbody>
</table>

**Table 3: Follows steps 1, 2 and 3**

<table>
<thead>
<tr>
<th></th>
<th>J₁</th>
<th>J₂</th>
<th>J₃</th>
<th>J₄</th>
<th>J₅</th>
<th>J₆</th>
<th>J₇</th>
<th>J₈</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>50</td>
<td>80</td>
<td>60</td>
<td>90</td>
<td>40</td>
<td>150</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>C₂</td>
<td>50</td>
<td>0</td>
<td>130</td>
<td>70</td>
<td>40</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>C₃</td>
<td>60</td>
<td>60</td>
<td>50</td>
<td>50</td>
<td>10</td>
<td>0</td>
<td>100</td>
<td>70</td>
</tr>
<tr>
<td>C₄</td>
<td>40</td>
<td>20</td>
<td>10</td>
<td>70</td>
<td>80</td>
<td>60</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>C₅</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 4: Follows step 4**

<table>
<thead>
<tr>
<th></th>
<th>J₁</th>
<th>J₂</th>
<th>J₃</th>
<th>J₄</th>
<th>J₅</th>
<th>J₆</th>
<th>J₇</th>
<th>J₈</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>40</td>
<td>80</td>
<td>60</td>
<td>50</td>
<td>90</td>
<td>150</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>C₂</td>
<td>40</td>
<td>0</td>
<td>130</td>
<td>50</td>
<td>70</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>C₃</td>
<td>10</td>
<td>60</td>
<td>50</td>
<td>60</td>
<td>50</td>
<td>0</td>
<td>100</td>
<td>70</td>
</tr>
<tr>
<td>C₄</td>
<td>80</td>
<td>20</td>
<td>10</td>
<td>40</td>
<td>70</td>
<td>60</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>C₅</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

By step 5; from the uncovered costs, choose the lowest cost (i.e. 10)
i. Deduct 10 from each exposed cost in the matrix above.
ii. To get Table 5, sum up 10 at each of the intersection points.

**Table 5**

<table>
<thead>
<tr>
<th></th>
<th>J₁</th>
<th>J₂</th>
<th>J₃</th>
<th>J₄</th>
<th>J₅</th>
<th>J₆</th>
<th>J₇</th>
<th>J₈</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>30</td>
<td>80</td>
<td>50</td>
<td>40</td>
<td>80</td>
<td>150</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>C₂</td>
<td>30</td>
<td>0</td>
<td>120</td>
<td>40</td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>C₃</td>
<td>0</td>
<td>60</td>
<td>40</td>
<td>50</td>
<td>40</td>
<td>0</td>
<td>100</td>
<td>60</td>
</tr>
<tr>
<td>C₄</td>
<td>70</td>
<td>0</td>
<td>0</td>
<td>30</td>
<td>60</td>
<td>60</td>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td>C₅</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Following steps 6 and 7, we allocate job J₃ to component M₁ and cross off the remaining zeros in the row corresponding to J₃; as a result, row four has just one zero. As stated in Table 6, allocate work J₇ to component M₄.

**Table 6**

<table>
<thead>
<tr>
<th></th>
<th>J₁</th>
<th>J₂</th>
<th>J₃</th>
<th>J₄</th>
<th>J₅</th>
<th>J₆</th>
<th>J₇</th>
<th>J₈</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>30</td>
<td>80</td>
<td>50</td>
<td>40</td>
<td>80</td>
<td>150</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>C₂</td>
<td>30</td>
<td>0</td>
<td>120</td>
<td>40</td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>C₃</td>
<td>0</td>
<td>60</td>
<td>40</td>
<td>50</td>
<td>40</td>
<td>0</td>
<td>100</td>
<td>60</td>
</tr>
<tr>
<td>C₄</td>
<td>70</td>
<td>0</td>
<td>0</td>
<td>30</td>
<td>60</td>
<td>60</td>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td>C₅</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
According to step 8, there is a tie in the 2nd and 3rd rows (containing two zeros). We allocate J_4 to component M_2 (in Table-7) because the cost associated with this position is the lowest in the original cost matrix.

Table 7

<table>
<thead>
<tr>
<th></th>
<th>J_1</th>
<th>J_2</th>
<th>J_3</th>
<th>J_4</th>
<th>J_5</th>
<th>J_6</th>
<th>J_7</th>
<th>J_8</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_1</td>
<td>30</td>
<td>80</td>
<td>50</td>
<td>40</td>
<td>80</td>
<td>150</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>C_2</td>
<td>30</td>
<td>0</td>
<td>120</td>
<td>40</td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>C_3</td>
<td>0</td>
<td>60</td>
<td>40</td>
<td>50</td>
<td>40</td>
<td>0</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>C_4</td>
<td>70</td>
<td>20</td>
<td>0</td>
<td>30</td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td>C_5</td>
<td>0</td>
<td>10</td>
<td>0 ×</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

Then following step 9, we get the final table-8.

Table 8

<table>
<thead>
<tr>
<th></th>
<th>J_1</th>
<th>J_2</th>
<th>J_3</th>
<th>J_4</th>
<th>J_5</th>
<th>J_6</th>
<th>J_7</th>
<th>J_8</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_1</td>
<td>30</td>
<td>80</td>
<td>50</td>
<td>40</td>
<td>80</td>
<td>150</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>C_2</td>
<td>30</td>
<td>0</td>
<td>120</td>
<td>40</td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>C_3</td>
<td>0</td>
<td>60</td>
<td>40</td>
<td>50</td>
<td>40</td>
<td>0 ×</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>C_4</td>
<td>70</td>
<td>20</td>
<td>0</td>
<td>30</td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td>C_5</td>
<td>0</td>
<td>10</td>
<td>0 ×</td>
<td>0</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

VI. RESULTS AND DISCUSSION

Table 9 shows the work assignment policy that reduces the overall cost.

<table>
<thead>
<tr>
<th>Component</th>
<th>Job</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_1</td>
<td>J_7</td>
<td>180</td>
</tr>
<tr>
<td>C_2</td>
<td>J_2, J_6</td>
<td>190,180</td>
</tr>
<tr>
<td>C_3</td>
<td>J_1</td>
<td>180</td>
</tr>
<tr>
<td>C_4</td>
<td>J_3</td>
<td>190</td>
</tr>
<tr>
<td>C_5</td>
<td>J_4, J_5, J_8</td>
<td>140, 210,200</td>
</tr>
</tbody>
</table>

Total Cost 1470

We find the total minimum cost in comparison to the other modified Hungarian methods like Kumar [26], Haragopal and Yadaiah [27], and Betts and Vasko [28].

VII. CONCLUSION

The study presents an improved Hungarian algorithm to solve a particular case (when the number of jobs is greater than the number of processing components) of an unbalanced assignment problem. Generally, most of the issues regarding assignments occur in the abovementioned case. The primary premise of our strategy is to allocate all tasks to be completed. If the created assignment plan is invalid, the virtual jobs will be changed, and the procedure will be continued until an actual optimal assignment strategy is discovered. We show that the assignment strategy found by the suggested approach is the best using mathematical analysis. It demonstrates that the revised algorithm is capable of determining the best assignment strategy.

REFERENCES RÉFÉRENCES REFERENCIAS


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On Bayesian Estimation of Loss of Estimators of Unknown Parameter of Binomial Distribution

By Randhir Singh
Ewing Christian College

Summary- This paper aims at the Bayesian estimation for the loss and risk functions of the unknown parameter of the binomial distribution under the loss function which is different from that given by Rukhin(1988). The estimation involves beta distribution, a natural conjugate prior density function for the unknown parameter. Estimators obtained are conservatively biased and have finite frequentist risk.

Keywords: Bayes Estimator, Loss Function, Risk Function, Binomial Distribution.

GJSFR-F Classification: DDC Code: 843.7 LCC Code: PQ2165.C5
On Bayesian Estimation of Loss of Estimators of Unknown Parameter of Binomial Distribution

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Keywords: Bayes Estimator, Loss Function, Risk Function, Binomial Distribution.

I. Introduction

Rukhin (1988) introduced a loss function given by,

\[ L(\theta, \delta, \gamma) = w(\theta, \delta) \gamma^{-\frac{1}{2}} + \gamma^{\frac{1}{2}} \] (1.1)

Where, \( \gamma \) is an estimator of the loss function \( w(\theta, \delta) \), which is non-negative. Guobing (2016) used this loss function and derived estimates of the loss and risk function of the parameter of Maxwell’s distribution. Singh (2021) took various forms of \( w(\theta, \delta) \) and derived estimates of the loss and risk function of the parameter of a continuous distribution which gives Half-normal distribution, Rayleigh distribution and Maxwell’s distribution as particular cases. Rukhin (1988) considered the Bayesian estimation of the unknown parameter \( \theta \) of the binomial distribution by taking

\[ w(\theta, \delta) = (\theta - \delta)^2 \] (1.2)

In this paper, Bayes estimate of the unknown parameter \( \theta \) of the binomial distribution has been obtained by replacing \( w(\theta, \delta) \) by \( w_1(\theta, \delta) \) given by

\[ w_1(\theta, \delta) = h(\theta)(\theta - \delta)^2 \] (1.3)

Where,

\[ h(\theta) = \frac{1}{\{\theta(1-\theta)\}} \] (1.4)

Author: Department of Statistics, Ewing Christian College, Prayagraj, India. e-mail: dr.singh.ecc@gmail.com
II. Estimation of Loss and Risk of the Parameter of Binomial Distribution

Let the random variable $X$ follows binomial distribution with parameters $n$ and $\theta$. Where $\theta$ is unknown satisfying $0 \leq \theta \leq 1$. The prior p.d.f. of $\theta$, denoted by $\pi_1(\theta)$ is as follows:

$$
\pi_1(\theta) = \begin{cases} 
\frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{B(\alpha, \beta)} & \text{if } \alpha \geq 0, \beta \geq 0, 0 < \theta < 1 \\
0 & \text{Otherwise}
\end{cases} 
$$

Under the assumption of prior probability density function (p.d.f.) for $\theta$ as above, Bayes estimates of $\theta$ derived by Rukhin (1988) were as follows: For $\alpha \geq 0, \beta \geq 0$

$$
\delta_B(X) = \frac{(X + \alpha)}{(n + \alpha + \beta)} 
$$

and for $\alpha = 0, \beta = 0$

$$
\delta_0(X) = \frac{X}{n} 
$$

It was shown that

$$
\mathbb{E}_\theta L(\theta, \delta_0, \gamma_0) = \infty
$$

Under $w_1(\theta, \delta)$ as above, the corresponding Bayes estimate is given by, For $\alpha \geq 0, \beta \geq 0$

$$
\delta_{1B}(X) = \frac{E\{\theta h(\theta)/X\}}{E\{h(\theta)/X\}}
$$

Or,

$$
\delta_{1B}(X) = \frac{(X + \alpha - 1)}{A - 2}
$$

On simplification, provided, $A = n + \alpha + \beta > 2$ and,

$$
\gamma_{1B}(X) = E\{\theta h(\theta)/X\} - \{\delta_{1B}(X)\}^2 E\{h(\theta)/X\}
$$
On Bayesian Estimation of Loss of Estimators of Unknown Parameter of Binomial Distribution

Or,
\[ \gamma_{1B}(X) = \frac{1}{A - 2} \quad (2.10) \]
on simplification, provided, \( A = n + \alpha + \beta > 2 \).

We see that, in this case \( \gamma_{1B}(X) \) does not depend upon \( X \) and is function of \( n, \alpha \) and \( \beta \)

\[ E_\theta L(\theta, \delta_{1B}, \gamma_{1B}) = E_\theta [h(\theta)(\theta - (X + \alpha - 1)(A - 2)^{-1})^2](A - 2)^{1/2} + (A - 2)^{-1/2} \quad (2.11) \]

Or,
\[ E_\theta L(\theta, \delta_{1B}, \gamma_{1B}) = [n + h(\theta)(1 - \alpha + \theta(\alpha + \beta - 2))]^2(A - 2)^{-3/2} + (A - 2)^{-1/2} < \infty \quad (2.12) \]

In this case,
\[ R(\theta, \delta_{1B}) = E_\theta \{h(\theta)(\theta - \delta_{1B})\}^2 \quad (2.13) \]

Or,
\[ R(\theta, \delta_{1B}) = [n + h(\theta)(1 - \alpha + \theta(\alpha + \beta - 2))]^2(A - 2)^{-2} \quad (2.14) \]

As mentioned by Keifer (1977), an estimator \( \gamma(X) \) is said to be conservatively biased if,
\[ E_\theta \{\gamma(X)\} \geq R(\theta, \delta) = E_\theta \{w(\theta, \delta)\} \quad (2.15) \]

In the light of this condition, \( \gamma_0(X) \) as given by Rukhin (1988) is not conservatively biased. In this case,
\[ E_\theta \{\gamma_{1B}(X)\} = \frac{1}{A - 2} \quad (2.16) \]

Let \( \delta_{0B}(X) \) and \( \gamma_{0B}(X) \) be values of \( \delta_{1B}(X) \) and \( \gamma_{1B}(X) \), respectively when, \( \alpha = \beta = 0 \).
If possible let,
\[ E_\theta \{\gamma_{0B}(X)\} \geq R(\theta, \delta_{0B}) \quad (2.17) \]

which holds if,
\[ -2\theta^2 + 2\theta - 1 \geq 0 \quad (2.18) \]

which is a contradiction, since \( 0 < \theta < 1 \) and maximum value of \(-2\theta^2 + 2\theta - 1 = -\frac{1}{2}\) which corresponds to \( \theta = 1/2 \). Moreover, \(-2\theta^2 + 2\theta - 1 = -1 \) for \( \theta = 1 \) and \( \theta = 0 \) Thus, \( \gamma_{0B}(X) \) is not conservatively biased.

When \( \alpha = \beta = 1 \), we have,
\[ E_\theta \{\gamma_{1B}(X)\} = R(\theta, \delta_{1B}) = \frac{1}{n} \quad (2.19) \]
When $\alpha = \beta > 1, \theta = 0.5$,

$$E_\theta \{ \gamma_{1B}(X) \} \geq R(\theta, \delta_{1B}) \quad (2.20)$$

When $\alpha = \beta > 1, \theta \neq 0.5$,

$$E_\theta \{ \gamma_{1B}(X) \} \geq R(\theta, \delta_{1B}) \quad (2.21)$$

which holds if

$$\alpha \leq 1 + g(\theta) \quad (2.22)$$

Where,

$$g(\theta) = \frac{2\theta(1-\theta)}{(2\theta-1)^2} \quad (2.23)$$

$g(\theta)$ is a monotonically increasing function of $\theta$ over the set $S = (0, 1) - \{0.5\}$. Hence, $\gamma_{1B}(X)$ as above, presents a valid 'frequentist report' as mentioned by Berger (1985).

The results are summarized in the following:

**THEOREM.** Let $(\delta_{1B}, \gamma_{1B})$ be Bayes estimators of the unknown parameter $\theta$ of the binomial distribution under the loss function $L(\theta, \delta, \gamma) = \frac{1}{\theta(1-\theta)}(\theta-\delta)^2\gamma^{-\frac{1}{2}} + \gamma^\frac{3}{2}$ and beta prior density with known parameters $\alpha$ and $\beta$. Then, the frequentist risk $E_\theta L(\theta, \delta_{1B}, \gamma_{1B})$ is finite for all values of $\alpha$ and $\beta$ provided $0 < \theta < 1$. For $\alpha = \beta = 0$, $\gamma_{1B}(X)$ is not conservatively biased. The estimator $\gamma_{1B}(X)$ is conservatively biased for $\alpha = \beta > 1$ and for $\alpha = \beta > 1$ satisfying $\alpha \leq 1 + \frac{2\theta(1-\theta)}{(2\theta-1)^2}, \theta \neq 0.5$. However, if $\alpha = \beta > 1, \theta = 0.5$, $\gamma_{1B}(X)$ is also conservatively biased.

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6. **Bookmarks are useful:** When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

7. **Revise what you wrote:** When you write anything, always read it, summarize it, and then finalize it.

8. **Make every effort:** Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

9. **Produce good diagrams of your own:** Always try to include good charts or diagrams in your paper to improve quality. Using several unnecessary diagrams will degrade the quality of your paper by creating a hodgepodge. So always try to include diagrams which were made by you to improve the readability of your paper. Use of direct quotes: When you do research relevant to literature, history, or current affairs, then use of quotes becomes essential, but if the study is relevant to science, use of quotes is not preferable.

10. **Use proper verb tense:** Use proper verb tenses in your paper. Use past tense to present those events that have happened. Use present tense to indicate events that are going on. Use future tense to indicate events that will happen in the future. Use of wrong tenses will confuse the evaluator. Avoid sentences that are incomplete.

11. **Pick a good study spot:** Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

12. **Know what you know:** Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

13. **Use good grammar:** Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice. Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

14. **Arrangement of information:** Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

15. **Never start at the last minute:** Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. **Multitasking in research is not good:** Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. **Never copy others' work:** Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. **Go to seminars:** Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. **Refresh your mind after intervals:** Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.
20. **Think technically:** Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

21. **Adding unnecessary information:** Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

22. **Report concluded results:** Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

23. **Upon conclusion:** Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium through which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

**Informal Guidelines of Research Paper Writing**

**Key points to remember:**
- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

**Final points:**

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

**The introduction:** This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

**The discussion section:**

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

**General style:**

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

**To make a paper clear:** Adhere to recommended page limits.
Mistakes to avoid:

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

Title page:

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

- Single section and succinct.
- An outline of the job done is always written in past tense.
- Concentrate on shortening results—limit background information to a verdict or two.
- Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.
The following approach can create a valuable beginning:

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

*Materials may be reported in part of a section or else they may be recognized along with your measures.*

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.
Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- Give details of all of your remarks as much as possible, focusing on mechanisms.
- Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

The Administration Rules

Administration Rules to Be Strictly Followed before Submitting Your Research Paper to Global Journals Inc.

Please read the following rules and regulations carefully before submitting your research paper to Global Journals Inc. to avoid rejection.

Segment draft and final research paper: You have to strictly follow the template of a research paper, failing which your paper may get rejected. You are expected to write each part of the paper wholly on your own. The peer reviewers need to identify your own perspective of the concepts in your own terms. Please do not extract straight from any other source, and do not rephrase someone else's analysis. Do not allow anyone else to proofread your manuscript.

Written material: You may discuss this with your guides and key sources. Do not copy anyone else's paper, even if this is only imitation, otherwise it will be rejected on the grounds of plagiarism, which is illegal. Various methods to avoid plagiarism are strictly applied by us to every paper, and, if found guilty, you may be blacklisted, which could affect your career adversely. To guard yourself and others from possible illegal use, please do not permit anyone to use or even read your paper and file.
**Criterion for Grading a Research Paper (Compilation)**

_**By Global Journals**_

Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals.

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<td><strong>Abstract</strong></td>
<td>A-B Clear and concise with appropriate content, Correct format. 200 words or below</td>
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<td></td>
<td>A-B Above 200 words</td>
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<td><strong>Introduction</strong></td>
<td>Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited</td>
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<td>Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads</td>
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<td><strong>Methods and Procedures</strong></td>
<td>Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake</td>
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<td><strong>Result</strong></td>
<td>Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited</td>
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