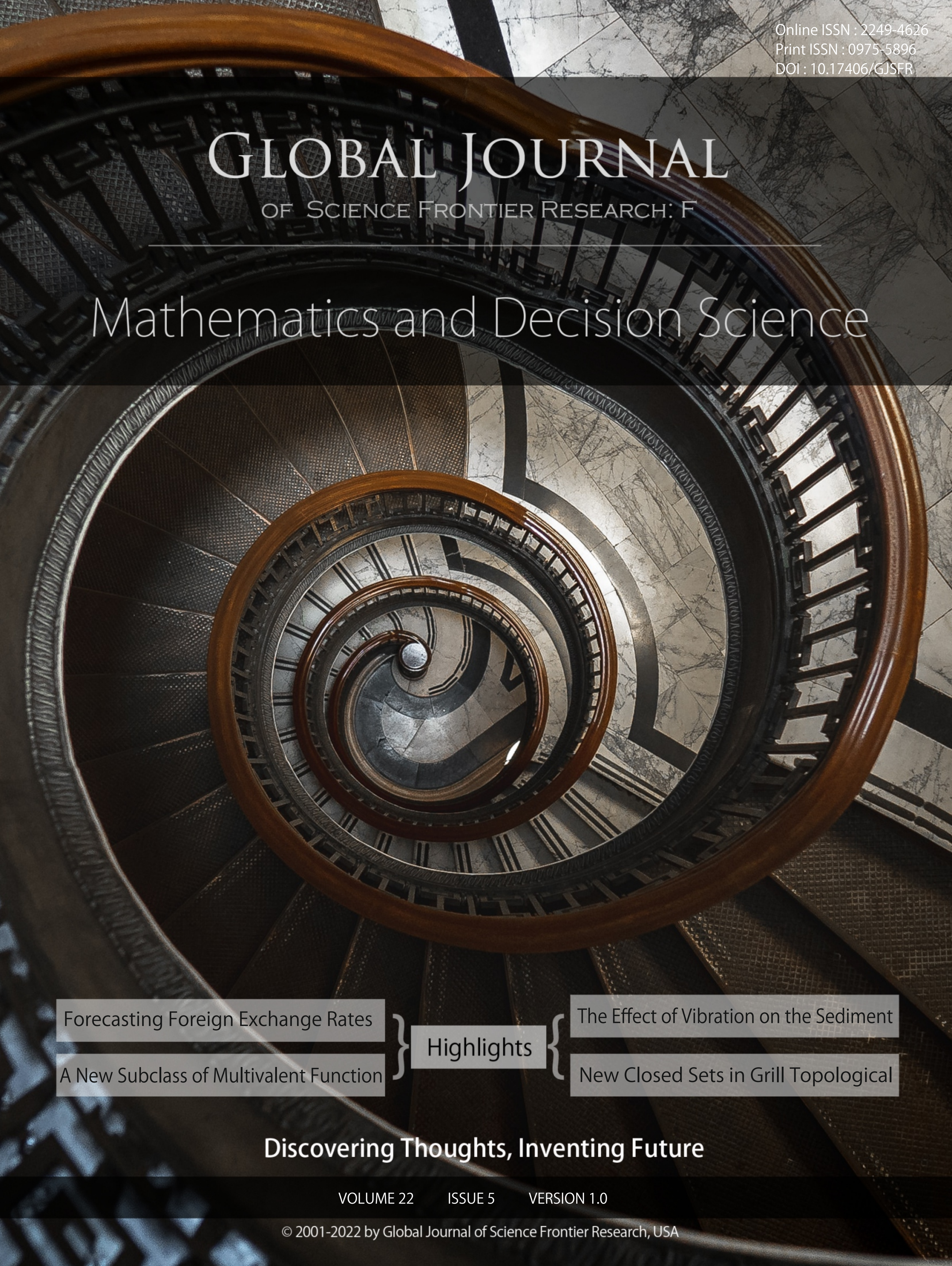

Online ISSN : 2249-4626
Print ISSN : 0975-5896
DOI : 10.17406/GJSFR

Global Journal

of Science Frontier Research: F

Mathematics and Decision Science

Highlights

- Forecasting Foreign Exchange Rates
- A New Subclass of Multivalent Function
- The Effect of Vibration on the Sediment
- New Closed Sets in Grill Topological

Discovering Thoughts, Inventing Future

VOLUME 22 ISSUE 5 VERSION 1.0

Global Journal of Science Frontier Research: F
Mathematics & Decision Sciences

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Mathematics & Decision Sciences

Volume 22 Issue 5 (Ver. 1.0)

Open Association of Research Society

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Yearly Subscription (Personal & Institutional)
250 USD (B/W) & 350 USD (Color)

Contents of the Issue

Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 22 Issue 5 Version 1.0 Year 2022
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

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By Shivani Indora & S. K. Bissu

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GJSFR-F Classification: *DDC Code: 621.381 LCC Code: TJ163.12*

ANEWSUBCLASSOFMULTIVALENTFUNCTIONDEFINEDBYUSINGJACKSONDERIVATIVEOPERATOR

A New Subclass of Multivalent Function Defined by using Jackson Derivative Operator

Shivani Indora [α] & S. K. Bissu [σ]

Abstract- In this paper the authors have used Jackson Derivative operator to form a new subclass of multivalent function and derived some results for a function belonging to new subclass of multivalent functions. The main emphasis is on coefficient estimate of functions belonging to new subclass of multivalent function, the radii of starlikeness, convexity and close to convexity properties of a function have also been discussed. The results reduces to the earlier known results of Silverman, Srivastava, Altintas and Khosravianarab by assuming some particular values of the parameters.

Keywords: jackson derivative operator, multivalent functions, coefficient estimate, radii of starlikeness.

I. Introduction

Let $\mathcal{H}(p)$ be the class of analytic and p-valent function $f(z)$. The function $f(z)$ can be expressed as

$$f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k \qquad (1.1)$$

where p is some natural number, $n \in \mathbb{N}$

The function $f(z)$ defined in (1.1) isan analytic function and p - valent function in the open unit disc

$$U_1 = \{z : |z| < 1\}$$

If a function $f(z) \in \mathcal{H}(p)$ satisfies the following condition

$$Re\left\{\frac{zf'(z)}{f(z)}\right\} > \varphi \qquad z \in U_1\ ,\ 0 \leq \varphi < p\ ,p \in \mathbb{N} \qquad (1.2)$$

then $f(z)$ is a p – valent starlike function of order φ
and if a function $f(z) \in \mathcal{H}(p)$ satisfies the following condition

Author α: Department of Mathematics, Sophia Girls' College (Autonomous), Ajmer. e-mail: shivaniindora15@gmail.com
Author σ: Department of Mathematics, Faculty of Sciences, Government College Beawar, Ajmer (Rajasthan), India.
e-mail: susilkbissu@gmail.com

$$Re\left\{1+\frac{zf''(z)}{f'(z)}\right\} > \varphi \quad z \in U_1 ,\ 0 \leq \ \varphi < p \ ,p \ \in \mathbb{N} \qquad (1.3)$$

then $f(z)$ is a $p-$valent convex function of order φ

To define a new subclass of multivalent function by using Jackson derivative, we use the following definitions

Definition 1: Let $f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k$ *and* $g(z) = z^p - \sum_{k=n+p}^{\infty} b_k z^k$ are the members of the class $\mathcal{H}(p)$, then their convolution product or Hadamard product is defined as

$$(f * g)(z) = (g * f)(z) \ = z^p - \sum_{k=n+p}^{\infty} a_k b_k z^k \qquad (1.4)$$

and generally the convolution product of functions $f(z)$ and $g(z)$ is denoted by $(f * g)(z)$ or $(g * f)(z)$.

Definition 2: The Jackson q- derivative of a function $f(z)$ is denoted by $D_q f(z)\ or\ D_{q,z} f(z)$ and it is defined as

$$D_{q,z} f(z) = \frac{f(z)-f(zq)}{z-zq} z \neq 0 \ and \ q \neq 1 \qquad (1.5)$$

The Jackson's q – derivative tends to ordinary derivative when q tends to 1.
The Jackson q- derivative can also be written as

$$D_{q,z}^{\mathrm{m}} z^r = \frac{\Gamma_q(1+\mathrm{r})}{\Gamma_q(1+\mathrm{r}-\mathrm{m})} z^{r-\mathrm{m}} \qquad where\ m \geq 0, r > -1 \qquad (1.6)$$

A new class of multivalent function form by using Jackson Derivative Operator is defined in the following definition.

Definition 5.3: A function $f(z) \in \mathcal{H}(p)$ is also belongs to new subclass $\Psi_{m,n,p}(\alpha, \beta, \lambda, \xi, \mathrm{q})$ if it follow the following condition

$$Re\left\{\frac{(z+\beta z^2)\left(D_{q,z}^{m+\xi+1} f(z)\right) + \lambda(z^2+\beta z)\left(D_{q,z}^{m+\xi+2} f(z)\right)}{(1-\lambda)\left(D_{q,z}^{m+\xi} f(z)\right) + \lambda(z+\beta z^2)\left(D_{q,z}^{m+\xi+1} f(z)\right)}\right\} > \alpha \qquad (1.7)$$

where $z \in U_1, m \in \mathbb{N} \cup \{0\}, 0 \leq \alpha < p,\ 0 \leq \beta < 1, 0 \leq \lambda \leq 1\ and\ 0 \leq \xi < 1$

By taking particular values of the parameters, $n, p, q, \beta, \lambda, \xi$ we get the previously defined subclasses of univalent and multivalent function. These classes were studied by Silverman [14], Srivastava [15], Altintas et. al [2] and Khosravianarb et. al [7].

Particular Cases:

1. If $m = 0, \beta = 0, \xi = 0, q \to 1$ then from (1.7) we get

$$Re\left\{\frac{z\left(D_{1,z} f(z)\right) + \lambda z^2 \left(D_{1,z}^2 f(z)\right)}{(1-\lambda)\left(D_{1,z}^0 f(z)\right) + \lambda z\left(D_{1,z} f(z)\right)}\right\} > \alpha$$

Ref

14. Silverman, H. (1975), *Univalent functions with negative coefficients*, Proceedings of the American mathematical society, 51(1), pp109-116. doi.org/10.1090/S0002-9939-1975-0369678-0

which is equivalent to

$$Re\left\{\frac{zf'(z) + \lambda z^2 f''(z)}{(1-\lambda)f(z) + \lambda z f'(z)}\right\} > \alpha$$

sowe get $\Psi_{0,n,p}(\alpha, 0, \lambda, 0, 1) \equiv T(n, p, \lambda, \alpha)$ and this classwas studied byAltintas et al. [2]

2. If $m = 0, \beta = 0, q \to 1$then from (1.7) we get

$$Re\left\{\frac{z\left(D_{1,z}^{\xi+1} f(z)\right) + \lambda z^2 \left(D_{1,z}^{\xi+2} f(z)\right)}{(1-\lambda)\left(D_{1,z}^{\xi} f(z)\right) + \lambda z \left(D_{1,z}^{\xi+1} f(z)\right)}\right\} > \alpha$$

so $\Psi_{0,n,p}(\alpha, 0, \lambda, \xi, 1) \equiv T(n, p, \lambda, \alpha, \xi)$ and this class was studied by Khosravianarb et al.[7]

3. If $m = 0, \beta = 0, \xi = 0, q \to 1, \lambda = 0$ then from (1.7) we get

$$Re\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha \qquad 0 \le \alpha < p$$

so $\Psi_{0,n,p}(\alpha, 0, 0, 0, 1) \equiv T^*(p, \alpha)$ and $T^*(p, \alpha)$ is the class of p valent starlike function of order α.

4. If $m = 0, \beta = 0, \xi = 0, q \to 1, \lambda = 0, p = 1$ then from (1.7) we get

$$Re\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha \qquad 0 \le \alpha < 1$$

so $\Psi_{0,n,1}(\alpha, 0, 0, 0, 1) \equiv T^*(1, \alpha)$, which was earlier studied by Srivastava et al. [15].

5. If $m = 0, \beta = 0, \xi = 0, q \to 1, \lambda = 0, p = 1, n = 1$ then from (1.7) we get

$$Re\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha \qquad 0 \le \alpha < 1$$

Then we get a class which was earlier discussed by Silverman [14].

6. If $m = 0, \beta = 0, \xi = 0, q \to 1, \lambda = 1$then from (1.7) we get

$$Re\left\{\frac{zf'(z) + z^2 f''(z)}{z f'(z)}\right\} > \alpha \qquad 0 \le \alpha < p$$

which is equivalent to $Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \alpha \qquad 0 \le \alpha < p$

so $\Psi_{0,n,p}(\alpha, 0, 1, 0, 1) \equiv C^*(p, \alpha)$ and $C^*(p, \alpha)$ represent a class of p valent convex function of order α.

Ref

2. Altintaş, O., Irmak, H., & Srivastava, H. M. (1995), *Fractional calculus and certain starlike functions with negative coefficients*, Computers & Mathematics with Applications, 30(2), pp 9-15. doi: 10.1016/0898-1221(95)00073-8

7. If $m = 0, \beta = 0, \xi = 0, q \to 1, \lambda = 1, p = 1$then from (1.7) we get

$$Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \alpha \qquad 0 \le \alpha < 1$$

sowe get $\Psi_{0,n,1}(\alpha, 0, 1, 0, 1) \equiv C^*(1, \alpha)$,which was earlier by studied Srivastava et al. [15].

8. If $m = 0, \beta = 0, \xi = 0, q \to 1, \lambda = 1, p = 1, n = 1$ then from (1.7) we get

$$Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \alpha \qquad 0 \le \alpha < 1$$

and this class of convex function was first introduced by Silverman [14].

II. Coefficient Estimate

In this part of the paper we derive the coefficient estimate of function $f(z)$, $f(z) \in \Psi_{m,n,p}(\alpha, \beta, \lambda, \xi, q)$

Theorem 1: A function $f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k$ and $f(z) \in \mathcal{H}(p)$ then $f(z)$ belong to the class $\Psi_{m,n,p}(\alpha, \beta, \lambda, \xi, q)$ if and only if

$$\sum_{k=n+p}^{\infty} E_{p,k}^{m,\xi} \left\{\frac{(1+\beta)[k-(m+\xi)]_q\left[1-\alpha\lambda+\lambda[k-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q\left[1-\alpha\lambda+\lambda[p-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}\right\} a_k \le 1 \qquad (2.1)$$

where $E_{p,k}^{m,\xi} = \frac{\Gamma_q(1+k)\Gamma_q(1+p-(m+\xi))}{\Gamma_q(1+p)\Gamma_q(1+k-(m+\xi))}$

$$z \in U_1, m \in \mathbb{N} \cup \{0\}, 0 \le \alpha < p, \qquad 0 \le \beta < 1, 0 \le \lambda \le 1 \text{ and } 0 \le \xi < 1$$

Proof: Let us consider that $f(z) \in \Psi_{m,n,p}(\alpha, \beta, \lambda, \xi, q)$ so we have

$$Re\left\{\frac{(z+\beta z^2)\left(D_{q,z}^{m+\xi+1} f(z)\right) + \lambda\,(z^2+\beta z)\left(D_{q,z}^{m+\xi+2} f(z)\right)}{(1-\lambda)\left(D_{q,z}^{m+\xi} f(z)\right) + \lambda(z+\beta z^2)\left(D_{q,z}^{m+\xi+1} f(z)\right)}\right\} > \alpha$$

Since $f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k$ and

$$D_{q,z}^{m+\xi} f(z) = \frac{\Gamma_q(1+p)}{\Gamma_q(1+p-(m+\xi))} z^{p-(m+\xi)} - \sum_{k=n+p}^{\infty} \frac{\Gamma_q(1+k)}{\Gamma_q(1+k-(m+\xi))} a_k z^{k-(m+\xi)} \qquad (2.2)$$

so we have

$$D_{q,z}^{m+\xi+1} = \frac{\Gamma_q(1+p)}{\Gamma_q(p-(m+\xi))} z^{p-(m+\xi+1)} - \sum_{k=n+p}^{\infty} \frac{\Gamma_q(1+k)}{\Gamma_q(k-(m+\xi))} a_k z^{k-(m+\xi+1)} \qquad (2.3)$$

$$D_{q,z}^{m+\xi+2} = \frac{\Gamma_q(1+p)}{\Gamma_q(p-(m+\xi+1))} z^{p-(m+\xi+2)} - \sum_{k=n+p}^{\infty} \frac{\Gamma_q(1+k)}{\Gamma_q(k-(m+\xi+1))} a_k z^{k-(m+\xi+2)} \qquad (2.4)$$

Ref

14. Silverman, H. (1975), *Univalent functions with negative coefficients*, Proceedings of the American mathematical society, 51(1), pp109-116. doi.org/10.1090/S0002-9939-1975-0369678-0

By using (2.2), (2.3) and (2.4) in (2.1) then we get numerator and denominator of (2.1) as numerator is denoted by N and denominator by D

$$\mathrm{N} = (z+\beta z^2)\left[\frac{\Gamma_q(1+p)}{\Gamma_q(p-(m+\xi))} z^{p-(m+\xi+1)} - \sum_{k=n+p}^{\infty} \frac{\Gamma_q(1+k)}{\Gamma_q(k-(m+\xi))} a_k\, z^{k-(m+\xi+1)}\right] +$$

$$\lambda(z^2+\beta z)\left[\frac{\Gamma_q(1+p)}{\Gamma_q(p-(m+\xi+1))} z^{p-(m+\xi+2)} - \sum_{k=n+p}^{\infty} \frac{\Gamma_q(1+k)}{\Gamma_q(k-(m+\xi+1))} a_k\, z^{k-(m+\xi+2)}\right]$$

$$\mathrm{D} = (1-\lambda)\left[\frac{\Gamma_q(1+p)}{\Gamma_q(1+p-(m+\xi))} z^{p-(m+\xi)} - \sum_{k=n+p}^{\infty} \frac{\Gamma_q(1+k)}{\Gamma_q(1+k-(m+\xi))} a_k\, z^{k-(m+\xi)}\right] +$$

$$\lambda\,(z+\beta z^2)\left[\frac{\Gamma_q(1+p)}{\Gamma_q\big(p-(m+\xi)\big)} z^{p-(m+\xi+1)} - \sum_{k=n+p}^{\infty} \frac{\Gamma_q(1+k)}{\Gamma_q\big(k-(m+\xi)\big)} a_k\, z^{k-(m+\xi+1)}\right]$$

solve above by using $[n]_q = \frac{\Gamma_q(1+n)}{\Gamma_q(n)}$ and on considering the value of z to be real and let $z \to 1$ then we get

$$\frac{\Gamma_q(1+p)}{\Gamma_q\big(1+p-(m+\xi)\big)}\left[(1+\beta)[p-(m+\xi)]_q\big[1-\alpha\lambda+\lambda[p-(m+\xi+1)]_q\big]-\alpha(1-\lambda)\right]$$

$$\geq \sum_{k=n+p}^{\infty} \frac{\Gamma_q(1+k)}{\Gamma_q\big(1+k-(m+\xi)\big)} a_k\left[(1+\beta)[k-(m+\xi)]_q\big[1-\alpha\lambda+\lambda[k-(m+\xi+1)]_q\big] - \alpha(1-\lambda)\right]$$

on simplifying we get,

$$\sum_{k=n+p}^{\infty} E_{p,k}^{m,\xi}\left\{\frac{(1+\beta)[k-(m+\xi)]_q\big[1-\alpha\lambda+\lambda[k-(m+\xi+1)]_q\big]-\alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q\big[1-\alpha\lambda+\lambda[p-(m+\xi+1)]_q\big]-\alpha(1-\lambda)}\right\} a_k \leq 1$$

where $E_{p,k}^{m,\xi} = \frac{\Gamma_q(1+k)\Gamma_q(1+p-(m+\xi))}{\Gamma_q(1+p)\Gamma_q(1+k-(m+\xi))}$

Conversely: Let us assume the inequality (2.1) is true

To Prove: $f(z) \in \Psi_{m,n,p}(\alpha,\beta,\lambda,\xi,q)$, for this we have to show that

$$Re\left\{\frac{(z+\beta z^2)\left(D_{q,z}^{m+\xi+1} f(z)\right) + \lambda\,(z^2+\beta z)\left(D_{q,z}^{m+\xi+2} f(z)\right)}{(1-\lambda)\left(D_{q,z}^{m+\xi} f(z)\right) + \lambda(z+\beta z^2)\left(D_{q,z}^{m+\xi+1} f(z)\right)}\right\} > \alpha$$

According to Lemma [4]

Ref

4. Aqlan, E.S. (2004), *Some Problems Connected with Geometric Function Theory*, Ph.D. Thesis, Pune University, Pune.

if $w = u + iv$ then $Re\, w \geq \alpha \Leftrightarrow |w - (1+\alpha)| \leq |w + (1-\alpha)|$ (2.5)

Let $L = |w - (1+\alpha)|$

and
$$w = \frac{(z+\beta z^2)\left(D_{q,z}^{m+\xi+1} f(z)\right) + \lambda\,(z^2+\beta z)\left(D_{q,z}^{m+\xi+2} f(z)\right)}{(1-\lambda)\left(D_{q,z}^{m+\xi} f(z)\right) + \lambda(z+\beta z^2)\left(D_{q,z}^{m+\xi+1} f(z)\right)} \tag{2.6}$$

$$L = \left|\frac{(z+\beta z^2)\left(D_{q,z}^{m+\xi+1} f(z)\right) + \lambda\,(z^2+\beta z)\left(D_{q,z}^{m+\xi+2} f(z)\right)}{(1-\lambda)\left(D_{q,z}^{m+\xi} f(z)\right) + \lambda(z+\beta z^2)\left(D_{q,z}^{m+\xi+1} f(z)\right)} - (1+\alpha)\right| \tag{2.7}$$

and $K = |w + (1-\alpha)|$

$$K = \left|\frac{(z+\beta z^2)\left(D_{q,z}^{m+\xi+1} f(z)\right) + \lambda\,(z^2+\beta z)\left(D_{q,z}^{m+\xi+2} f(z)\right)}{(1-\lambda)\left(D_{q,z}^{m+\xi} f(z)\right) + \lambda\,(z+\beta z^2)\left(D_{q,z}^{m+\xi+1} f(z)\right)} + (1-\alpha)\right| \tag{2.8}$$

From (2.7) and (2.8), $K - L > 0$
i.e. $|w + (1-\beta)| - |w - (1+\beta)| > 0$ which implies $Re(w) > \alpha$

Hence the inequality
$$Re\left\{\frac{(z+\beta z^2)\left(D_{q,z}^{m+\xi+1} f(z)\right) + \lambda\,(z^2+\beta z)\left(D_{q,z}^{m+\xi+2} f(z)\right)}{(1-\lambda)\left(D_{q,z}^{m+\xi} f(z)\right) + \lambda(z+\beta z^2)\left(D_{q,z}^{m+\xi+1} f(z)\right)}\right\} > \alpha$$

which implies $f(z) \in \Psi_{m,n,p}(\alpha, \beta, \lambda, \xi, q)$
so, the proof of theorem 1 is completed

Corollary 1: Let the function $f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k$ is a member of new subclass $\Psi_{m,n,p}(\alpha, \beta, \lambda, \xi, q)$ of multivalent function then

$$a_k \leq \left\{\frac{(1+\beta)[p-(m+\xi)]_q\,[1-\alpha\lambda + \lambda[p-(m+\xi+1)]_q] - \alpha(1-\lambda)}{(1+\beta)[k-(m+\xi)]_q\,[1-\alpha\lambda + \lambda[k-(m+\xi+1)]_q] - \alpha(1-\lambda)}\right\} \frac{1}{E_{p,k}^{m,\xi}} \tag{2.9}$$

where $k = n + p$, p is some natural number, n is a natural number.

III. Property of New Subclass Related to Radii of Star Likeness, Convexity and Close to Convexity

In this part of the paper, we derive some results related to Radii of starlikeness, convexity and close to convexity for the function $f(z)$ belonging to the new subclass $\Psi_{m,n,p}(\alpha, \beta, \lambda, \xi, q)$

Theorem 2: Let the function $f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k$ and $f(z)$ belong to $\Psi_{m,n,p}(\alpha, \beta, \lambda, \xi, q)$ then the function $f(z)$ is p-valent close to convex of order $\varphi\,; 0 \leq \varphi < p$ in $|z| < r_1^*$, where

Notes

$$r_1^* = \inf_{k \geq n+p} \left\{ \left(\frac{p-\varphi}{k}\right) \left\{\frac{(1+\beta)[k-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[k-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[p-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}\right\} E_{p,k}^{m,\xi} \right\}^{\frac{1}{k-p}} \quad (3.1)$$

Proof: Let $f(z) \in \Psi_{m,n,p}(\alpha, \beta, \lambda, \xi, q)$ *and* $f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k$

To prove $f(z)$ is p-valent close to convex of order φ ; $0 \leq \varphi < p$ in $|z| < r_1^*$ for this we have to show that

$$\left|\frac{f'(z)}{z^{p-1}} - p\right| \leq p - \varphi \qquad |z| < r_1^* \quad (3.2)$$

$$\left|\frac{f'(z)}{z^{p-1}} - p\right| = \left|\frac{pz^{p-1} - \sum_{k=n+p}^{\infty} ka_k z^{k-1}}{z^{p-1}} - p\right|$$

$$= \left|\frac{\sum_{k=n+p}^{\infty} ka_k z^{k-1}}{z^{p-1}}\right|$$

$$\leq \sum_{k=n+p}^{\infty} ka_k |z|^{k-p} \quad (3.3)$$

The inequality (3.2) is less than or equal to $p-\varphi$ if

$$\sum_{k=n+p}^{\infty} \left(\frac{k}{p-\varphi}\right) a_k |z|^{k-p} \leq 1 \quad (3.4)$$

we know that $f(z) \in \Psi_{m,n,p}(\alpha, \beta, \lambda, \xi, q)$ if and only if

$$\sum_{k=n+p}^{\infty} E_{p,k}^{m,\xi} \left\{\frac{(1+\beta)[k-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[k-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[p-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}\right\} a_k \leq 1$$

The inequality (3.2) is hold true if

$$\left(\frac{k}{p-\varphi}\right)|z|^{k-p}$$

$$\leq E_{p,k}^{m,\xi} \left\{\frac{(1+\beta)[k-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[k-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[p-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}\right\}$$

or, we have

$$|z|^{k-p} \leq \left(\frac{p-\varphi}{k}\right) E_{p,k}^{m,\xi} \left\{\frac{(1+\beta)[k-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[k-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[p-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}\right\} \quad (3.5)$$

so we get the required result

$|z| < r_1^*$

$$= \inf_{k \geq n+p} \left\{ \left(\frac{p-\varphi}{k}\right) \left\{ \frac{(1+\beta)[k-(m+\xi)]_q \left[1-\alpha\lambda + \lambda[k-(m+\xi+1)]_q - \alpha(1-\lambda)\right]}{(1+\beta)[p-(m+\xi)]_q \left[1-\alpha\lambda + \lambda[p-(m+\xi+1)]_q - \alpha(1-\lambda)\right]} \right\} E_{p,k}^{m,\xi} \right\}^{\frac{1}{k-p}}$$

Hence, the given function $f(z)$ is p-valent close to convex of order φ

Theorem 3: Let the function $f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k$ and $f(z) \in \Psi_{m,n,p}(\alpha, \beta, \lambda, \xi, q)$ then the function $f(z)$ is a p-valent starlike of order φ ; $0 \leq \varphi < p$ in $|z| < r_2^*$,where

$$r_2^* = \inf_{k \geq n+p} \left\{ \left(\frac{p-\varphi}{k-\varphi}\right) \left\{ \frac{(1+\beta)[k-(m+\xi)]_q \left[1-\alpha\lambda+\lambda[k-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q \left[1-\alpha\lambda+\lambda[p-(m+\xi+1)]_q\right]-\alpha(1-\lambda)} \right\} E_{p,k}^{m,\xi} \right\}^{\frac{1}{k-p}} \quad (3.6)$$

Proof: Let $f(z) \in \Psi_{m,n,p}(\alpha, \beta, \lambda, \xi, q)$ *and* $f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k$

To prove the function $f(z)$ is p-valent starlike of order φ; $0 \leq \varphi < p$ in $|z| < r_2^*$ for this we have to show that

$$\left|\frac{zf'(z)}{f(z)} - p\right| \leq p - \varphi \qquad |z| < r_2^* \quad (3.7)$$

Now we take the L.H.S. part of the inequality (3.7)

$$\left|\frac{zf'(z)}{f(z)} - p\right| = \left|\frac{z(pz^{p-1} - \sum_{k=n+p}^{\infty} ka_k z^{k-1})}{z^p - \sum_{k=n+p}^{\infty} a_k z^k} - p\right|$$

$$= \left|\frac{\sum_{k=n+p}^{\infty}(k-p)a_k z^k}{z^p - \sum_{k=n+p}^{\infty} a_k z^k}\right|$$

$$\leq \frac{\sum_{k=n+p}^{\infty}(k-p)a_k |z|^{k-p}}{1-\sum_{k=n+p}^{\infty} a_k |z|^{k-p}} \quad (3.8)$$

The inequality (3.7) is less than or equal to $p-\varphi$ if

$$\sum_{k=n+p}^{\infty} \frac{(k-\varphi)}{(p-\varphi)} a_k |z|^{k-p} \leq 1 \quad (3.9)$$

we know that $f(z) \in \Psi_{m,n,p}(\alpha, \beta, \lambda, \xi, q)$ if and only if

$$\sum_{k=n+p}^{\infty} E_{p,k}^{m,\xi} \left\{ \frac{(1+\beta)[k-(m+\xi)]_q \left[1-\alpha\lambda + \lambda[k-(m+\xi+1)]_q\right] - \alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q \left[1-\alpha\lambda + \lambda[p-(m+\xi+1)]_q\right] - \alpha(1-\lambda)} \right\} a_k \leq 1$$

The inequality (3.9) is hold true if

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$$\left(\frac{k-\varphi}{p-\varphi}\right)|z|^{k-p}$$

$$\leq E_{p,k}^{m,\xi}\left\{\frac{(1+\beta)[k-(m+\xi)]_q\left[1-\ \alpha\lambda\ +\ \lambda[k-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q\left[1-\alpha\lambda\ +\ \lambda[p-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}\right\}$$

or, we have

$$|z|^{k-p}\leq\left(\frac{p-\varphi}{k-\varphi}\right)E_{p,k}^{m,\xi}\left\{\frac{(1+\beta)[k-(m+\xi)]_q\left[1-\alpha\lambda+\lambda[k-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q\left[1-\alpha\lambda+\lambda[p-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}\right\} \quad (3.10)$$

so we get the required result

$|z|<r_2^*$

$$=\inf_{k\geq n+p}\left\{\left(\frac{p-\varphi}{k-\varphi}\right)\left\{\frac{(1+\beta)[k-(m+\xi)]_q\left[1-\alpha\lambda\ +\ \lambda[k-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q\left[1-\alpha\lambda\ +\ \lambda[p-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}\right\}E_{p,k}^{m,\xi}\right\}^{\frac{1}{k-p}}$$

Hence, the given function $f(z)$ is p-valent starlike of order φ

Theorem 4: Let the function $f(z)=z^p-\sum_{k=n+p}^{\infty}a_k z^k$ and $f(z)\in\Psi_{m,n,p}(\alpha,\beta,\lambda,\xi,q)$ then the given function $f(z)$ is a p-valent convex function of order $\varphi\,;0\leq\varphi<p$ in $|z|<r_3^*$, where

$$r_3^*=\inf_{k\geq n+p}\left\{\frac{p}{k}\left(\frac{p-\varphi}{k-\varphi}\right)\left\{\frac{(1+\beta)[k-(m+\xi)]_q\left[1-\alpha\lambda+\lambda[k-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q\left[1-\alpha\lambda+\lambda[p-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}\right\}E_{p,k}^{m,\xi}\right\}^{\frac{1}{k-p}} \quad (3.11)$$

Proof: Let $f(z)\in\Psi_{m,n,p}(\alpha,\beta,\lambda,\xi,q)$ *and* $f(z)=z^p-\sum_{k=n+p}^{\infty}a_k z^k$

To prove the function $f(z)$ is p-valent convex function of order $\varphi\,;0\leq\varphi<p$ in $|z|<r_3^*$ for this we have to show that

$$\left|\frac{zf''(z)}{f'(z)}+(1-p)\right|\leq p-\varphi \qquad |z|<r_3^* \quad (3.12)$$

Taking the L.H.S. part of the inequality (3.12)

$$\left|\frac{zf''(z)}{f'(z)}+(1-p)\right|=\left|\frac{z(p(p-1)z^{p-2}-\sum_{k=n+p}^{\infty}k(k-1)a_k z^{k-2})}{pz^{p-1}-\sum_{k=n+p}^{\infty}ka_k z^{k-1}}+(1-p)\right|$$

$$=\left|\frac{\sum_{k=n+p}^{\infty}k(k-p)a_k z^{k-p}}{p-\sum_{k=n+p}^{\infty}ka_k z^{k-p}}\right|$$

$$\leq \frac{\sum_{k=n+p}^{\infty} k(k-p)a_k|z|^{k-p}}{p-\sum_{k=n+p}^{\infty} ka_k|z|^{k-p}}$$

The inequality (3.12) is less than or equal to $p-\varphi$ if

$$\sum_{k=n+p}^{\infty} \frac{k(k-\lambda)}{p(p-\lambda)} a_k |z|^{k-p} \leq 1 \quad (3.13)$$

we know that $f(z) \in \Psi_{m,n,p}(\alpha,\beta,\lambda,\xi,q)$ if and only if

$$\sum_{k=n+p}^{\infty} E_{p,k}^{m,\xi} \left\{\frac{(1+\beta)[k-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[k-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[p-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}\right\} a_k \leq 1$$

The inequality (3.12) is hold true if

$$\frac{k}{p}\left(\frac{k-\varphi}{p-\varphi}\right)|z|^{k-p}$$

$$\leq E_{p,k}^{m,\xi} \left\{\frac{(1+\beta)[k-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[k-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[p-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}\right\}$$

or, we have

$$|z|^{k-p} \leq \frac{p}{k}\left(\frac{p-\varphi}{k-\varphi}\right) E_{p,k}^{m,\xi} \left\{\frac{(1+\beta)[k-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[k-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[p-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}\right\} \quad (3.14)$$

so we get the required result

$$|z| < r_3^*$$

$$= \inf_{k \geq n+p} \left\{\frac{p}{k}\left(\frac{p-\varphi}{k-\varphi}\right)\left\{\frac{(1+\beta)[k-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[k-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}{(1+\beta)[p-(m+\xi)]_q\left[1-\alpha\lambda + \lambda[p-(m+\xi+1)]_q\right]-\alpha(1-\lambda)}\right\} E_{p,k}^{m,\xi}\right\}^{\frac{1}{k-p}}$$

Hence, the given function $f(z)$ is p-valent convex function of order φ

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Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 22 Issue 5 Version 1.0 Year 2022
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Unbranched Riemann Domains over Q-Complete Spaces

By Youssef Alaoui

Abstract- It is proved that if $\Pi : X \to \Omega$ is an unbranched Riemann domain and locally r-complete morphism over a q-complete space Ω, then X is cohomo- logically $(q + r - 1)$-complete, if $q \geq 2$. We have shown in [1] that if $\Pi : X \to \Omega$ is an unbranched Riemann domain and locally q-complete morphism over a Stein space Ω, then X is cohomologically q-complete with respect to the struc- ture sheaf. In section 4 of this article, we prove by means of a counterexample that that there exists for each integer $n \geq 3$ an open subset $\Omega \subset Cn$ which is locally $(n - 1)$-complete but Ω is not $(n - 1)$-complete. The counterexample we give is obtained by making a slight modification of a recent example given by the author [2].

In 1962, Andreotti and Grauert [3] showed finiteness and vanishing the- orems for cohomology groups of analytic spaces under geometric conditions of q-convexity. Since then the question whether the reciprocal statements of these theorems are true have been subject to extensive studies.

GJSFR-F Classification: *MSC 1991: 32E10, 32E40*

Strictly as per the compliance and regulations of:

Global Journal of Science Frontier Research (F) Volume XXII Issue V Version I Year 2022

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Ref

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Unbranched Riemann Domains over Q-Complete Spaces

Youssef Alaoui

Abstract

It is proved that if $\pi : X \to \Omega$ is an unbranched Riemann domain and locally r-complete morphism over a q-complete space Ω, then X is cohomologically $(q+r-1)$-complete if $q \geq 2$. We have shown in [1] that if $\pi : X \to \Omega$ is an unbranched Riemann domain and locally q-complete morphism over a Stein space Ω, then X is cohomologically q-complete for the structure sheaf $\mathcal{O}_X$. In section 4 of this article, we prove using a counter-example that there exists for each integer $n \geq 3$ an open subset $\Omega \subset \mathbb{C}^n$ which is locally $(n-1)$-complete, but Ω is not $(n-1)$-complete. The counter-example we give is based on a recent example given by the author [2].

By the theory of Andreotti and Grauert [3] it is known that a q-complete complex space is always cohomologically q-complete. A counter-example to the converse of this theorem was given in [2], where it is shown that there exists for each integer $n \geq 3$ a domain $\Omega \subset \mathbb{C}^n$ which is cohomologically $(n-1)$-complete but Ω is not $(n-1)$-complete. Since then, the question of whether the joint statements of these theorems are factual has been subject to extensive studies. For example, it was shown that if X is a Stein manifold and if $D \subset X$ is an open subset that has a C^2 boundary such that $H^p(D, \mathcal{O}_D) = 0$ for all $p \geq q$, then D is q-complete.

In this article, we prove that for any pair of integers (n, q), $2 \leq q < n$, there exists an open subset Ω of $\mathbb{C}^n$ which is cohomologically $(\tilde{q}-1)$-complete but Ω is not $(\tilde{q}-1)$-complete if $n = mq+1$, where $m = [\frac{n}{q}]$ denotes as usual the integral part of $\frac{n}{q}$ and $\tilde{q} = n - m + 1$.

I. Introduction

Let $\pi : X \to Y$ be a holomorphic map of complex spaces. Then π is said to be locally r-complete if there exists for every $x \in Y$ an open neighborhood U in Y such that $\pi^{-1}(U)$ is r-complete.

A Riemann domain over a complex space Y is a pair (X, Π), where $\pi : X \to Y$ is a holomorphic map which is non-degenerate at every point of X, i.e., $\pi^{-1}(\pi(x))$ is a discrete set at each point $x \in X$. The pair (X, π) is called unbranched or unramified if $\pi : X \to Y$ is locally biholomorphic.

Let X and Y be complex spaces and $\pi : X \to Y$ an unbranched Riemann domain such that Y is q-complete and π a locally r-complete morphism.

Author: Y. Alaoui. Department of Fundamental Sciences, Hassan II Institute of Agronomy and Veterinary Sciences, B.P. 6202, Rabat-Instituts, 10101. Morocco. e-mails: y.alaoui@iav.ac.ma, comp5123ster@gmail.com

Does it follow that X is $(q+r-1)$-complete?

It was shown in [4] that this problem has a positive answer when $q=r=1$ and X and Y have isolated singularities.

It is known from [9] that if $\pi : X \to \Omega$ is an unbranched Riemann domain between two complex spaces with isolated singularities, Ω q-complete, and π is locally 1-complete, then X is q-complete.

We have shown in [1] that if $\pi : X \to \Omega$ is a locally q-complete unbranched Riemann domain over an n-dimensional Stein complex space Ω, then X is cohomologically q-complete with respect to the structure $\mathcal{O}_X$.

As a result, the author has provided a positive answer to the local Steiness problem : he has proved that if X is a Stein space and if $\Omega \subset X$ is a locally Stein open subset of X, then Ω is Stein. (See [1]).

In this article, we prove that if $\pi : X \to \Omega$ is a locally r-complete unbranched Riemann domain over a q-complete n-dimensional complex space Ω, then for any coherent analytic sheaf $\mathcal{F}$ on X, the cohomology group $H^l(X, \mathcal{F})$ vanishes for all $l \geq r+q-1$, if $q \geq 2$.

In particular, we obtain the interesting conclusion.

Corollary. If X is a q-complete complex space of dimension n and if $\Omega \subset X$ is a locally r-complete open subset of X, then

(a) Ω is cohomologically $(q+r-1)$-complete if $q \geq 2$.

(b) Ω is cohomologically r-complete with respect to the structure sheaf if X is a Stein space $(q=1)$.

It should be mentioned [13] that if Y is q-complete and if $\pi : X \to Y$ is a locally r-complete morphism, then the space X is cohomologically $(q+r)$-complete. But in general, $H^{q+r-1}(X, \mathcal{O}_X)$ does not vanish, even when $\pi : X \to Y$ is locally 1-complete and $q=1$ [12] (See also [6]).

The above question generalizes the following classical problem:

Is a locally q-complete open subset Ω of a Stein space X necessarily q-complete?

A counter-example to this problem is not known. One can easily verify that Ω is cohomologically $(q+1)$-complete. It is easy to see that a cohomologically q-complete open subset $\Omega \subset \mathbb{C}^n$ is always q-complete with corners. But it is unknown if these two conditions are equivalent.

By the theory of Andreotti and Grauert [3], it is known that if X is a q-complete complex space, then for every coherent analytic sheaf $\mathcal{F}$ on X, the cohomology group $H^p(X, \mathcal{F}) = 0$ for all $p \geq q$. But it is not known if these two conditions are equivalent except when X is a Stein manifold, $\Omega \subset X$ is cohomologically q-complete with respect to the structure sheaf $\mathcal{O}_\Omega$ and Ω has a smooth boundary [7]. In [2], we have shown that there exists for each $n \geq 3$ an open subset $\Omega \subset \mathbb{C}^n$ which is cohomologically $(n-1)$-complete, but Ω is not $(n-1)$-complete.

In section 4 of this article, we prove that for each $n \geq 3$, there exists an integer q with $2 \leq q < n$ such that for any coherent analytic sheaf $\mathcal{F}$, the cohomology group $H^p(\Omega, \mathcal{F})$ vanishes for all $p \geq q$ but Ω is not q-complete.

II. Preliminaries

We start by recalling some definitions and results concerning q-complete spaces.

Let Ω be an open set in $\mathbb{C}^n$ with complex coordinates $z_1, \cdots, z_n$. Then it is known that a function $\phi \in C^\infty(\Omega)$ is q-convex if for every point $z \in \Omega$, the Levi form.

$$L_z(\phi, \xi) = \sum_{i,j} \frac{\partial^2 \phi(z)}{\partial z_i \partial \bar{z}_j} \xi_i \bar{\xi}_j, \quad \xi \in \mathbb{C}^n$$

Has at most $q-1$ negative or zero eigenvalues.

Ref

1. Y. Alaoui, On the cohomology groups of unbranched Riemann domains over Stein spaces. To appear in Rendiconti del Seminario Matematico. Politecnico di Torino

A smooth real-valued function ϕ on a complex space X is called q-convex if every point $x \in X$ has a local chart $U \to D \subset \mathbb{C}^n$ such that $\phi|_U$ has an extension $\hat{\phi} \in C^\infty(D, \mathbb{R})$ which is q-convex on D.

Two q-convex functions ϕ, ψ on X have the exact positivity directions if, for each point $x \in X$, there exists an open neighborhood U of x that can be identified to a closed analytic subset B of a domain D of some $\mathbb{C}^n$, and a complex vector subspace E of $\mathbb{C}^n$ of dimension $\geq n-q+1$ such that the Levi forms of $L_z(\phi,\xi)$ and $L_z(\psi,\xi)$, $z \in U$, are positive definite when restricted to E.

We say that X is q-complete if there exists a q-convex function $\phi \in C^\infty(X, \mathbb{R})$ which is exhaustive on X, i.e. $\{x \in X; \phi(x) < c\}$ is relatively compact for any $c \in \mathbb{R}$.

A complex space X is said to be cohomologically q-complete if the cohomology groups $H^p(X, \mathcal{F})$, $\mathcal{F} \in Coh(X)$, vanish for all $p \geq q$.

An open subset D of Ω is called q-Runge, if for every compact set $K \subset D$, there is a q-convex exhaustion function $\phi \in C^\infty(\Omega)$ such that

$$K \subset \{x \in \Omega : \phi(x) < 0\} \subset\subset D$$

This generalizes the classical notion of Runge pairs of Stein spaces.
It is shown in [3] that if D is q-Runge in Ω, then for every coherent analytic sheaf $\mathcal{F}$ on Ω, the cohomology groups $H^p(D, \mathcal{F})$ vanish for $p \geq q$ and the restriction map

$$H^p(\Omega, \mathcal{F}) \longrightarrow H^p(D, \mathcal{F})$$

has a dense image for all $p \geq q-1$.

A holomorphic map $\pi : X \to \Omega$ of complex spaces is called a q-complete morphism if there exists a q-convex function $\quad : X \to \mathbb{R}$ such that for every real number $\mu \in \mathbb{R}$, the restriction of Π from $\{x \in X; \quad (x) \leq \mu\}$ to Ω is proper. The canonical topologies on $H^p(X, \mathcal{F})$ are separated for all $p \geq q+1$ and for every coherent analytic sheaf $\mathcal{F}$ on X.

III. Unbrunched Riemann Domains Over Q-Complete Spaces

Theorem 1. *Let X and Y be two n-dimensional complex spaces such that Y is q-complete and $\pi : X \to Y$ is an unbranched Riemann domain and locally r-complete morphism. Then X is cohomologically $(q+r-1)$-complete.*

Proof. Since Y is q-complete, there exists, according to [14], a smooth q-convex function $\phi : Y \to]0, +\infty[$ such that for every real number λ, $Y(\lambda) = \{y \in Y : \phi(y) < \lambda\}$ is relatively compact in Y and $\{y \in Y : \phi(y) \leq \lambda\} \setminus \partial Y(\lambda)$ contains at most one point. Put $p = q+r-1$ and let $\mathcal{F}$ be a coherent analytic sheaf on X. We define $X(\lambda) = \Pi^{-1}(Y(\lambda))$ and consider the set A of all real numbers λ such that $H^p(X(\lambda), \mathcal{F}) = 0$.
To prove that $H^p(X(\lambda), \mathcal{F}) = 0$ for every $\lambda \in \mathbb{R}$, it will be sufficient to show that
(a) $A \neq \emptyset$ and, if $\lambda \in A$ and $\lambda' < \lambda$, then $\lambda' \in A$.
(b) if $\lambda_j \to \lambda$ and $\lambda_j \in A$ for all j, then $\lambda \in A$.
(c) if $\lambda_0 \in A$, there exists $\varepsilon_0 > 0$ such that $\lambda_0 + \varepsilon_0 \in A$.

We first prove assertion (a). Clearly, A is not empty. Indeed if
$\lambda_0 = min\{\phi(x); x \in Y\}$, then $]-\infty, \lambda_0] \subset A$. Also, if $\lambda \in A$ and $\lambda' < \lambda$, then by theorem 1 of [13], the restriction map

Ref

3. A. Andreotti and H. Grauert, Théorèmes de finitude de la cohomologie des espaces complexes. Bull. Soc. Math. France 90 (1962;) 193 – 259.

$$H^p(X(\lambda),\mathcal{F}) \xrightarrow{\rho} H^p(X(\lambda'),\mathcal{F})$$

has a dense range. Moreover, ρ is, in addition, injective. In fact, let

$$H^p(X(\lambda'),\mathcal{F}) \xrightarrow{\rho'} H^p(X(\mu),\mathcal{F})$$

be the restriction map, where μ is any real number with $\mu < Min(\lambda',\lambda_0)$. Then the compostion $\rho' o\rho$ is obviously injective. This implies that the restriction ρ is injective, which means that $H^p(X(\lambda),\mathcal{F}) = 0$ and $\lambda' \in A$.

Notes

To prove (c), we fix some $\lambda_0 \in A$ and suppose that $\{y \in Y : \phi(y) = \lambda_0\}\backslash\partial Y(\lambda_0) = \{y_0\}$ for some $y_0 \in Y$.

Let U be a Stein open neighborhood of y_0 such that $\Pi^{-1}(U)$ is r-complete and $\overline{U} \cap \overline{Y(\lambda_0)} = \emptyset$. There exist finitely many Stein open sets $U_i \subset\subset Y$, $1 \le i \le k$, disjoint from U such that $\partial Y(\lambda_0) \subset \bigcup_{i=1}^{k} U_i$ and $\Pi^{-1}(U_i)$ are r-complete. Let $\theta_i \in C_0^\infty(U_i,\mathbb{R}^+)$ be smooth compactly supported functions such that $\sum_{i=1}^{k}\theta_i(\xi)) > 0$ at every point $\xi \in \partial Y(\lambda_0)$. We can therefore choose sufficiently small numbers $c_i > 0$, $0 \le i \le k$, so that the functions $\phi_i : Y \to \mathbb{R}$, $1 \le i \le k$, defined by

$$\phi_0 = \phi, \quad \phi_i = \phi - \sum_{j=1}^{i} c_j\theta_j$$

Are q-convex with the same positivity directions. If we set

$$Y_i = \{x \in Y : \phi_i(x) < \lambda_0\} \text{ and } Y_0 = Y(\lambda_0), \quad \text{then}$$

$$Y_0 \subset Y_1 \subset Y_2 \subset \cdots \subset Y_k, Y_0 \subset\subset Y_k, Y_i \setminus Y_{i-1} \subset\subset U_i \text{ for } 1 \le i \le k$$

Moreover, since ϕ is exhaustive, there exists $\varepsilon_0 > 0$ such that $Y(\lambda_0+\varepsilon_0) \subset Y_k \cup U$. We define for an arbitrary real number λ with $\lambda_0 < \lambda < \lambda_0 + \varepsilon_0$ and integer $j = 0,\cdots,k$, the sets $Y_j(\lambda) = Y_j \cap Y(\lambda)$ and $X_j(\lambda) = \Pi^{-1}(Y_j(\lambda))$.

Since $Y(\lambda) = (Y(\lambda)\cap Y_k) \cup (Y(\lambda)\cap U)$, then $X(\lambda) = X_k(\lambda) \cup V(\lambda)$, where $V(\lambda) = \Pi^{-1}(Y(\lambda)\cap U) = \{x \in \Pi^{-1}(U) : \phi o\Pi(x) < \lambda\}$ is p-complete, because $\Pi^{-1}(U)$ is r-complete and $\phi o\Pi$ is q-convex. Moreover, $X_k(\lambda)\cap V(\lambda)$ is p-Runge in $V(\lambda)$. Therefore

$$H^p(X(\lambda),\mathcal{F}) = H^p(X_k(\lambda),\mathcal{F}) \oplus H^p(V(\lambda),\mathcal{F}) = H^p(X_k(\lambda),\mathcal{F})$$

To prove (c), we show inductively on j that $H^p(X_j(\lambda),\mathcal{F}) = 0$. For $j = 0$ this is clearly satisfied since $X_0(\lambda) = X(\lambda_0)$ and $\lambda_0 \in A$. Assume now that $j \ge 1$ and that $H^p(X_{j-1}(\lambda),\mathcal{F}) = 0$. Since $Y_j = Y_{j-1} \cup (Y_j \cap U_j)$, then $X_j(\lambda) = X_{j-1}(\lambda) \cup V_j(\lambda)$, where

$$V_j(\lambda) = \Pi^{-1}(U_j \cap Y_j(\lambda)) = \{x \in \Pi^{-1}(U_j) : \phi o\Pi(x) < \lambda, \phi_j o\Pi(x) < \lambda_0\}$$

is p-complete since $\Pi^{-1}(U_j)$ is r-complete and $\phi o\Pi$ and $\phi_j o\Pi$ are q-convex with the same positivity directions. Furthermore, as $X_{j-1}(\lambda) \cap V_j(\lambda)) = X_{j-1}(\lambda) \cap \Pi^{-1}(U_j) = \{x \in \Pi^{-1}(U_j) : \phi_{j-1} o\Pi(x) < \lambda_0, \phi o\Pi(x) < \lambda\}$ is clearly p-Runge in $\Pi^{-1}(U_j)$, then the restriction map

$$H^s(\Pi^{-1}(U_j), \mathcal{F}) \xrightarrow{\rho'} H^s(X_{j-1}(\lambda) \cap V_j(\lambda), \mathcal{F})$$

has a dense image for all $s \geq p-1$. Since ρ' is clearly injective and $p-1 \geq r$, then $H^{p-1}(X_{j-1}(\lambda) \cap V_j(\lambda), \mathcal{F}) = 0$. Therefore from the Mayer-Vietoris sequence for cohomology

$$\cdots \to H^{p-1}(X_{j-1}(\lambda) \cap V_j(\lambda), \mathcal{F}) \to H^p(X_j(\lambda), \mathcal{F}) \to H^p(X_{j-1}(\lambda), \mathcal{F}) \to \cdots,$$

we deduce that $H^p(X_j(\lambda), \mathcal{F}) = 0$.

To prove statement (b), it is sufficient to show that if $\lambda_j \nearrow \lambda$ and $\lambda_j \in A$ for all j, then

$$H^{p-1}(X(\lambda_{j+1}), \mathcal{F}) \longrightarrow H^{p-1}(X(\lambda_j), \mathcal{F})$$

has a dense image.

To complete the proof of theorem 1, it is, therefore, enough, according to (Cf. [3], p. 250), to show the following lemma.

Lemma 1. For every pair of real numbers $\mu < \lambda$, the restriction map

$$H^{p-1}(X(\lambda), \mathcal{F}) \to H^{p-1}(X(\mu), \mathcal{F})$$

has a dense range.

Proof. We consider the set T of all real numbers λ such that

$$H^{p-1}(X(\lambda), \mathcal{F}) \to H^{p-1}(X(\mu), \mathcal{F})$$

has a dense range for all $\mu \leq \lambda$.

To see that T is not empty, we choose $\lambda_0 = min\{\phi(y); y \in Y\}$. Then clearly $]-\infty, \lambda_0] \subset T$.

To prove that T is open in $]-\infty, +\infty[$ it is, therefore, sufficient to show that if $\lambda \in T$, there exists $\varepsilon > 0$ such that $\lambda + \varepsilon \in T$. For this, we consider a finite covering $(U_i)_{1 \leq i \leq k}$ of $\{y \in Y : \phi(z) = \lambda\}$ by Stein open sets $U_i \subset\subset Y$ and compactly supported functions $\theta_i \in C_o^\infty(U_i)$, $\theta_j \geq 0$, $j = 1, \cdots, k$ such that $\Pi^{-1}(U_i)$ is r-complete and $\sum_{i=1}^{k} \theta_i(x) > 0$ at any point of $\partial Y(\lambda)$. Define $Y_j = \{z \in Y : \phi_j(z) < \lambda\}$ where

Ref

3. A. Andreotti and H. Grauert, Théorèmes de finitude de la cohomologie des espaces complexes. Bull. Soc. Math. France 90 (1962;) 193 – 259.

$\phi_j(z) = \phi(z) - \sum_{1}^{j} c_i\theta_i$, with $c_i > 0$ sufficiently small so that $\phi_j(z)$ are still q-convex With the same positivity directions for $1 \leq j \leq k$.

If we consider the following sets defined in the lemma 2

$Y(\lambda) = \{y \in Y : \phi(y) < \lambda\}$, $X(\lambda) = \Pi^{-1}(Y(\lambda))$, $Y_i = \{x \in Y : \phi_i(x) < \lambda_0\}$ and $Y_0 = Y(\lambda_0)$, then

$$Y_0 \subset Y_1 \subset Y_2 \subset \cdots \subset Y_k, Y_0 \subset\subset Y_k, Y_i \setminus Y_{i-1} \subset\subset U_i \quad \text{for} \quad 1 \leq i \leq k$$

and $X_j(\lambda) = \Pi^{-1}(Y_j \cap Y(\lambda)) = X_{j-1}(\lambda) \cup V_j(\lambda)$, where

$$V_j(\lambda) = \Pi^{-1}(U_j \cap Y_j(\lambda)) = \{x \in \Pi^{-1}(U_j) : \phi o \Pi(x) < \lambda, \phi_j o \Pi(x) < \lambda_0\}$$

Now since $X_{j-1}(\lambda) \cap V_j(\lambda)$ is p-Runge in the p-complete set $V_j(\lambda)$ and $H^p(X_j(\lambda), \mathcal{F}) = 0$, it follows from the long exact sequence of cohomology

$$\cdots \to H^{p-1}(X_j(\lambda), \mathcal{F}) \to H^{p-1}(X_{j-1}(\lambda), \mathcal{F}) \oplus H^{p-1}(V_j(\lambda), \mathcal{F}) \to$$

$$H^{p-1}(X_{j-1}(\lambda) \cap V_j(\lambda), \mathcal{F}) \to H^p(X_j(\lambda), \mathcal{F}) \to \cdots$$

that the restriction map

$$H^{p-1}(X_j(\lambda), \mathcal{F}) \to H^{p-1}(X_{j-1}(\lambda), \mathcal{F})$$

has a dense range.

Moreover, since ϕ is exhaustive, there exists $\varepsilon > 0$ such that $Y(\lambda + \varepsilon) \subset Y_k$. We deduce that the restriction map

$$H^{p-1}(X(\lambda + \varepsilon), \mathcal{F}) \to H^{p-1}(X(\lambda), \mathcal{F})$$

has a dense image, which implies that $\lambda + \varepsilon \in T$.

Let now $\lambda_j \in T$, $j \geq 0$, such that $\lambda_j \nearrow \lambda$, and let $\mathcal{U} = (U_i)_{i \in I}$ be a countable base of Stein open covering of X. Then the restriction map between spaces of cocycles

$$Z^{p-1}(\mathcal{U}|_{X_{\lambda_{j+1}}}, \mathcal{F}) \to Z^{p-1}(\mathcal{U}|_{X_{\lambda_j}}, \mathcal{F})$$

has dense image for $j \geq 0$. Let $\lambda' < \lambda$ and $j \in \mathbb{N}$ such that $\lambda' < \lambda_j$. By [1, p.246], the restriction map $Z^{n-2}(\mathcal{U}|_{X_\lambda}, \mathcal{F}) \to Z^{n-2}(\mathcal{U}|_{X_{\lambda_j}}, \mathcal{F})$ has a dense image. Since $\lambda_j \in T$, then $Z^{n-2}(\mathcal{U}|_{X_{\lambda_j}}, \mathcal{F}) \to Z^{n-2}(\mathcal{U}|_{X_{\lambda'}}, \mathcal{F})$ has also a dense image, and hence $\lambda \in T$.

Now since $H^p(X(j), \mathcal{F}) = 0$ for all $j \in \mathbb{N}$ and $H^{p-1}(X(j+1), \mathcal{F})$ has a dense image in $H^{p-1}(X(j), \mathcal{F})$ for all $j \geq 0$, it follows from ([3], p. 250) that

Ref

3. A. Andreotti and H. Grauert, Théorèmes de finitude de la cohomologie des espaces complexes. Bull. Soc. Math. France 90 (1962;) 193 – 259.

$$H^p(X,\mathcal{F}) \to H^p(X(0),\mathcal{F})$$

is bijective, which shows that $H^p(X,\mathcal{F}) = 0$.

IV. A Counter-Example to the Andreotti-Grauert Conjecture

Theorem 2. There exists for each integer $n \geq 3$ a cohomologically q-complete open subset $\Omega \subset \mathbb{C}^n$, $2 \leq q < n$, which is not q-complete.

We consider the following example due to Diederich and Forness [4]. Let (n,q) be a pair of integers with $2 \leq q < n$ and such that $n = mq+1$, where $m = [\frac{n}{q}]$ is the integral part of $\frac{n}{q}$. We define the functions.

$$\phi_j(z) = \sigma_j(z) + \sum_{i=1}^{m} \sigma_i(z)^2 + N||z||^4 - \frac{1}{4}||z||^2, \quad j = 1, \cdots, m,$$

and

$$\phi_{m+1}(z) = -\sigma_1(z) - \cdots - \sigma_m(z) + \sum_{i=1}^{m} \sigma_i(z)^2 + N||z||^4 - \frac{1}{4}||z||^2,$$

where $\sigma_j(z) = Im(z_j) + \sum_{i=m+1}^{n} |z_i|^2 - (m+1) \sum_{i=m+(j-1)(q-1)+1}^{m+j(q-1)} |z_i|^2$, for $j = 1, \cdots, m$

$z = (z_1, z_2, \cdots, z_n)$, and $N > 0$ a positive constant. Then, if N is large enough, the functions $\phi_1, \cdots \phi_2$ are q-convex on $\mathbb{C}^n$ and, if $\rho = Max(\phi_1, \cdots, \phi_{m+1})$, then, for $\varepsilon_o > 0$ small enough, the set $D_{\varepsilon_o} = \{z \in \mathbb{C}^n : \rho(z) < -\varepsilon_o\}$ is relatively compact in the unit ball $B = B(0,1)$ if N is sufficiently large. (See [4]).

We fix some $\varepsilon > \varepsilon_0$ and consider a covering $(U_i)_{i\in\mathbb{N}}$ of ∂D_ε, by Stein open subsets $U_i \subset\subset D_{\varepsilon_0}$ and functions $\theta_i \in C_0^\infty(\mathbb{C}^n, \mathbb{R})$ such that

$$\theta_j \geq 0, \quad Supp(\theta_j) \subset\subset U_j, \sum_{i=1}^{k} \theta_j(x) > 0 \text{ at any point } x \in \partial D_\varepsilon.$$

We can therefore choose sufficiently small positive numbers $c_1, \cdots, c_k$ so that the functions $\phi_{i,j} = \phi_i - \sum_{l=1}^{j} c_l\theta_l$ are q-convex for $i = 1, \cdots, m+1$ and $1 \leq j \leq k$.

We define $\phi_{i,0} = \phi_i$ for $i = 1, \cdots, m+1$, $D_0 = D_\varepsilon$ and $D_j = \{z \in D_{\varepsilon_0} : \rho_j(z) < -\varepsilon\}$, where $\rho_j(z) = \rho - \sum_{i=1}^{j} c_i\theta_i$ for $j = 1, \cdots, k$. Then ρ_j are q-convex with corners and it is clear that

$$D_0 \subset D_1 \subset \cdots \subset D_k,\ D_0 \subset\subset D_k \subset\subset D_{\varepsilon_0} \text{ and } D_j \setminus D_{j-1} \subset\subset U_j \text{ for } j = 1, \cdots = k.$$

Lemma 2. In the situation described above, for any coherent analytic sheaf $\mathcal{F}$ on D_{ε_0}, the restriction map $H^p(D_{j+1},\mathcal{F}) \to H^p(D_j,\mathcal{F})$ is surjective for all $p \geq \tilde{q}-1$ and all $0 \leq j \leq k-1$. In particular, $dim_{\mathbb{C}} H^p(D_j,\mathcal{F}) < \infty$, if $p \geq \tilde{q}-1$.

Ref

4. M. Coltoiu and K. Diederich, The levi problem for Riemann domains over Stein spaces with isolated singularities. Math. Ann. (2007) 338: 283 – 289.

Proof. We first prove that the cohomology group $H^p(D_j \cap U_l, \mathcal{F}) = 0$ for all $p \geq \tilde{q}-1$, $0 \leq j \leq k$, and $1 \leq l \leq k$. In fact, the set $D_j \cap U_l$ can be written in the form $D_j \cap U_l = D'_1 \cap \cdots \cap D'_{m+1}$, where $D'_i = \{z \in U_l : \phi_{i,j}(z) < -\varepsilon\}$ are clearly q-complete. Then for every $i_1, \cdots, i_m \in \{1, \cdots, m+1\}$, $D'_{i_1} \cap \cdots \cap D'_{i_m}$ are $(\tilde{q}-1)$-complete. Therefore, by using Proposition 1 of [11], we obtain

$$H^p(D_j \cap U_l, \mathcal{F}) \cong H^{p+m}(D'_1 \cup \cdots \cup D'_{m+1}, \mathcal{F})$$

if $p \geq \tilde{q}-1$, which implies that $H^p(D_j \cap U_l, \mathcal{F}) = 0$ for all $p \geq \tilde{q}-1$.

Now since $D_{j+1} = D_j \cup (D_{j+1} \cap U_{j+1})$, it follows from the Mayer-Vietoris sequence for cohomology

$$\to H^p(D_{j+1}, \mathcal{F}) \to H^p(D_j, \mathcal{F}) \oplus H^p(D_{j+1} \cap U_{j+1}, \mathcal{F}) \to H^p(D_j \cap U_{j+1}, \mathcal{F}) \to H^{p+1}(D_{j+1}, \mathcal{F}) \to$$

that the restriction map

$$H^p(D_{j+1}, \mathcal{F}) \longrightarrow H^p(D_j, \mathcal{F})$$

is surjective when $p \geq \tilde{q}-1$.

Let now A be the set of all real numbers $\varepsilon \geq \varepsilon_0$ such that $H^p(D_\varepsilon, \mathcal{F}) = 0$ for all $p \geq \tilde{q}-1$.

Lemma 3. *-The set A is not empty and, if $\varepsilon \in A$, $\varepsilon > \varepsilon_0$, then there exits $\varepsilon' \in A$ such that $\varepsilon_0 \leq \varepsilon' < \varepsilon$.*

Proof. In fact, if $\mu_0 = Min_{z\in\overline{B}}\{\phi_i(z), i = 1, \cdots, m+1\}$, then one sees easily that $[-\mu_0, +\infty[\subset A$.

For the proof of the second assertion, if with the notations of lemma 1 we set $D_0 = D_\varepsilon$, we obtain $D_0 \subset D_1 \subset \cdots \subset D_k$, $D_0 \subset\subset D_k \subset\subset D_{\varepsilon_0}$ and $D_j \setminus D_{j-1} \subset\subset U_j$ for $j = 1, \cdots = k$.

We fix some $1 \leq j \leq k$ and $1 \leq l \leq k$, and set $D_j \cap U_l = D'_1 \cap \cdots D'_{m+1}$, where $D'_i = \{z \in U_l : \phi_{i,j}(z) < -\varepsilon\}$, then D'_i are q-complete and q-Runge in U_l. Therefore because of the proof of lemma 2, one obtains

$$H^p(D_j \cap U_l, \mathcal{F}) \cong H^{p+m}(D'_1 \cup \cdots \cup D'_{m+1}, \mathcal{F}) = 0$$

for $p \geq \tilde{q}-1$ and, consequently, the restriction map

$$H^p(D_{j+1}, \mathcal{F}) \longrightarrow H^p(D_j, \mathcal{F})$$

is surjective for all $p \geq \tilde{q}-1$.

We now show inductively on j that $H^{\tilde{q}-1}(D_j, \mathcal{F}) = 0$. For $j = 0$, this is clearly satisfied since $D_0 = D_\varepsilon$ and $\varepsilon \in A$. Assume now that this property has already

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been proved for $j < k$. Since for every $i_1, \cdots, i_m$, in $\{1, \cdots, m+1\}$, the open set $D'_{i_1} \cap \cdots \cap D'_{i_m}$ is $(\tilde{q}-1)$-Runge in U_l, then the restriction map

$$H^p(U_l, \mathcal{F}) \longrightarrow H^p(D'_{i_m} \cap \cdots \cap D'_{i_m}, \mathcal{F})$$

has a dense range for $p \geq \tilde{q}-2$. Since the canonical topologies on $H^i(D'_{i_m} \cap \cdots \cap D'_{i_m}, \mathcal{F})$ are obviously separated for $i \geq 2$, then $H^p(D'_{i_m} \cap \cdots \cap D'_{i_m}, \mathcal{F}) = 0$ for all $p \geq \tilde{q}-2$. We know from Proposition 1 of [11] that $H^p(D_j \cap U_l, \mathcal{F}) \cong H^{p+m}(D'_1 \cup \cdots \cup D'_{m+1}, \mathcal{F})$ for $p \geq \tilde{q}-2 = n-m-1$. We can choose the covering $(U_i)_{1 \leq i \leq k}$ of ∂D_ε such that $U_l \setminus D'_1 \cup \cdots \cup D'_{m+1}$ has no compact connected components, so it follows from the mean theorem of [5], that the restriction $H^p(U_l, \mathcal{F}) \longrightarrow H^p(D'_1 \cup \cdots \cup D'_{m+1}, \mathcal{F})$ has a dense image for $p \geq n-1$. This proves that

$$H^p(D_j \cap U_l, \mathcal{F}) \cong H^{p+m}(D'_1 \cup \cdots \cup D'_{m+1}, \mathcal{F}) = 0 \text{ for all } p \geq \tilde{q}-2.$$

Now since $H^{\tilde{q}-2}(D_j \cap U_{j+1}, \mathcal{F}) = H^{\tilde{q}-1}(D_{j+1} \cap U_{j+1}, \mathcal{F}) = H^{\tilde{q}-1}(D_j, \mathcal{F}) = 0$, it follows from the Mayer-Vietoris sequence for cohomolgy

$$\to H^{\tilde{q}-2}(D_j \cap U_{j+1}, \mathcal{F}) \to H^{\tilde{q}-1}(D_{j+1}, \mathcal{F}) \to H^{\tilde{q}-1}(D_j, \mathcal{F}) \oplus H^{\tilde{q}-1}(D_{j+1} \cap U_{j+1}, \mathcal{F}) \to$$

that $H^{\tilde{q}-1}(D_{j+1}, \mathcal{F}) = 0$.

On the other hand, since ρ is proper, there exists $\varepsilon' > 0$ such that $\varepsilon - \varepsilon' > \varepsilon_o$ and $D_{\varepsilon-\varepsilon'} = \{z \in D_{\varepsilon_o} : \rho(z) < \varepsilon' - \varepsilon\} \subset\subset D_k$.

Since $H^{\tilde{q}-1}(D_k, \mathcal{F}) \to H^{\tilde{q}-1}(D_{\varepsilon-\varepsilon'}, \mathcal{F})$ is surjective, $H^{\tilde{q}-1}(D_k, \mathcal{F}) = 0$ and $dim_{\mathbb{C}} H^{\tilde{q}-1}(D_{\varepsilon-\varepsilon'}, \mathcal{F}) < \infty$, then $H^{\tilde{q}-1}(D_{\varepsilon'-\varepsilon'}, \mathcal{F}) = 0$, whence $\varepsilon - \varepsilon' \in A$.

Lemma 4. The open set D_{ε_0} is cohomologically $(\tilde{q}-1)$-complete.

Proof. For this, we consider the set A of all real numbers $\varepsilon \geq \varepsilon_0$ such that $H^p(D_\varepsilon, \mathcal{F}) = 0$ for all $p \geq \tilde{q}-1$. Then by lemma 3, A is not empty and open in $[\varepsilon_0, \infty[$. Moreover, if $\varepsilon = Inf(A)$, there exists a decreasing sequence of real numbers $\varepsilon_j \in A$, $j \geq 1$, such that $\varepsilon_j \searrow \varepsilon$. Since $H^p(D_{\varepsilon_j}, \mathcal{F}) = 0$ for $p \geq \tilde{q}-1$ and, by lemma 1, the restriction map $H^p(D_{\varepsilon_{j+1}}, \mathcal{F}) \to H^p(D_{\varepsilon_j}, \mathcal{F})$ is surjective for all $p \geq \tilde{q}-2$, then by ([3], p. 250), the restriction map

$$H^p(D_\varepsilon, \mathcal{F}) \longrightarrow H^p(D_{\varepsilon_1}, \mathcal{F})$$

is an isomorphism for $p \geq \tilde{q}-1$, which shows that $\varepsilon \in A$.

Assume now that $\varepsilon > \varepsilon_0$. Then there exists, according to lemma 1, $\varepsilon' \in A$ such that $\varepsilon_0 < \varepsilon' < \varepsilon$, which contradicts the fact that $\varepsilon = Inf(A)$. We conclude that $\varepsilon = \varepsilon_0 \in A$, and hence D_{ε_0} is cohomologically $(\tilde{q}-1)$-complete.

End of the proof of theorem 2

We have shown that D_{ε_0} is cohomologically $(\tilde{q}-1)$-complete. We are now going to prove that for a good choice of the contants ε_0 and N, we can find an $\varepsilon > \varepsilon_0$ such that D_ε is cohomologically $(\tilde{q}-1)$-complete but Ω not $(\tilde{q}-1)$-complete.

Ref

11. K. Matsumoto. On the cohomological completeness of q-complete domains with corners. Nagoya Math. J. Vol. 165 (2002), 105-112.

In fact, it was shown by Diederich-Forness [4] that if $\delta > 0$ is small enough, then the topological sphere of real dimension $n + \tilde{q} - 2$

$$S_\delta = \{z \in \mathbb{C}^n : x_1^2 + \cdots + x_m^2 + |z_{m+1}|^2 + \cdots + |z_n|^2 = \delta,$$

$$y_j = -\sum_{i=m+1}^{n} |z_i|^2 + (m+1) \sum_{i=m+(j-1)(q-1)+1}^{m+j(q-1)} |z_i|^2 \ \text{ for } \ j = 1, \cdots, m\}$$

is not homologous to 0 in D_{ε_0}. This follows from the fact that the set $E = \{z \in \mathbb{C}^n : x_1 = z_2 = \cdots = z_n = 0\}$ does not intersect D_{ε_0}, since on E

$$\phi_j = y_j + \frac{3}{4}\sum_{i=1}^{m} y_i^2 + N(\sum_{i=1}^{m} y_i^2)^2 \ \text{ for } \ j = 1, \cdots, m$$

and

$$\phi_{m+1} = -y_1 - \cdots - y_m + \frac{3}{4}\sum_{i=1}^{m} y_i^2 + N(\sum_{i=1}^{m} y_i^2)^2$$

such that $\rho \geq 0$ on E. So the following real form of degree $n + \tilde{q} - 2$

$$\omega = (\sum_{i=1}^{n} x_i^2 + \sum_{i=m+1}^{n} y_i^2)^{-2n+m} (\sum_{i=1}^{n} (-1)^i x_i dx_1 \wedge \cdots \widehat{dx_i} \wedge \cdots \wedge dx_n \wedge dy_{m+1} \wedge \cdots \wedge$$

$$dy_n + \sum_{i=1}^{n-m} (-1)^{n+i} y_{m+i} dx_1 \wedge \cdots \wedge dx_n \wedge dy_{m+1} \wedge \cdots \wedge \widehat{dy_{m+i}} \wedge \cdots \wedge dy_n)$$

is well-defined and d-closed on D_{ε_0}. Since ω does not depend on $y_1, \cdots, y_m$, then by the standard argument $\int_{S_\delta} \omega \neq 0$. Therefore S_δ is not homologous to 0 in D_{ε_0}.

Let $\mathcal{E}_q$ be the sheaf of germs of C^∞ q-forms on $\mathbb{C}^n$ and $\mathcal{T}_q$ the sheaf of germs of C^∞ d-closed q-forms. Then we have an exact sequence of sheaf homomorphisms

$$0 \to \mathcal{T}_q \to \mathcal{E}_q \stackrel{d}{\to} \mathcal{T}_{q+1} \to 0$$

Since by the de Rham theorem for every $p \geq 1$, the cohomology group $H^p(D_{\varepsilon_0}, \mathbb{C})$ is isomorphic to

$$\frac{\{\omega \in \Gamma(D_{\varepsilon_0}, \mathcal{E}_p) : df = 0\}}{\{d\omega : \omega \in \Gamma(D_{\varepsilon_0}, \mathcal{E}_{p-1})\}},$$

it follows from Stokes formula that $H^{n+\tilde{q}-2}(D_{\varepsilon_0}, \mathbb{C})$ does not vanish.

We are going to show that $H^r(D_{\varepsilon_0}, \mathcal{O}_{D_{\varepsilon_0}}) = 0$ for all r with $1 \leq r \leq \tilde{q} - 3$.

We first assert that we can choose N, ε_0, and $\varepsilon > \varepsilon_0$ such that, if, with the notations of Proposition 1, we set

$$\phi_j(z) = \sigma_j(z) + \sum_{i=1}^{m} \sigma_i(z)^2 + N||z||^4 - \frac{1}{4}||z||^2, \ \ j = 1, \cdots, m,$$

and

$$\phi_{m+1}(z) = \sigma(z) + \sum_{i=1}^{m} \sigma_i(z)^2 + N||z||^4 - \frac{1}{4}||z||^2, \ \text{ where } \ \sigma(z) = -\sum_{i=1}^{m} \sigma_i(z),$$

Ref

4. M. Coltoiu and K. Diederich, The levi problem for Riemann domains over Stein spaces with isolated singularities. Math. Ann. (2007) 338: 283 – 289.

$$\sigma_j(z) = Im(z_j) + \sum_{i=m+1}^{n} |z_i|^2 - (m+1) \sum_{i=m+(j-1)(q-1)+1}^{m+j(q-1)} |z_i|^2, \text{ for } j = 1, \cdots, m,$$

$$(z) = N||z||^4 - \frac{1}{4}||z||^2 + \varepsilon_0 \text{ and } \rho(z) = Max(\phi_1(z), \cdots, \phi_{m+1}) + \sum_{i=1}^{m} \sigma_i(z)^2 + \ (z) - \varepsilon_0,$$

then we obtain

$$D_\varepsilon = \{z \in D_\varepsilon : \phi(z) < \varepsilon_0 - \varepsilon\}$$

where $m' = Min_{z \in \overline{D}_{\varepsilon_0}} \ (z)$, and

$$\phi(z) = \sigma(z) + \sum_{i=1}^{m} \sigma_i(z)^2 + m'$$

In fact, we can choose $\varepsilon > \varepsilon_0$ sufficiently big and $\lambda > 0$ small enough so that $\varepsilon_0 - \varepsilon < m' < (1+\lambda).Min_{z \in \overline{D}_\varepsilon} \ (z)$ and $\lambda\varepsilon - (1+\lambda)\varepsilon_0 > 0$.

On the other hand, if $\delta = Min_{z \in \overline{D}_{\varepsilon_0}} ||z||^2$, then we have

$$0 < \delta \leq ||z||^2 < \frac{1}{4N} - \frac{\varepsilon_0}{N} \text{ for every } z \in \overline{D}_{\varepsilon_0}$$

Therefore by suitable choice of ε_0, ε and N we can also achieve that

$$(N||z||^4 - \frac{1}{4}||z||^2) - Min_{z \in \overline{D}_{\varepsilon_0}}(N||z||^4 - \frac{1}{4}||z||^2) < Min(\frac{\varepsilon - \varepsilon_0}{2}, \lambda\varepsilon - (1+\lambda)\varepsilon_0),$$

and

$$Max_{z \in \overline{D}_{\varepsilon_0}}(N||z||^4 - \frac{1}{4}||z||^2) - |(N||z||^4 - \frac{1}{4}||z||^2) < Min(\frac{\varepsilon - \varepsilon_0}{2}, \lambda\varepsilon - (1+\lambda)\varepsilon_0),$$

for every $z \in \overline{D}_\varepsilon$.

Because $\ (z) < \varepsilon_0 - \varepsilon$ on $\overline{D}_\varepsilon$, then clearly we obtain

$$\phi(z) = \sigma(z) + \sum_{i=1}^{m} \sigma_i(z)^2 + m' < \sigma(z) + \sum_{i=1}^{m} \sigma_i(z)^2 + (1+\lambda).\psi(z)) < (1+\lambda)(\varepsilon_0 - \varepsilon), \text{ if } z \in \overline{D}_\varepsilon,$$

which shows that

$$D_\varepsilon = \{z \in D_\varepsilon : \phi(z) < \varepsilon_0 - \varepsilon\}$$

We are now going to show that for every none-positive real number α with $\alpha < \varepsilon_0 - \varepsilon$, the open sets

$$B_\alpha = \{z \in D_\varepsilon : \phi(z) < \alpha\}$$

are relatively compact in D_ε.

To see this, we consider a sequence $(z_j)_{j \geq 0} \subset B_\alpha$, which converges to a point $z \in \overline{D}_\varepsilon$. Then one has for every sufficiently large integer j

$$\rho(z_j) = Max(\sigma_1(z_j), \cdots, \sigma_m(z_j), \sigma(z_j)) + \sum_{i=1}^{m} \sigma_i(z_j)^2 + N||z_j||^4 - \frac{1}{4}||z_j||^2 < -\varepsilon$$

Since

$$\phi(z_j) < \varepsilon_0 - \varepsilon + \lambda\psi(z_j) < (1+\lambda)(\varepsilon_0 - \varepsilon)$$

and

$$N||z_j||^4 - \frac{1}{4}||z_j||^2 - Min_{z\in \overline{D}_{\varepsilon_0}}(N||z||^4 - \frac{1}{4}||z||^2) < \lambda\varepsilon - (1+\lambda)\varepsilon_0$$

then

$$\rho(z_j) = \phi(z_j) + N(||z_j||^4 - \frac{1}{4}||z_j||^2) - m < \varepsilon_0 - \varepsilon + \lambda\psi(z_j) + \lambda\varepsilon - (1+\lambda)\varepsilon_0$$

A passage to the limit shows that

$$\rho(z) \leq \varepsilon_0 - \varepsilon + \lambda\psi(z) + \lambda\varepsilon - (1+\lambda)\varepsilon_0 < (1+\lambda)(\varepsilon_0 - \varepsilon) + \lambda\varepsilon - (1+\lambda)\varepsilon_0 = -\varepsilon,$$

because $\ (z) < \varepsilon_0 - \varepsilon$, which implies that $z \in D_\varepsilon$. We conclude that with such a choice of ε_0, N, and ε the limit $z \in D_\varepsilon$, and hence the open set

$$B_\alpha = \{z \in D_\varepsilon : \phi(z) < \alpha\}$$

is relatively compact in D_ε for all real numbers α, with $\alpha < \varepsilon_0 - \varepsilon$.

Now since ϕ is in addition $(m+2)$-convex, then a similar proof of theorem 15 of [3] shows that, if Ω^i is the sheaf of germs of holomorphic i-forms on $\mathbb{C}^n$, $i \geq 0$, $(\Omega^0 = \mathcal{O}_{\mathbb{C}^n})$, and $B_c = \{z \in D_\varepsilon : \phi(z) < c\}$ for $c \leq \varepsilon_0 - \varepsilon$, then the map

$$H^r(D_\varepsilon, \Omega^i) \longrightarrow H^r(D_\varepsilon \setminus B_c, \Omega^i)$$

is injective for every $r < n-m-1$ and $c < \varepsilon_0 - \varepsilon$. Then obviously $H^r(D_\varepsilon, \Omega^i) = 0$ for $1 \leq r \leq n-m-2$ and $i \geq 0$. In fact, let $c_0 = Max_{z\in\overline{D}_\varepsilon}\phi(z)$. Then there exists $z_1 \in \partial D_\varepsilon$ such that $\phi(z_1) = c_0$. Since $c_0 = \phi(z_1) = \sigma(z_1) + \sum_{i=1}^{m}\sigma_i(z_1)^2 + m' < \rho(z_1) + \varepsilon_0 \leq \varepsilon_0 - \varepsilon$, then $B_{c_0} = D_\varepsilon$, and hence $H^r(D_\varepsilon, \Omega^i) = 0$ for $1 \leq r \leq n-m-2$.

Now if we suppose that D_ε is $(\tilde{q}-1)$-complete, then there exists a C^∞ strictly $(\tilde{q}-1)$-convex function $\ : D_\varepsilon \to \mathbb{R}$ such that $D_{\varepsilon,c} = \{z \in D_\varepsilon : \ (z) < c\}$ is relatively compact in D_ε for every $c \in \mathbb{R}$.

We now consider the resolution of the constant sheaf $\mathbb{C}$ on D_ε

$$0 \to \mathbb{C} \to \mathcal{O} \overset{d}{\to} \Omega^1 \overset{d}{\to} \cdots \to \overset{d}{\to} \Omega^n \to 0$$

If we set $Z^j = Im(\Omega^{j-1} \overset{d}{\to} \Omega^j)$ for $1 \leq j \leq n-1$, then we get short exact sequences

$$0 \to \mathbb{C} \to \mathcal{O} \to Z^1 \to 0$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$

$$0 \to Z^j \to \Omega^j \to Z^{j+1} \to 0$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$

$$0 \to Z^{n-2} \to \Omega^{n-2} \to Z^{n-1} \to 0$$

$$0 \to Z^{n-1} \to \Omega^{n-1} \to \Omega^n \to 0$$

Since, by Proposition 1, D_ε is cohomologically $(\tilde{q}-1)$-complete, then $H^r(D_\varepsilon, \Omega^i) = 0$ for all $r \geq \tilde{q}-1$ and $i \geq 0$. So we obtain the isomorphisms

$$H^{\tilde{q}-1}(D_\varepsilon, Z^{n-1}) \cong \cdots \cong H^{2n-m-2}(D_\varepsilon, Z^1) \cong H^{2n-m-1}(D_\varepsilon, \mathbb{C})$$

and the exact sequence

$$\cdots \to H^{\tilde{q}-2}(D_\varepsilon, \Omega^n) \to H^{\tilde{q}-1}(D_\varepsilon, Z^{n-1}) \to H^{\tilde{q}-1}(D_\varepsilon, \Omega^{n-1}) = 0$$

Ref

3. A. Andreotti and H. Grauert, Théorèmes de finitude de la cohomologie des espaces complexes. Bull. Soc. Math. France 90 (1962;) 193 – 259.

We deduce that the map

$$H^{\tilde{q}-2}(D_\varepsilon, \Omega^n) \xrightarrow{\varphi} H^{n+\tilde{q}-2}(D_\varepsilon, \mathbb{C})$$

is surjective. The map φ is defined as follows : If a differential form $\omega \in C^\infty_{n,\tilde{q}-2}(D_\varepsilon)$ satisfies the equation $\overline{\partial}\omega = 0$, then ω is also d-closed and therefore defines a cohomology class in $H^{n+\tilde{q}-2}(D_\varepsilon, \mathbb{C})$.

Moreover, since, by theorem 1 in [8], every d-closed differential form $\omega \in C^\infty_{n,\tilde{q}-2}(D_\varepsilon)$ is cohomologous to a $\overline{\partial}$-closed $(n, \tilde{q}-2)$ differntial form $\omega' \in C^\infty_{n,\tilde{q}-2}(D_\varepsilon)$, it follows that the map

$$H^{\tilde{}-2}(D_\varepsilon, \Omega^n) \xrightarrow{\varphi} H^{n+\tilde{q}-2}(D_\varepsilon, \mathbb{C})$$

is bijective.

Now if we suppose that D_ε is $(\tilde{q}-1)$-complete, then there exists a C^∞ strictly $(\tilde{q}-1)$-convex function $\ : D_\varepsilon \to \mathbb{R}$ such that $D_{\varepsilon,c} = \{z \in D_\varepsilon : \ (z) < c\}$ is relatively compact in D_ε for every $c \in \mathbb{R}$.

Notice that for the given ε, if $\delta > 0$ is small enough, the topological sphere

$$S_\delta = \{z \in \mathbb{C}^n : x_1^2 + |z_2|^2 + \cdots + |z_n|^2 = \delta,\ \sigma_1(z) = 0\} \subset D_\varepsilon$$

Since is exhaustive on D_ε, there exists $c' > 0$ such that S_δ is not homologuous to 0 in $D_{\varepsilon,c'}$. Let $c > c'$. Then $D_{\varepsilon,c}$ and $D_{\varepsilon,c'}$ are $(\tilde{q}-1)$-complete and, similarly $H^p(D_{\varepsilon,c}, \Omega^i) = H^p(D_{\varepsilon,c'}, \Omega^i) = 0$ for $1 \leq p \leq n-m-2$ and $i \geq 0$. Also the maps $H^{\tilde{q}-2}(D_{\varepsilon,c}, \Omega^n) \to H^{n+\tilde{q}-2}(D_{\varepsilon,c}, \mathbb{C})$ and $H^{\tilde{q}-2}(D_{\varepsilon,c'}, \Omega^n) \to H^{n+\tilde{q}-2}(D_{\varepsilon,c'}, \mathbb{C})$ are bijective. Moreover, since the levi form of has at least $m+1$ strictly positive eingenvalues, then by using Morse theory (See for instance [7]) we find that

$$H^{n+\tilde{q}-2}(D_{\varepsilon,c}, \mathbb{C}) \cong H^{n+\tilde{q}-2}(D_{\varepsilon,c'}, \mathbb{C})$$

It follows from the commutative diagram of continuous maps

$$\begin{array}{ccc} H^{\tilde{q}-2}(D_{\varepsilon,c}, \Omega^n) & \to & H^{n+\tilde{q}-2}(D_{\varepsilon,c}\mathbb{C}) \\ \downarrow & & \downarrow \\ H^{\tilde{q}-2}(D_{\varepsilon,c'}, \Omega^n) & \to & H^{n+\tilde{q}-2}(D_{\varepsilon,c'}, \mathbb{C}) \end{array}$$

that the restriction homomorphism

$$H^{\tilde{q}-2}(D_{\varepsilon,c}, \Omega^n) \to H^{\tilde{q}-2}(D_{\varepsilon,c'}, \Omega^n)$$

is bijective. Since in addition $D_{\varepsilon,c'}$ is relatively compact in $D_{\varepsilon,c}$, the function being exhaustive on D_ε, then, according to theorem 11 of [1], one obtains

$$dim_{\mathbb{C}} H^{\tilde{q}-2}(D_{\varepsilon,c'}, \Omega^n) < \infty$$

Since the sheaf Ω^n is isomorphic to $\mathcal{O}_{D_\varepsilon}$, then we have also $dim_{\mathbb{C}} H^{\tilde{q}-2}(D_{\varepsilon,c'}, \mathcal{O}_{D_\varepsilon}) < \infty$. Furthermore, since $D_{\varepsilon,c'}$ is cohomologically $(\tilde{q}-1)$-complete and $H^r(D_\varepsilon, \mathcal{O}_{D_\varepsilon}) = 0$ for $1 \leq r \leq n-m-2$, it follows from theorem 1 of [6] that $D_{\varepsilon,c'}$ is Stein, which is in contradiction with the fact that $H^{n+\tilde{q}-2}(D_{\varepsilon,c'}, \mathbb{C}) \neq 0$, since $S_\delta \subset D_{\varepsilon,c'}$ is not homologous to 0 in $D_{\varepsilon,c'}$. We conclude that D_ε is cohomologically $(\tilde{q}-1)$-complete but not $(\tilde{q}-1)$-complete.

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7. H. Diederich, J. E. Fornaess, Smoothing q-convex functions and vanishing theorems. Invent. Math. 82. 291-305 (1985).

Theorem 3. There exists for each integer $n \geq 3$ a cohomologically $(n-1)$-complete open subset Ω of $\mathbb{C}^n$ which is locally $(n-1)$-complete in $\mathbb{C}^n$ but Ω is not $(n-1)$-complete.

Proof. We consider for $n \geq 3$ the functions $\phi_1, \phi_2 : \mathbb{C}^n \to \mathbb{R}$ defined by

$$\phi_1(z) = \sigma_1(z) + \sigma_1(z)^2 + N||z||^4 - \frac{1}{4}||z||^2,$$

$$\phi_2(z) = -\sigma_1(z) + \sigma_1(z)^2 + N||z||^4 - \frac{1}{4}||z||^2,$$

where $\sigma_1(z) = Im(z_1) + \sum_{i=3}^{n} |z_i|^2 - |z_2|^2$, $z = (z_1, z_2, \cdots, z_n)$, and $N > 0$ a positive constant. Then, if N is large enough, the functions ϕ_1 and ϕ_2 are $(n-1)$-convex on $\mathbb{C}^n$ and, if $\rho = Max(\phi_1, \phi_2)$, then, for $\varepsilon_o > 0$ small enough, the set $D_{\varepsilon_o} = \{z \in \mathbb{C}^n : \rho(z) < -\varepsilon_o\}$ is relatively compact in the unit ball $B = B(0,1)$, if N is sufficiently large.

According to ([2], p. 20), we can choose $\varepsilon_0 > 0$ such that if $\delta = Min_{z \in \overline{D}_{\varepsilon_0}} ||z||^2$, then we have

$$0 < \delta \leq ||z||^2 < \frac{1}{4N} - \frac{\varepsilon_0}{N} \quad \text{for every} \quad z \in \overline{D}_{\varepsilon_0}$$

and that by a suitable choice of $\varepsilon > \varepsilon_0$,

$$D_\varepsilon = \{z \in \mathbb{C}^n : \rho(z) < -\varepsilon\}$$

is cohomologically $(n-1)$-complete but not $(n-1)$-complete.

Now if we suppose that at a boundary point $z_0 \in \partial D_\varepsilon$, we have $\phi_1(z_0) = \phi_2(z_0)$, then $\sigma_1(z_0) = 0$ and, hence $N|z_0|^4 - \frac{|z_0|^2}{4} = \varepsilon$. This implies $|z_0|^2 = \frac{1}{8N}(1 + \sqrt{1 + 64N\varepsilon} < \frac{1}{4N}$. Therefore $\frac{1}{2}\sqrt{1 + 64N\varepsilon} < \frac{1}{2}$, which is a contradiction. This implies that $\phi_1(z) \neq \phi_2(z)$ at every boundary point $z \in \partial D_\varepsilon$. We conclude that with such a choice of ε_0 , N and ε, D_ε is obviously locally $(n-1)$-complete in $\mathbb{C}^n$.

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Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 22 Issue 5 Version 1.0 Year 2022
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Forecasting Foreign Exchange Rates of Different Countries using Non Linear Models

By Boina Jhansi Rani & Dr. S. A. Jyothi Rani
Osmania University

Abstract- This Paper investigates the behavior of daily exchange rates between USD/INR, GBP/INR, CHF/INR and JPY/INR. This Paper attempts to examine the performance of ARCH/GARCH model in forecasting the currencies traded in Indian foreign exchange markets. The accuracy of the forecast is compared with Mean Error (ME), Mean Absolute Error (MAE), Mean Percentage error (MPE), Mean Absolute Percentage Error (MAPE) and Root Mean Squared Error (RMSE), Mean Absolute Square Error. Study the performance indicators of model by using AIC and BIC values. This paper attempts to examine the performance of ARCH/GARCH model in forecasting the currencies traded in Indian foreign exchange markets. Study the Forecasting value of next 5 years exchange rates USD American dollar, GBP British Pound, CHF Swiss Franc and JPY Japanese JPY currencies with INR Indian data from that investigates the behavior of daily exchange rates between USD/INR, GBP/INR, CHF/INR and JPY/INR.

Keywords: forex, forecasting, ARCH, GRACH.

GJSFR-F Classification: DDC Code: 823.914 LCC Code: PR6052.R246

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Forecasting Foreign Exchange Rates of Different Countries using Non Linear Models

Boina Jhansi Rani [α] & Dr. S. A. Jyothi Rani [σ]

Abstact- This Paper investigates the behavior of daily exchange rates between USD/INR, GBP/INR, CHF/INR and JPY/INR. This Paper attempts to examine the performance of ARCH/GARCH model in forecasting the currencies traded in Indian foreign exchange markets. The accuracy of the forecast is compared with Mean Error (ME), Mean Absolute Error (MAE), Mean Percentage error (MPE), Mean Absolute Percentage Error (MAPE) and Root Mean Squared Error (RMSE), Mean Absolute Square Error. Study the performance indicators of model by using AIC and BIC values. This paper attempts to examine the performance of ARCH/GARCH model in forecasting the currencies traded in Indian foreign exchange markets. Study the Forecasting value of next 5 years exchange rates USD American dollar, GBP British Pound, CHF Swiss Franc and JPY Japanese JPY currencies with INR Indian data from that investigates the behavior of daily exchange rates between USD/INR, GBP/INR, CHF/INR and JPY/INR.

Comparison of daily exchange rates of USD American dollar, GBP British Pound, CHF Swiss Franc and JPY Japanese JPY currencies with INR Indian currency data taken from 15th May 2011-13th May 2021 analyzed is collected from the official website (http://www.rbi.org.in) of Reserve Bank of India (RBI).

After choosing the best model, serial correlation and heteroscedasticity is checked. Also, by using the Jarque-Bera test it can be stated if the residuals are normally distributed or not. At last the conditional standard deviation is plotted. Autoregressive conditional heteroskedasticity (ARCH) models are used for modelling observed time series. Also they are used in order to characterize various observed time series. An ARCH (q) model is estimated using ordinary least squares.

Keywords: forex, forecasting, ARCH, GRACH.

I. Introduction

Generally, we se ARMA model for evaluate conditional mean and ARCH to evaluate Conditional variance or Volatility or Dispersion. We are more interested to find out conditional variance, because we want to use the past history to forecast the variance. In financial market depends on "Volatility is clustering" implies time varying conditional variance.

In financial marketing risk has become an important part both for risk management and for regulatory purposes. Different investors have different levels of risk that they willing to take volatility is perceived as a measure of risk.

ARMA models are mainly focused on to model the conditional expectation of a process given the past, but in an ARMA model the conditional variance given the past is constant. What does this mean for, say, modeling stock returns? Suppose we have noticed that recent daily returns have been unusually volatile. We might forecast that tomorrow's return is also more variable than usual. However, an ARMA model cannot capture this type of behavior because its conditional variance is constant. So we need improved time series models if we desire to model the non constant volatility. In this

Author α σ: Department of Statistics, Osmania University, Hyderabad. e-mails: rani0130@gmail.com, jyothi2263@gmail.com

Global Journal of Science Frontier Research (F) Volume XXII Issue V Version I Year 2022

paper we look at ARCH and GARCH time series models that are becoming widely used in econometrics and finance because they have randomly varying volatility. ARCH is an acronym meaning Auto Regressive Conditional Heteroscedasticity. In ARCH models the conditional variance has a structure very similar to the structure of the conditional expectation in an AR model. We first study the ARCH (1) model, which is the simplest GARCH model and similar to an AR (1) model. Then we look at ARCH (p) models that are analogous to AR (p) models. Finally, we look at GARCH (Generalized ARCH) models that model conditional variances much as the conditional expectation is modeled by an ARMA model.

Econometricians are being asked to forecast and analyze the size of the errors of the model. In this case the questions are about volatility and the standard tools have become the ARCH/GARCH models. The basic version of the least squares model assumes that, the expected value of all error terms when squared is the same at any given point. This assumption is called homoskedasticity and it is this assumption that is the focus of ARCH/GARCH models.

ARCH models were introduced by Engle (1982) generalized autoregressive conditional heteroscedasticity (GARCH) (Bollerslev, 1986) were developed early in stock market forecasting. However, these models are generally not effective tools for forecasting due to the non-linearity of data and the occurrence of shocks (Sharma et al. 2016). The model assumes that the variance of the current error term is related to the size of previous period error terms.

Emerson Rodolfo Abraham *et al* (2020) study the Time Series Prediction with Artificial Neural Networks: An Analysis Using Brazilian Soybean Production. In this paper they were collected the data 1961-2016 regarding soybean production in Brazil. The results reveal that ANN is the best approach to predict soybean harvest area and production while classical linear function remains more effective to predict soybean yield.

The work of Babu et *al* (2014) proposed a hybrid system based on ARIMA and GARCH models. The authors collected data from January 2010 to January 2011 to define the first dataset (TD1) used for evaluating the performance of their model. As reported in this, they compared results between the proposed approach and other autoregressive models such as ARIMA, GARCH etc. The corresponding errors measures (MAPE, MAE, MaxAPE and RMSE) show that the proposed approach outperforms other models. Furthermore, the authors considered SBI shares from January 2010 to December 2010 to test the performance of proposed approach. The error performance measures (MAE, MaxAPE, etc.) confirmed that the proposed method obtains better results among others model (ARIMA and GARCH single scenario etc.). Also, the proposed hybrid system minimizes error performance. Despite achieving considerable results, autoregressive models present several limitations for stock price prediction compared to ML-based techniques.

Hamilton (1994) proposed the increasing interest in predicting the future behavior of complex systems by involving a temporal component. Researchers have investigated this problem modeling a convenient representation for financial data, the so-called time series (i.e., numerical data points observed sequentially through time). Previous studies have highlighted the difficulty studying financial time series accurately due to their non-linear and non-stationary patterns.

Dinesh et *al* (2021) studied the Integration of genetic algorithm with artificial neural network for stock market forecasting. In this they were proposed an intelligent forecasting method based on a hybrid of an Artificial Neural Network (ANN) and a

Notes

Genetic Algorithm (GA) and uses two US stock market indices, DOW30 and NASDAQ100, for forecasting. The data were partitioned into training, testing, and validation datasets. The model validation was done on the stock data of the COVID-19 period. The experimental findings obtained using the DOW30 and NASDAQ100 reveal that the accuracy of the GA and ANN hybrid model for the DOW30 and NASDAQ100 is greater than that of the single ANN (BPANN) technique, both in the short and long term.

Various ARCH models have been applied by researchers to analyze the volatility of exchange rates in different countries. For example some studies are: (Benavides, 2006) in which the author analyses the volatility forecast for the Mexican Peso – U.S. Dollar exchange rate, (Trenca et. al., 2011) which analyzes the evolution of the exchange rate for: Euro/RON, dollar/RON, yen/RON, British pound/RON, Swiss franc/RON for a period of five years from 2005 until 2011, (Alam et. al., 2012) in which the authors analyze exchange rates of Bangladeshi Taka (BDT) against the U.S. Dollar (USD) for the period of July 03, 2006 to April 30, 2012, (Musa et. al, 2014) forecast the exchange rate volatility between Naira and US Dollar using GARCH models.

II. Literature Review

a) Arch Model

ARCH is as AR component except we use lag or variance rather than lag of dependent variable.

The following reasons for the ARCH success:

- ARCH models are simple and easy to handle
- ARCH models take care of clustered errors
- ARCH models take care of nonlinearities
- ARCH models take care of changes in the econometrician's ability to forecast. The basic equation of estimation of parameters in ARCH model is given in (4.2.2)

Let Y_t be N (0, 1). The process Z_t is an ARCH (q) process if it is stationary and if it satisfies for all't' and some strictly positive valued process σ_t, the equations

$$Z_t = \sigma_t Y_t \quad \dots (1)$$

$$\sigma_t^2 = \varphi_0 + \xi_1 Y_{t-1}^2 + \xi_2 Y_{t-2}^2 + \dots + \xi_q Y_{t-q}^2 \quad \dots (2)$$

Constraints on parameters:

- variance has to be positive:

$$\varphi_0 > 0, \xi_1, \xi_2, \dots, \xi_{q-1} \geq 0, \xi_q > 0$$

- Stationary:

$$\xi_1 + \xi_2 + \dots + \xi_q < 1$$

b) Garch Model

GARCH is as MA component except we use past error of variance equation rather than past error of mean equation. In ARCH model computational problems may arise when the polynomial presents a high order. To facilitate such computation, Bollerslev (1986) proposed a Generalized Autoregressive Conditional Heteroskedasticity

(GARCH) model. GARCH (p,q) model (generalized autoregressive conditional heteroskedasticity) - equation for variance σ_t^2.

$$\sigma_t^2 = \varphi_0 + \xi_1 Y_{t-1}^2 + \xi_2 Y_{t-2}^2 + \ldots + \xi_q Y_{t-q}^2 + \delta_1 \sigma_{t-1}^2 + \delta_2 \sigma_{t-2}^2 + \ldots + \delta_q \sigma_{t-p}^2 \quad \ldots (3)$$

It is quite obvious the similar structure of Autoregressive Moving Average (ARMA) and GARCH processes: a GARCH (p, q) has a polynomial δ (L) of order "p" - the autoregressive term, and a polynomial ξ (L) of order "q" - the moving average term.

Notes

c) Estimation of Value at Risk (VAR)

Value at Risk (VaR) estimates the maximum expected loss on an investment, over a given period of time and given a specified degree of confidence. A value at risk statistic has three components: a time period, a confidence level and a loss amount (expressed either in currency or loss percentage). Value at risk answers question like what is the worst I can, with a 95% or 99% level of confidence, expect to lose in investment over the next day, month or year. According to Glyn (2014) value at risk is given by

$$Var = \mu + Z_\alpha \, \sigma \quad (4)$$

In this research, value at risk (VaR) will be estimated using GARCH models identified between USD/INR, GBP/INR, CHF/INR and JPY/INR on the foreign exchange rate.

d) Methodology

ARCH and GARCH models have become important tools in the analysis of time series data, particularly in financial application. These models are especially useful when the goal of the study is to analyse and forecast volatility. This study investigates the volatility in foreign exchange rate.

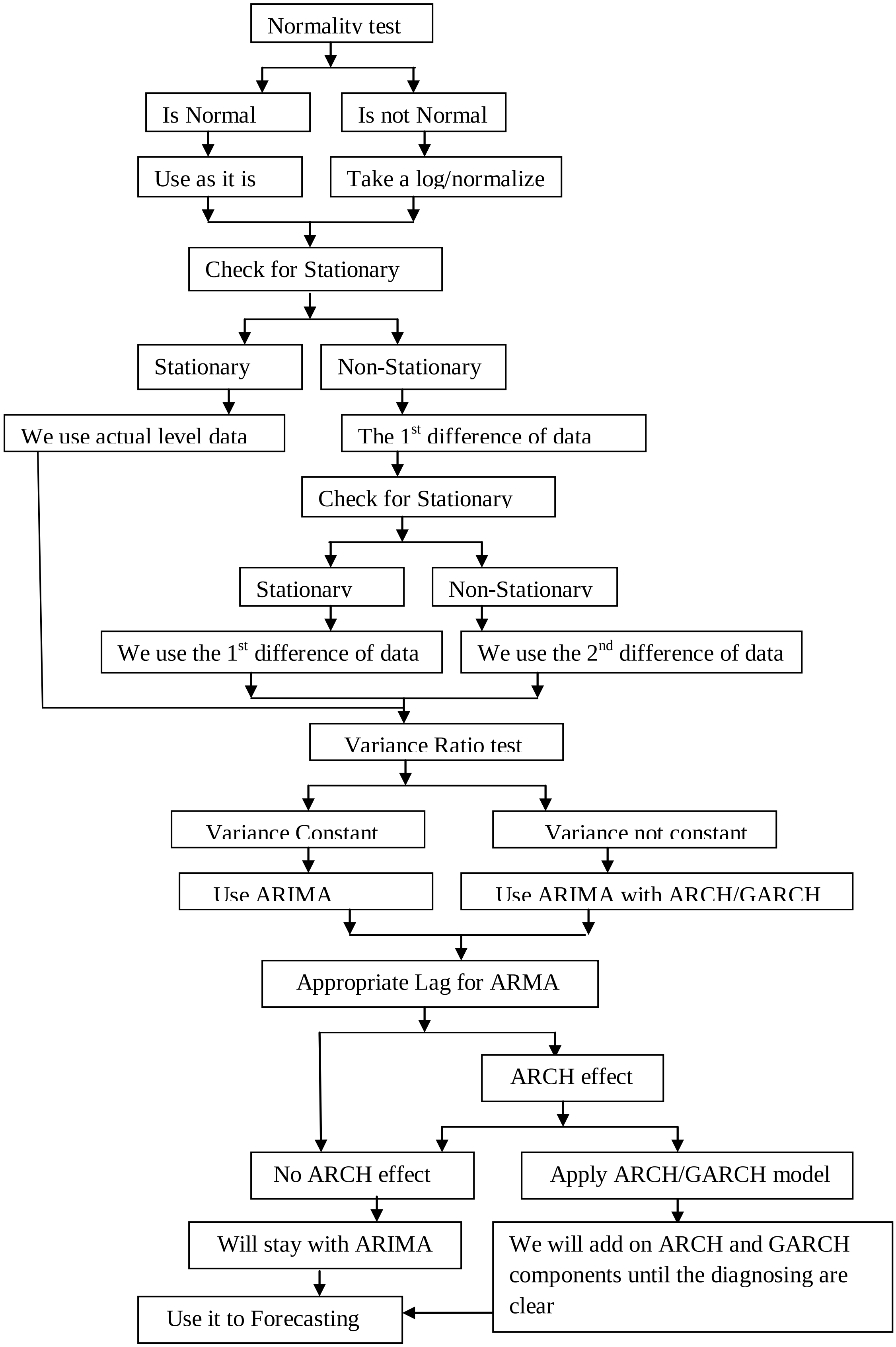
This Paper investigates the behavior of daily exchange rates between USD/INR, GBP/INR, CHF/INR and JPY/INR. This Paper attempts to examine the performance of ARCH/GARCH model in forecasting the currencies traded in Indian foreign exchange markets. This paper attempts to examine the performance of ARCH/GARCH model in forecasting the currencies traded in Indian foreign exchange markets. Study the Forecasting value of next 5 years exchange rates USD American dollar, GBP British Pound, CHF Swiss Franc and JPY Japanese JPY currencies with INR Indian data from that investigates the behavior of daily exchange rates between USD/INR, GBP/INR, CHF/INR and JPY/INR.

Comparison of daily exchange rates of USD American dollar, GBP British Pound, CHF Swiss Franc and JPY Japanese JPY currencies with INR Indian currency data taken from 15th May 2011-13th May 2021 analyzed is collected from the official website (http://www.rbi.org.in) of Reserve Bank of India (RBI).

After choosing the best model, serial correlation and heteroscedasticity is checked. Also, by using the Jarque-Bera test it can be stated if the residuals are normally distributed or not. At last the conditional standard deviation is plotted. Autoregressive conditional heteroskedasticity (ARCH) models are used for modelling observed time series. Also they are used in order to characterize various observed time series. An ARCH (q) model is estimated using ordinary least squares.

Objectives: Constructing ARCH/GARCH model for the investigated time series includes the following flowchart:

Notes


Normality test
Is Normal
Is not Normal
Use as it is
Take a log/normalize
Check for Stationary
Stationary
Non-Stationary
We use actual level data
The 1st difference of data
Check for Stationary
Stationary
Non-Stationary
We use the 1st difference of data
We use the 2nd difference of data
Variance Ratio test
Variance Constant
Variance not constant
Use ARIMA
Use ARIMA with ARCH/GARCH
Appropriate Lag for ARMA
ARCH effect
No ARCH effect
Apply ARCH/GARCH model
Will stay with ARIMA
We will add on ARCH and GARCH components until the diagnosing are clear
Use it to Forecasting

Notes

e) Check for Stationarity

Check for stationarity of daily exchange rates between USD/INR, GBP/INR, CHF/INR and JPY/INR data.

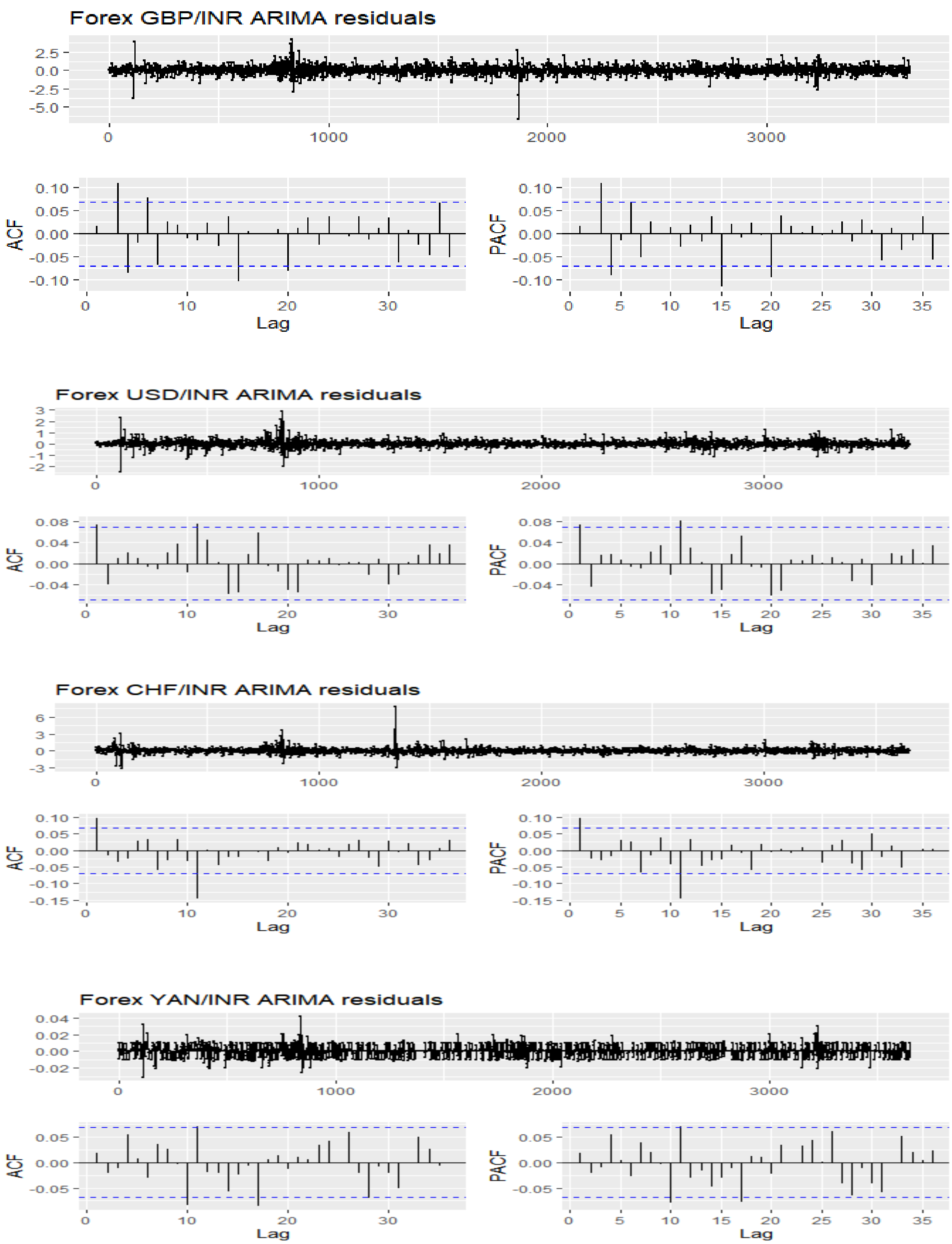


Figure 1.2.1: Residual Plots of the Daily Exchange Rates between USD/INR, GBP/INR, CHF/INR and JPY/INR

Notes

Observation of above Residual plots of the daily exchange rates between USD/INR, GBP/INR, CHF/INR and JPY/INR all are in the form of non-stationary, so we preferred ARCH and GARCH Models to test the diagnostic checking.

f) ARCH, GARCH Comparison and Analysis

We shall be developing two types of models (ARCH - autoregressive conditional heteroskedastic model, GARCH - generalized autoregressive conditional heteroskedastic model and compare them in order to find the best model.

i. *Arch Analysis*

Table 1.2.1: Chi- Squared and P-Values for the Fitted Model based on Daily Exchange Rates between USD/INR, GBP/INR, CHF/INR and JPY/INR

	Chi-squared	p-value
USD/INR	200.77	¡ 2.2e-16
GBP/INR	41.134	=1.422e-10
CHF/INR	97.26	¡ 2.2e-16
JPY/INR	70.278	¡ 2.2e-16

Small p-values (less than 0.05) suggest that the data is stationary and doesn't need to be differenced, From Table 1.2.1 above, it is clearly seen that all the p-values of ADF test are less than 0.05, suggesting that the series of the foreign exchange rates of USD/INR, GBP/INR, CHF/INR and JPY/INR are stationary in mean but not in variance. Also, the USD/INR, GBP/INR, CHF/INR and JPY/INR foreign exchange rates show a clear evidence of ARCH effect since the p-values of Chi-square test are less than$\alpha = 0.05$. This signifies that the variances of the foreign exchange rates are non-constant for all the periods specified. The Box and Jenkins procedure is applied to the variance series (Zi^{2}) obtained from difference logged series of the closing prices which will lead to the model building process.

ii. *Garch Analysis*

The model building procedures begins with the identified GARCH processes for the USD/INR, GBP/INR, CHF/INR and JPY/INR foreign exchange rates. The GARCH process based on following tests for checking its diagnostic Jarque Bera Test, Box-Ljung test. From Table 1.2.6, GARCH (1,1) process was identified for USD/INR foreign exchange rates, GARCH(1,1) process was identified for GBP/INR foreign exchange rates, GARCH (2,2) process was identified for CHF/INR foreign exchange rates and GARCH (1,1) process was identified for JPY/INR foreign exchange rates.

Below table present the GARCH processes and their parameters for USD/INR, GBP/INR, CHF/INR and JPY/INR foreign exchange rates.

Table 1.2.1: Summary of Garch Model Fit for USD/INR, GBP/INR, CHF/INR and JPY/INR Foreign Exchange Rates

Countries	Model Identified	Parameters			
		α0	α1	β1	β2
USD/INR	GARCH(1,1)	0.0020391	0.0642354	0.9040804	NIL
GBP/INR	GARCH(1,1)	0.018310	0.084582	0.841950	NIL
CHF/INR	GARCH(2,2)	0.0041388	0.1077998	0.2238257	0.6525444
JPY/INR	GARCH(1,1)	2.259e-05	5.000e-02	5.000e-02	NIL

The volatility model identified for USD/INR, GBP/INR, CHF/INR and JPY/INR foreign exchange rates are:

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$$\sigma_t^2 = 0.0020391 + 0.00203\ Z_{t-1}^2 + 0.9040804\ \sigma_{t-1}^2 \quad ...\ (5)$$

$$\sigma_t^2 = 0.018310 + 0.084582\ Z_{t-1}^2 + 0.84195\ \sigma_{t-1}^2 \quad ...\ (6)$$

$$\sigma_t^2 = 0.0041388 + 0.1077998\ Z_{t-1}^2 + 0.2238257\sigma_{t-1}^2 + 0.6525444\sigma_{t-1}^2 \quad ...\ (7)$$

$$\sigma_t^2 = 2.259e\text{-}05 + 5.000e\text{-}02\ Z_{t-1}^2 + 5.000e\text{-}02\ \ \sigma_{t-1}^2 \quad ...\ (8)$$

Notes

Table 1.2.2: Evaluation of Garch Model Fit for USD/INR Foreign Exchange Rates

GARCH(1,1)	Parameters		
	α0	**α1**	**β1**
Standard Error	0.0001203	0.0043000	0.0050571
t-value	16.95	14.94	178.78
Pr((>—t—)	<2e-16	<2e-16	<2e-16

Table 1.2.3: Evaluation of Garch Model Fit for GBP/INR Foreign Exchange Rates

GARCH(1,1)	Parameters		
	α0	**α1**	**β1**
Standard Error	0.002088	0.004876	0.012585
t-value	8.77	17.34	66.90
Pr((>—t—)	<2e-16	<2e-16	<2e-16

Table 1.2.4: Evaluation of Garch Model Fit for CHF/INR Foreign Exchange Rates

GARCH(1,1)	Parameters			
	α0	**α1**	**β1**	**β2**
Standard Error	0.0005061	0.0072744	0.0537764	0.0507097
t-value	8.177	14.819	4.162	12.868
Pr((>—t—)	2.22e-16	< 2e-16	3.15e-05	< 2e-16

Table 1.2.5: Evaluation of Garch Model Fit for JPY/INR Foreign Exchange Rates

GARCH(1,1)	Parameters		
	α0	**α1**	**β1**
Standard Error	3.782e-06	6.694e-03	1.495e-01
t-value	5.973	7.470	0.334
Pr((¿—t—)	2.33e-09	8.04e-14	0.738

Table 1.2.6: Diagnostic Tests of Garch Model Fit for USD/INR, GBP/INR, CHF/INR and JPY/INR Foreign Exchange Rates

Countries	Jarque Bera Test		Box-Ljung test	
	Chi squared	p-value	Chi squared	p-value
USD/INR	21112	¡ 2.2e-16	7.5815	0.005897
GBP/INR	3442.1	¡ 2.2e-16	1.7723	0.1831
CHF/INR	234670	¡ 2.2e-16	0.0041978	0.9483
JPY/INR	2310.1	¡ 2.2e-16	7.6918	0.005547

Notes

g) Volatility USD/INR, GBP/INR, CHF/INR and JPY/INR Foreign Exchange Rates

The volatility is measured by the conditional standard deviation and it is presented in figure 1.2.2

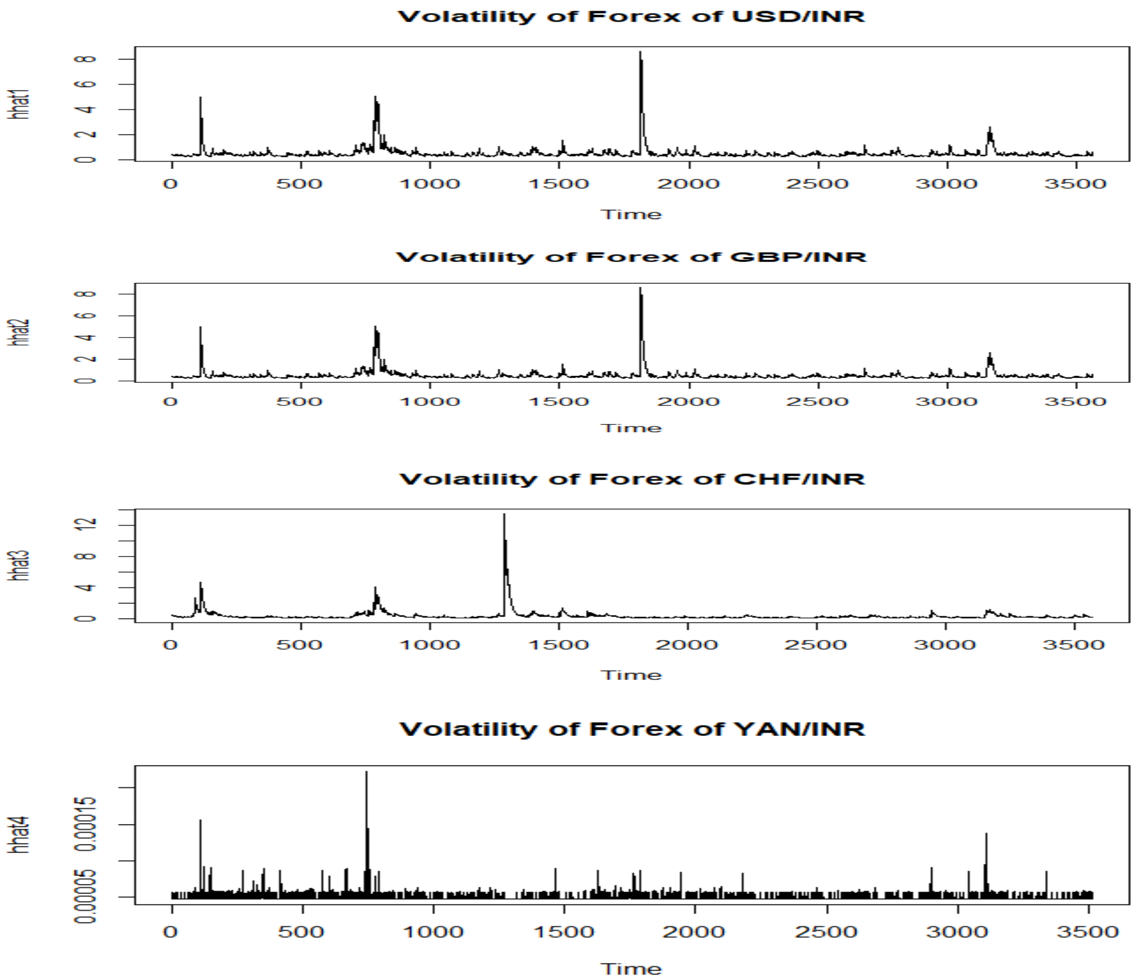

Figure 1.2.2: Conditional Standard Deviation Plot of USD/INR, GBP/INR, CHF/INR And JPY/INR Foreign Exchange Rates

III. Conclusion

1. The test p-values from tables 1.2.1 (shown in the second column) are more than 5%, so in this model there is no ARCH effect, meaning that we have a good model.
2. From the p-value of the Jarque - Bera test we can state that the residuals are not normally distributed, which is not desirable. So, the only problem of the model is that the residuals are not normally distributed. The volatility is measured by the conditional standard deviation and it is presented in figure 1.2.2
3. The Box-Ljung test indicates that the ARCH (1) model is dynamically adequate with white noise error.
4. The volatility experienced by the insurance returns series were modeled using univariate Generalized Autoregressive Conditional Heterskedastic (GARCH) model and observed that GARCH (1,1) was identified as the best model for USD/INR and GBR/INR and YAN/INR. GARCH(2,2) was identified as the best model for USD/INR CHF/INR.

5. The Jarque Bera Test indicates the USD/INR, GBP/INR, CHF/INR and JPY/INR foreign exchange rates are significant.
6. The Box-Ljung test indicates that the USD/INR and JPY/INR foreign exchange rates are significant remaining GBP/INR, CHF/INR foreign exchange rates are insignificant.
7. The study has shown that GARCH models are better models for analyzing foreign exchange rates data.

Notes

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Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 22 Issue 5 Version 1.0 Year 2022
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

On New Closed Sets in Grill Topological Spaces

By Dr. V. V. S. Ramachandram

Abstract- In the present work, the main intention is to introduce new type of closed sets namelyGg**-closed sets in Grill Topological spaces and study some of their properties.

Keywords: topological space, grill, operator ϕ, g-closed set, Gg-closed set, Gg-closed set, Gg**-closed set.*

GJSFR-F Classification: DDC Code: 514 LCC Code: QA611

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Keywords: topological space, grill, operator ϕ, g-closed set, Gg-closed set, Gg-closed set, Gg**-closed set.*

I. Introduction

N. Levine[1] introduced the concept of generalized closed sets in topological spaces. Later many authors introduced new types closed sets in topological spaces and established their properties. They also studied the relationship with other types of closed sets in topological spaces. The concept of Grill was first introduced by Choquet [2] in the year 1947.Some authors introduced the concept of generalized closed set in Grill topological spaces in later years. In 2012, Dhananjoy Mandal and M. N. Mukherjee[3], introduced the concept of Gg-closed set in Grill topological spaces and studied their properties. In the year 2017, M. Kaleswari and others[4] introduced the concept of Gg*-closed sets in Grill topological spaces and established some of their properties. With their inspiration the concept of Gg**-closed set in a Grill topological space was introduced in the present work and studied some of their properties.

II. Preliminaries

Definition 2.1

A Grillon a topological space (X,τ) is a nonempty collection G of nonempty subsets of X such that

(i) $A \in G, A \subseteq B \subseteq X \Rightarrow B \in G$

(ii) $A \subseteq X, B \subseteq X, A \cup B \in G \Rightarrow A \in G$ or $B \in G$.

If G is a Grill on a topological space (X,τ), then it is called a Grill topological space denoted with (X,τ,G).

Author: *Professor in Mathematics, Dept. of BS&H, ISTS Engineering College for Women, Eastgonagudem, Andhrpradesh.*
e-mail: vvsrch@gmail.com

Ref

1. N. Levine, Generalized closed sets in topological spaces, Rend. Circ. Mat. Palermo 19 (1970), 89–96.

Definition 2.2

Let (X,τ,G) be a Grill topological space and A is any subset of X. The operator $\phi : P(A) \to P(A)$ is defined as $\phi(A) = \{x \in X \,/ U \cap A \in G, \forall U \in \tau(x)\}$ where $\tau(x)$ denotes the neighbourhood of x in the space X.

Definition 2.3:

A subset A of a Grill topological space (X,τ,G) is said to be Gg-closed if $\phi(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X. The complement of a Gg-closed set is a Gg-open set.

Definition 2.4: [6]

A subset A of a Grill topological space (X,τ,G) is said to be Gg*-closed if $\phi(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in X. The complement of a Gg*-closed set is a Gg*-open set.

Throughout the paper, by a space X we always mean a topological space (X,τ) with no separation axioms assumed. For any subset A of the space X, the closure of A is denoted with $cl(A)$ and interior of the subset A is denoted with $\text{int}(A)$.

Theorem 2.5:[5]

Let (X,τ,G) be a Grill topological space. Then for any $A \subseteq X, B \subseteq X$ the following hold:

(a) $A \subseteq B \Rightarrow \phi(A) \subseteq \phi(B)$

(b) $\phi(A \cup B) = \phi(A) \cup \phi(B)$

(c) $\phi(\phi(A)) \subseteq \phi(A) = cl(\phi(A)) \subseteq cl(A)$

III. Gg**-Closed Sets in Grill Topological Spaces

In this section a new type of closed set was defined in a Grill topological space with an example.

Definition 3.1:

A subset A of a Grill topological space (X,τ,G) is said to be Gg**-closed set if $\phi(A) \subseteq U$ whenever $A \subseteq U$ and U is g*-open in X. The complement of a Gg**-closed set is a Gg**-open set.

Example 3.2:

Consider $X = \{a,b,c\}$, $\tau = \{\varnothing, X, \{a\}, \{a,b\}, \{a,c\}, \{b\}\}$ and $G = \{X, \{a\}, \{a,c\}$. Then, (X,τ,G) is a Grill topological space. In this space, g*-closed sets are $\{\varnothing, X, \{b\}, \{c\}, \{b,c\}, \{a,c\}\}$. Take the set $A = \{a,c\}$ in the space. Then, $\tau(a) = \{X, \{a\}, \{a,b\}, \{a,c\}$, $\tau(b) = \{X, \{b\}, \{a,b\}\}$ and $\tau(c) = \{X, \{a,c\}\}$.

Now, $\{a\} \cap A = \{a\} \in G, \{a,b\} \cap A = \{a\} \in G, \{a,c\} \cap A = \{a,c\} \in G, X \cap A = A \in G$. This shows that $a \in \phi(A)$. In a similar way we can check $b \notin \phi(A), c \in \phi(A)$. So, $\phi(A) = \{\text{a}, \text{c}\}$.

Also, the g*-open sets containing A are $\{X, \{a,c\}\}$ and each of the sets contain $\phi(A)$. Hence, the set $A = \{a,c\}$ is a Gg**-closed set in the Grill topological space (X,τ,G).

Notes

IV. Properties of Gg**-Closed Sets

This section is dedicated to study some simple properties of Gg**-closed sets.

Theorem 4.1:

In a Grill topological space (X,τ,G), every non-member of G is Gg**-closed.

Notes

Proof:

Let A be any non-member of G and U be a g*-open set containing A. Then, $A\cap U = A \notin G$. This shows that $\phi(A) = \{\ \} \subseteq U$ and hence A is Gg**-closed set.

Remark:

The converse of the above theorem need not be true. This can be seen from the following example.

Example 4.2:

Consider the Grill topological space (X,τ,G) defined by the sets $X=\{a,b,c\}$, $\tau=\{\varnothing, X,\{a\},\{a,b\},\{a,c\},\{b\}\}$, $G=\{X,\{a\},\{a,c\}\}$.In this space $A=\{a,c\}$ is a Gg**-closed set but it is a member of the grill G.

Theorem 4.3:

In a Grill topological space (X,τ,G), every closed set is a Gg**-closed set.

Proof:

Let A be any closed set in the Grill topological space (X,τ,G). Then, $A=cl(A)$. Let U be any g*-open set containing A. Then, U is a g*-open set containing $cl(A)$. We claim that $\phi(A)\subseteq U$ .Suppose $x\in\phi(A)$.Then, $A\cap U\in G, \forall U\in\tau(x)$. This implies that $x\in cl(A)$ and so $x\in U$. Hence, $\phi(A)\subseteq U$ as we claimed.

Remark:

The converse of the above theorem need not be true. This can be seen from the following example.

Example 4.4:

Let $X=\{a,b,c\}, \tau=\{\varnothing, X,\{a\},\{a,b\},\{a,c\},\{b\}\}$ and $G=\{X,\{a\},\{a,c\}$ be a grill. Then (X,τ,G) is a Grill topological space. In the space, $A=\{b,c\}$ is a Gg**-closed set but it is not a closed set.

Theorem 4.5:

In a grill topological space (X,τ,G), every $g*-$ closed set is a Gg**-closed set.

Proof:

Let A be ag*-closed set in the Grill topological space (X,τ,G) and U be any g*-open set containing A .Then, $\phi(A)\subseteq U$. Hence, A is a Gg**- closed set.

Remark:

The converse of the above theorem need not be true. This can be seen from the following example.

Example 4.6:

Let $X=\{a,b,c\},\tau=\{\varnothing,X,\{a\},\{a,b\},\{a,c\},\{b\}\}$ and $G=\{X,\{a\},\{a,c\}\}$ be a grill. Then, (X,τ,G) is a Grill topological space. In the space, $A=\{a,b\}$ is a Gg**-closed set, but it is not a g*-closed set.

Theorem 4.7:

In a Grill topological space (X,τ,G), every Gg*-closed set is a Gg**-closed set.

Proof:

Let A be any Gg*-closed set in the Grill topological space (X,τ,G) and U be any g*-open set containing A .Then, $\phi(A)\subseteq U$ .Hence, A is a Gg**-closed set.

Remark:

The converse of the above theorem need not be true. This can be seen from the following example.

Example 4.8:

Let $X=\{a,b,c\},\tau=\{\varnothing,X,\{a\},\{a,b\},\{a,c\},\{b\}\}$ and $G=\{X,\{a\},\{a,c\}\}$ be a grill. Then, (X,τ,G) is a grill topological space. In the space, $A=\{a,b\}$ is a Gg**-closed setbut it is not a Gg*-closed set.

Theorem 4.9:

In a Grill topological space (X,τ,G) union of any two Gg**-closed setsis aGg**-closed set.

Proof:

Let A,B be any two Gg**-closed sets in a Grill topological space (X,τ,G). Let U be any g*-open set containing $A\cup B$. Since $A\subseteq A\cup B, B\subseteq A\cup B$, U is a g*-open set containing A and B also. Since, both the sets are Gg**-closed, $\phi(A)\subseteq U,\phi(B)\subseteq U$. But $\phi(A\cup B)=\phi(A)\cup\phi(B)\subseteq \mathrm{U}$. This shows that the set $A\cup B$ is a Gg**-closed set.

Remark:

In a Grill topological space (X,τ,G), intersection of two Gg**-closed sets need not be a Gg**-closed set. This can be seen in the following example.

Example 4.10:

Let $X=\{a,b,c\}$, $\tau=\{\varnothing,X,\{a\},\{a,b\},\{a,c\},\{b\}\}$ and $G=\{X,\{a\},\{a,c\}\}$ be a grill. Then, (X,τ,G) is a Grill topological space. In the space, the subsets $A=\{a,b\}$ and $B=\{a,c\}$ are Gg**-closed sets, but the intersection $\{a\}$ is not a Gg**-closed set.

Theorem 4.11:

If A,B are two subsets of a grill topological space (X,τ,G) such that A is a Gg**-closed setand $A\subseteq B\subseteq\phi(A)$, then B is a Gg**-closed set.

Proof:

Let U be any g*-open set containing B in the Grill topological space (X,τ,G). This implies, U is ag*-open set containing A also. Since A is a Gg**-closed, $\phi(A)\subseteq U$.

Notes

But, $B \subseteq \phi(A) \Rightarrow \phi(B) \subseteq \phi(\phi(A)) = \phi(A)$. This shows that $\phi(B) \subseteq U$ and hence B is a Gg**-closed set.

V. Conclusion

In this paper an attempt was made to introduce the concept of Gg**-closed sets in a Grill topological space. Some basic properties of these sets were discussed. In continuation to this, continuity using these closed sets can be studied in future work.

Notes

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Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 22 Issue 5 Version 1.0 Year 2022
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

An Extension of Fermatean b-Derived Fuzzy Soft Ternary Subgroup

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GJSFR-F Classification: MSC 2010: 03E72, 20N15, 20N25, 20E15

ANEXTENSIONOFFERMATEANBDERIVEDFUZZYSOFTTERNARYSUBGROUP

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Global Journal of Science Frontier Research (F) Volume XXII Issue V Version I Year 2022

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Keywords: soft set, fermatean uncertainty set, n-dimensional, extension, subgroup, k-cut, extension homomorphism, ternary group, b-derived, fermatean uncertainty ternary subgroup.

I. Introduction

The idea of fuzzy uncertainty as a major role occurs in all fields of science and engineering domains and it was initially developed by Zadeh [17]. Many scientists concentrated the fuzzy uncertainty to predict the fuzzified solution of the applications. The idea of fuzzification in group into uncertainty subgroup is introduced by Rosenfeld [14]. This is the first uncertainty filation of any quantity of algorithmic structures and newly expressed in another dimension in the different fields of science and engineering. The uncertainty is initially discussed through the n-ary systems by Kasner [7] and Dudek [2]. Furthermore, extension of this uncertainty on n-ary groups introduced by Dornte [6]. The uncertainty of n-ary groups as a generalization of uncertainty subgroup discussed by Davvz in [5]. Atanassov [1] introduced the axioms of set properties with notations in uncertainty sets. Dudek [3] has established the axioms of set properties along with basic notations using n-ary systems. Investigate of uncertainty n-ary subgroups introduced by [13]. The fermatean uncertainty sets was initially introduced by Senapathi and Yager [15]. D. Molodtsov's introduced the concept of soft sets in [11]. Followed by, the cubic fermatean uncertainty soft ideal structures, defining basic notions in soft sets along with suitable applications were presented in [9]. R.Nagarajan studied fermatean uncertainty multi-group over multi-homomorphism [12].

In this paper, an extension of the fermatean uncertainty soft subgroup structures under a norm. Also, the cubic fermatean uncertainty soft ideal structures and fermatean uncertainty multi-group over multi-homomorphism are discussed in detail.

The rest of the article is structured as follows: Section 2 explained the preliminaries related to the present study. Section 3 presented an extension fermatean uncertainty under normal subgroups such important theoretical proofs, corollaries and numerical examples related to these results. Conclusions and future research directions are presented in Section 4.

Ref

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Author α: Department of Mathematics, Bharath Institute of Science and Technology, Bharath Higher Education and Research, Chennai, India. e-mail: svmanemaran@gmail.com

Author σ: Department of Science and Humanities, J.J College of Engineering and Technology, (Sowdambikaa Group of Institutions), Tiruchirappalli, India. e-mail: rajenagarajan1970@gmail.com

II. Preliminaries

Let G be a non-empty set. Define (G,ρ) be an n-dimensional groupoid as a mapping $\rho: G_n \to G,\ n \geq 2$. Let $y_i, y_{i+1}, y_{i+2}, \ldots, y_j$ be the sequence of elements in n-dimensional groupoids denoted by $y_i^{\,j}$. The symbol $y_{i+1}^{(t)}$ is a set of symbols $y_{i+1}, y_{i+2}, \ldots, = y_{i+t} = y$ commonly denoted by $y^{(t)}$. Mathematically, we write $\rho(y_1, y_2, \ldots, y_n) = \rho(y_1^{\,n})$ and $\rho(y_1, y_2, \ldots, y_i, y, \ldots, y,\ y_{i+t+1}, \ldots, y_n) = \rho(y_i^{\,i}, y^{(t)}, y^{n}{}_{i+t+1})$.

For any $y_1, \ldots, y_{2n-1} \in G$ then n-dimensional groupoid (G,ρ) is called $(i,j)-associative$ if it satisfies

$$\rho\left(y_i^{\,i-1}, \rho\left(y_i^{\,n+i-1}\right), y_{n+1}^{\,2n-1}\right) = \rho\left(y_1^{\,j-1}, \rho\left(y_j^{\,n+j-1}\right), y_{n+j}^{\,2n-1}\right)$$

For all $1 \leq i \leq j \leq n$, the above operation ρ is associative then (G,ρ) is said to be an n-dimensional semi group. Moreover, the n-dimensional groupoid satisfies associative condition if and only if it is called $(1,j)-associative$ for all $j = 2, \ldots n$. In binary case, the value of n=2 is called usual semi group. Let $y_0 y_1, y_2, \ldots y_n \in G$ with an integer $i \in \{1,2,3,\ldots n\}$, there exists an object z in G such that

$$\rho\left(y_i^{\,i-1}, z, \ldots, y_{i+1}^{\,n}\right) = y_0 \tag{1}$$

The above Eqn. (1) is known as i-solvable or solvable at the place 'i'. If the solution is unique then Eqn. (1) is uniquely i-solvable.

For $i = 1, \ldots, n$, an n-dimensional groupoid (G,ρ) is uniquely solvable then it is called an n-dimensional quasigroup. An n-dimensional quasigroup satisfies the associative property then it is called n-dimensional group.

For $n \geq 3$, the element $c_2^{\,n-2}$ in an n-dimensional operation ρ defines binary operation $y \bullet x = \rho\left(y, c_2^{\,n-2}\right)$. If (G,ρ) is an n-dimensional group the $(G,\bullet)$ is a group. Followed by the nature of different groups were obtained based on the element $c_2^{\,n-2}$. In all cases, these groups are isomorphic to each other [4]. So, we can consider only the groups of the form $ret_c(G,\rho) = (G,\bullet)$ satisfies the operation $y \bullet x = \rho\left(y, c_2^{\,(n-2)}, x\right)$. In this group, take $e = c$ then $y^1 = \rho\left(\bar{c}, c^{(n-3)}, y, \bar{c}\right)$.

Dudek and Michalski et. al [4] proved the result is as follows: For the elements $a, y_1^{\,n} \in G$ in an n-dimensional group (G,ρ), there exists a group $(G,\bullet)$ with automorphism ϕ satisfies

$$\rho\left(y_1^{\,n}\right) = y_1 \bullet \phi(x_2) \bullet \phi^2(x_3) \bullet \ldots \bullet \phi^{n-1}(x_n) \bullet a \tag{2}$$

Senapati and Yager et. al [15]: Let X be a universe of discourse. The operation F in X is a fermatean uncertainty set (FUS) defined by $F = \{\langle x,\ m_F(x),\ n_F(x)\rangle / x \in X\}$ with corresponding

Ref

15. Senapati. T., Yager. R.R, Fermatean Fuzzy Sets, 2019a; Communicated.

membership functions $m_F(x): X \to [0,1]$ and $n_F(x): X \to [-1,0]$ satisfies $0 \le (m_F(x))^3 + (n_F(x))^3 \le 1 \ \forall x \in X$. Herein, $m_F(x), n_F(x)$ denotes the degree of membership and non-membership of an element $x \in F$.

Senapati and Yager et. al [16]: For any FUS, the elements $x, F \in X$ the degree of an indeterminacy is given by $\prod F(x) = \sqrt[3]{1-(m_F(x))^3 - (n_F(x))^3}$. The ordered pair of the membership functions $(m_F(x), n_F(x))$ is the fermatean uncertainty number (FUN) denoted by $F = (m_F, n_F)$.

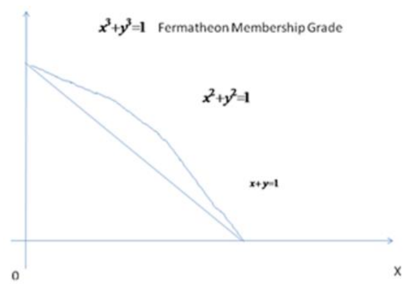

Figure 2.1: Fermatean membership grade

Senapati and Yager et. al [15]: The degree of the set of fuzzy membership functions are more than the degrees of both set of Pythagorean membership grades and bi-uncertainty membership grades. These membership grades are graphically plotted in Figure 2.2.

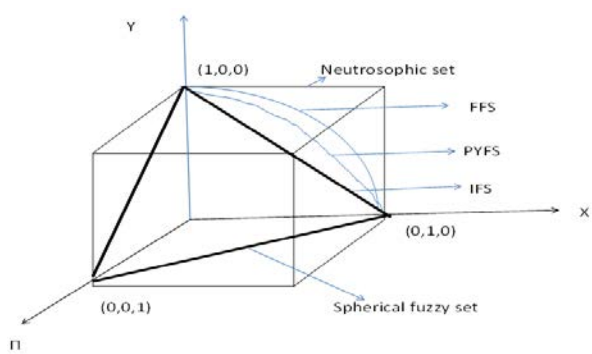

Figure 2.2: Types of membership functions

In Figure 2.1, the set of points on the line $x+y \le 1$ shows the bi-uncertainty membership values. Moreover, the set of points that covers from the line $x+y \le 1$ above to atmost $x^2+y^2 \le 1$ shows the degree of Pythagorean membership values.

Davvaz and Dudek et. al [5]: An extension uncertainty subgroup (EUSG) in an n-dimensional group on (G, ρ) satisfies following axioms

$$EUSG_1 : \text{For all } y_1^{\,n} \in G,\ \mu\left(\rho y_1^{\,n}\right) \ge \min\{\mu y_1, \ldots, \mu y_1\},$$

$$EUSG_2 : \text{For all } y_1^{\,n} \in G,\ \mu\left(\overset{\circ}{y}\right) \ge \mu(y).$$

Put n=3 in the above condition ($EUSG_2$) we get $\overset{=}{y} = y$ is

Ref

15. Senapati. T., Yager. R.R, Fermatean Fuzzy Sets, 2019a; Communicated.

$$EUSG_3: \text{For all } y \in G, \left(\mu\left(\overset{\circ}{y}\right) = \mu(y)\right)$$

For every $y \in G$, there exists a natural number p such that $\overline{y}^{(p)} = y$ where $\overline{y}^{(p)}$ indicates the skew elements $\overline{y}^{(p-1)}$ and $\overline{y}^{(0)} = y$. W.A. Dudek et. al: describes an uncertainty over n-dimensional subgroups satisfies $\mu(\overline{y}) > \mu(y)\ \forall\ y \in G$.

Followed by the above backgrounds scope of the study, we discussed the theorems in Fermatean uncertainty of normal subgroups with suitable numerical examples.

III. Extension Fermatean Uncertainty of Normal Subgroups

In this section, an extension fermatean uncertainty (T, S) normal subgroup with related theorems, corollary with suitable examples are discussed in detail.

Definition 3.1: A collection of Fermatean uncertainty $A = \left(m_A{}^3, n_A{}^3\right)$ in G is said to be an extension fermatean uncertainty (T, S) under normal subgroup of (G, ρ) satisfies the following axioms:

$$EFUSG_1: \text{For all } y_1{}^n \in G \Rightarrow \left(m_A{}^3\left(\rho\left(y_1{}^n\right)\right) \geq T\left\{m_A{}^3(y_1), \ldots, m_A{}^3(y_n)\right\}\right)$$

$$EFUSG_2: \text{For all } y_1{}^n \in G \Rightarrow \left(n_A{}^3\left(\rho\left(y_1{}^n\right)\right) \leq S\left\{n_A{}^3(y_1), \ldots, n_A{}^3(y_n)\right\}\right)$$

$$EFUSG_3: \text{For all } y \in G \Rightarrow \left(m_A{}^3(y) \geq m_A{}^3(y)\right)$$

$$EFUSG_4: \text{For all } y \in G \Rightarrow \left(n_A{}^3(y) \leq n_A{}^3(y)\right)$$

$$EUSG_2: \text{For all } y_1{}^n \in G \Rightarrow \mu(\overline{y}) \geq \mu(y)$$

Example 3.2: Let (Z_4, ρ) be an extension Fermatean uncertainty subgroup. Define a mapping $\rho: Z_4{}^4 \to Z_4$ defined by $\rho(y_1, y_2, y_3, y_4) = S(y_1, y_2, y_3, y_4)$.

Consider, (Z_4, ρ) is a 4-dimensional additive subgroup of an integer modulo 4. Let $A = \left(m_A{}^3, n_A{}^3\right)$ be the fermatean uncertainty collection in (Z_4, ρ) given by

$$m_A{}^3 = \begin{cases} 0.8, & \text{if } y = 0 \\ 0.3, & \text{if } y = 1,2,3 \end{cases} \quad \text{and} \quad n_A{}^3 = \begin{cases} 0.3, & \text{if } y = 0 \\ 0.8, & \text{if } y = 1,2,3 \end{cases}$$

The above fermatean uncertainty collection $A = \left(m_A{}^3, n_A{}^3\right)$ satisfies an extension fermatean uncertainty subgroup of (Z_4, ρ).

Theorem 3.3: If $\{A_i / i \in I\}$ is an arbitrary family of an extension fermatean uncertainty subgroup of (G, ρ) then $\cap A_i$ is also an extension fermatean uncertainty subgroup of (G, ρ) given by

$$\cap A_i = \left\{y, \left(\wedge m_A{}^3(y), \vee n_A{}^3(y)\right) / y \in G\right\}$$

Notes

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Proof: According to Definition 3.1: the proof of this theorem obviously satisfied.

Theorem 3.4: Let $A=\left(m_A{}^3, n_A{}^3\right)$ be a collection of fermatean uncertainty in G is an extension fermatean uncertainty subgroup of (G,ρ) with operation $\lozenge$ satisfies $A=\left\{y, \left(m_A{}^3(y), 1-n_A{}^3(y)\right)/y \in G\right\}$.

Proof: From Definition 3.1: the membership function $\overline{m}_A$ satisfies the axioms $EFUSG_2$ and $EFUSG_4$.

Let $y_1{}^n \in G$ then the membership function can be written as

$$\overline{m}_A{}^3\left(\rho\left(y_1{}^n\right)\right) = 1 - m_A{}^3\left(\rho\left(y_1{}^n\right)\right)$$

$$\leq 1 - T\left\{m_A{}^3(y_1), \ldots, m_A{}^3(y_n)\right\}$$

$$= S\left\{m_A{}^3(y_1), m_A{}^3(y_2), \ldots, m_A{}^3(y_{1-n})\right\}$$

And

$$\overline{m}_A{}^3\left(\overline{y}\right) = 1 - m_A{}^3(y) \leq 1 - m_A{}^3(y) = m_A{}^3\left(\overline{y}\right)$$

Hence, the notion $\lozenge A$ is an extension of fermatean uncertainty subgroup of (G,ρ).

Definition 3.5: Let $A=\left(m_A{}^3, n_A{}^3\right)$ be a fermatean uncertainty collection in G. Let $k \in [0,1]$ with the bounds of membership function on G is defined by

$$U\left(m_A{}^3; k\right) = \left\{y \in G/m_A{}^3(y) \geq k\right\},$$

$$L\left(n_A{}^3; k\right) = \left\{y \in G/n_A{}^3(y) \leq k\right\}$$

Herein, the symbol $m_A{}^3$ and $n_A{}^3$ denotes the *level k-cut* in G.

The consequence of the following theorems to be proved based on the results discussed in [Dudek].

Theorem 3.6: Let A be the fermatean uncertainty collection in G with the image $\mathrm{Im}\left(m_A{}^3\right)=\left\{k_i/i \in I\right\}$ and $\mathrm{Im}\left(n_A{}^3\right)=\left\{k_j/j \in I\right\}$ is also an extension fermatean uncertainty subgroup of (G,ρ). If and only if the $m_A{}^3$-level k-cut and $n_A{}^3$-level k-cut of G are extension subgroups of (G,ρ) for every $k \in [0,1]$ such that $k \in \mathrm{Im}\left(m_A{}^3\right) \cap \mathrm{Im}\left(n_A{}^3\right)$, which are known as $m_A{}^3$-level extension subgroups and $n_A{}^3$-level extension subgroups respectively.

Proof: Let A be an extension fermatean uncertainty subgroup of (G,ρ). If $y_1 \in G$ *and* $k \in [0,1]$, then $m_A{}^3(y_i) \geq k$ *for all* $i = 1,2,3,\ldots,n$ thus

$$m_A^{3}\left(\rho\left(y_1^{\,n}\right)\right)\geq T\left\{m_A^{3}(y_1),\ldots,m_A^{3}(y_n)\right\}\geq k$$

this implies $\left(\rho\left(y_1^{\,n}\right)\right)\in U\left(m_A^{3};k\right)$ *and* $\left(\rho\left(y_1^{\,n}\right)\right)\in U\left(m_A^{3};k\right)$ and

$$n_A^{3}\left(\rho\left(y_1^{\,n}\right)\right)\leq S\left\{n_A^{3}(y_1),\ldots,m_A^{3}(y_n)\right\}\leq k$$

implies $\left(\rho\left(y_1^{\,n}\right)\right)\in L\left(n_A^{3};k\right)$.

Moreover, for some $y\in U\left(m_A^{3};k\right)$ *and* $y\in L\left(n_A^{3};k\right)$ we have

$$m_A^{3}\left(\bar{y}_1\right)\geq m_A^{3}(y_1)\geq k \text{ and } n_A^{3}\left(\bar{y}_1\right)\leq n_A^{3}(y_1)\leq k,$$

implies $y\in U\left(m_A^{3};k\right)$ *and* $y\in L\left(n_A^{3};k\right)$.

Thus, m_A^{3}-level k-cut and n_A^{3}-level k-cut are also an extension subgroup of (G,ρ).

Conversely, assume that both m_A^{3}-level k-cut and n_A^{3}-level k-cut are extension subgroup of (G,ρ).

Define

$$k_0=T\left\{m_A^{3}(y_1),\ldots,m_A^{3}(y_n)\right\} \text{ and } k_1=S\left\{n_A^{3}(y_1),\ldots,n_A^{3}(y_n)\right\}$$

For some $y_1^{\,n}\in G$ then $y_1^{\,n}\in U\left(m_A^{3};k_0\right)$ and $y_1^{\,n}\in L\left(n_A^{3};k_1\right)$,

Consequently $\rho\left(y_1^{\,n}\right)\in U\left(m_A^{3};k_0\right)$ and $\rho\left(y_1^{\,n}\right)\in L\left(n_A^{3};k_1\right)$.

Thus, $m_A^{3}\left(\rho\left(y_1^{\,n}\right)\right)\geq k_0=T\left\{m_A^{3}(y_1),\ldots,m_A^{3}(y_n)\right\}$

$$n_A^{3}\left(\rho\left(y_1^{\,n}\right)\right)\leq k_1=S\left\{n_A^{3}(y_1),\ldots,m_A^{3}(y_n)\right\}.$$

Taking $y_1^{\,n}\in U\left(m_A^{3};k\right)$ and $y_1^{\,n}\in L\left(n_A^{3};k\right)$ then

$$m_A^{3}(y)=k_0\geq k \text{ and } n_A^{3}(y)=k_1\leq k.$$

Thus, $y\in U\left(m_A^{3};k_0\right)$ *and* $y\in L\left(n_A^{3};k_1\right)$.

Since, by the assumption, $\bar{y}\in U\left(m_A^{3};k_0\right)$ and $\bar{y}\in L\left(n_A^{3};k_1\right)$,

Where $m_A^{3}\left(\bar{y}\right)\geq k_0=m_A^{3}(y)$ and $n_A^{3}\left(\bar{y}\right)\geq k_1=n_A^{3}(y)$.

This completes the proof.

Notes

Theorem 3.7: Let A be the fermatean uncertainty collection in G is an extension fermatean uncertainty subgroup of (G,ρ) if and only if $m_A{}^3 - level\ k\text{-}cut$ and $n_A{}^3 - level\ k\text{-}cut$ of G are also extension subgroup of (G,ρ). For each $i=1,2,3,\dots,n$ and $y_1{}^n \in G$ satisfies the following conditions:

$$(i)\ m_A{}^3\left(\rho\left(y_1{}^n\right)\right) \geq T\left\{m_A{}^3(y_1),\dots,m_A{}^3(y_n)\right\}$$

$$(ii)\ n_A{}^3\left(\rho\left(y_1{}^n\right)\right) \leq S\left\{n_A{}^3(y_1),\dots,n_A{}^3(y_n)\right\}$$

$$(iii)\ m_A{}^3(y_i) \geq T\left\{m_A{}^3(y_i),\dots,m_A{}^3(y_{i-1}),m_A{}^3\left(\rho\left(y_{1_j}\right)^n\right),m_A{}^3(y_{i-1})m_A{}^3(y_n)\right\}$$

$$(iv)\ n_A{}^3(y_i) \leq S\left\{n_A{}^3(y_i),\dots,n_A{}^3(y_{i-1}),n_A{}^3\left(\rho\left(y_{1_j}\right)^n\right),n_A{}^3(y_{i-1})n_A{}^3(y_n)\right\}.$$

Proof: According to Theorem 3.6, for each non-empty level subsets both $U\left(m_A{}^3;k_0\right)$ and $L\left(n_A{}^3;k_1\right)$ are closed under ρ operation in (G,ρ) with $y_1{}^n \in U\left(m_A{}^3;k_0\right)$ and $y_1{}^n \in L\left(n_A{}^3;k_1\right)$ satisfies $\rho\left(y_1{}^n\right) \in U\left(m_A{}^3;k_0\right)$ and $\rho\left(y_1{}^n\right) \in L\left(n_A{}^3;k_1\right)$

For some $i=1,2,\dots,n$ and $z \in G$ we have $y_0, y_i{}^{i-1}, y_{i+1}{}^n$, where $\rho\left(y_i{}^{i-1}, z, y_{i+1}{}^n\right)$ satisfies $y_0 \in U\left(m_A{}^3;k_0\right)$ and $y_0 \in L\left(n_A{}^3;k_1\right)$. From the above conditions (iii) and (iv), we have $m_A{}^3(z) \geq k$ and $n_A{}^3(z) \leq k$. Therefore, $z \in m_A{}^3(k)$ and $z \in n_A{}^3(z)$ is a solution of Eqn.(1). Conclude that both level k-cuts $m_A{}^3$ and $n_A{}^3$ is also an extension subgroups in G.

Conversely, assume that both level k-cuts $m_A{}^3$ and $n_A{}^3$ in G be an extension subgroups then the following conditions in (i) and (ii) are proved.

For $y_1{}^n \in G$ we consider

$$k_0 = T\left\{m_A{}^3(y_i),\dots,m_A{}^3(y_{i-1}),m_A{}^3\left(\rho\left(y_1{}^{n_j}\right)\right),m_A{}^3(y_{i-1})m_A{}^3(y_n)\right\}$$

$$k_1 = S\left\{n_A{}^3(y_i),\dots,n_A{}^3(y_{i-1}),n_A{}^3\left(\rho\left(y_1{}^{n_j}\right)\right),n_A{}^3(y_{i-1})n_A{}^3(y_n)\right\}$$

then it satisfies $y_i{}^{i-1}, y_{i+1}{}^n$, $\rho\left(y_1{}^n\right) \in U\left(m_A{}^3;k_0\right)$ and $y_i{}^{i-1}, y_{i+1}{}^n$, $\rho\left(y_1{}^n\right) \in L\left(n_A{}^3;k_1\right)$

According to Definition 3.1: then we obtained as $y_i \in U\left(m_A{}^3;k_0\right)$ and $y_i \in L\left(n_A{}^3;k_1\right)$.

Thus, $m_A{}^3(y_i) \geq k_0$ and $n_A{}^3(y_i) \leq k_1$ satisfies the conditions (iii) and (iv).

Definition 3.8: The subgroups (G,ρ) and (G^1,ρ) are under fermatean extension and for any $y_1{}^n \in G$ define a homomorphism map: $\alpha : G \to G^1$ defined by

$$\alpha\left(\rho\left(y_1{}^n\right)\right) = \rho\left(\alpha^n\left(y_1{}^n\right)\right) \text{ where } \alpha^n\left(y_1{}^n\right) = \alpha(y_1),\dots,\alpha(y_n).$$

The above homomorphism map is said to be a fermatean extension homomorphism. For any fermatean uncertainty collection $A \in G^1$, we define a pre image of A under α is denoted by $\alpha^{-1}(A)$. Also, the fermatean uncertainty collection in G is defined by

Notes

$$\alpha^{-1}(A) = \left(m_A{}^3\alpha^{-1}(A),\ n_A{}^3\alpha^{-1}(A)\right),$$

Where, $m_A{}^3\alpha^{-1}(A)(y) = m_A{}^3(\alpha(y))$ and $n_A{}^3\alpha^{-1}(A)(y) = n_A{}^3(\alpha(y))\ \forall\ y \in G$.

For any $A \in G$, the symbol $\alpha(A)$ as the image of A under α is also a fermatean uncertainty collection in G^1 defined by $\alpha(A) = \left(\alpha_{\sup}\left(m_A{}^3\right), \alpha_{\inf}\left(n_A{}^3\right)\right)$.

$$\text{For all } y \in G \text{ and } x \in G^1 \text{ then } \alpha_{\sup}(x) = \begin{cases} \sup\limits_{y \in \alpha^{-1}(x)} m_A{}^3(y), & \textit{if } \alpha^{-1}(x) \neq \phi, \\ 0, & \textit{Otherwise} \end{cases}$$

$$\alpha_{\inf}(x) = \begin{cases} \inf\limits_{y \in \alpha^{-1}(x)} n_A{}^3(y), & \textit{if } \alpha^{-1}(x) \neq \phi, \\ 0, & \textit{elsewhere} \end{cases}$$

Theorem 3.9: For all $y \in G$, an into map $\alpha : G \to G^1$ is defined by $\alpha(\bar{y}) = \alpha(y)$. If A be an extension fermatean uncertainty subgroup of (G^1, ρ) then $\alpha^{-1}(A)$ is also an extension fermatean uncertainty subgroup under (G, ρ).

Proof:

Let $y_1{}^n \in G$ we have
$$m_{\alpha^{-1}(A)}{}^3\left(\rho\left(y_1{}^n\right)\right) = m_A{}^3\left(\alpha\left(y_1{}^n\right)\right)$$
$$= m_A{}^3\left(\alpha\, \rho\left(\alpha^n\left(y_1{}^n\right)\right)\right)$$
$$\geq T\left\{m_A{}^3\left(\alpha(y_1)\right), \ldots, m_A{}^3\left(\alpha(y_n)\right)\right\}$$
$$= T\left\{m_{\alpha^{-1}(A)}{}^3(y_1), \ldots, m_{\alpha^{-1}(A)}{}^3(y_n)\right\}$$

$$n_{\alpha^{-1}(A)}{}^3\left(\rho\left(y_1{}^n\right)\right) = n_A{}^3\left(\alpha\left(y_1{}^n\right)\right)$$
$$= n_A{}^3\left(\alpha\, \rho\left(\alpha^n\left(y_1{}^n\right)\right)\right)$$
$$\leq S\left\{n_A{}^3\left(\alpha(y_1)\right), \ldots, n_A{}^3\left(\alpha(y_n)\right)\right\}$$
$$= S\left\{n_{\alpha^{-1}(A)}{}^3(y_1), \ldots, n_{\alpha^{-1}(A)}{}^3(y_n)\right\}$$

$$m_{\alpha^{-1}(A)}{}^3(\bar{y}) = m_A{}^3\left(\alpha(\bar{y})\right) \geq m_A{}^3\left(\alpha(y)\right) = m_{\alpha^{-1}(A)}{}^3(y)$$
$$n_{\alpha^{-1}(A)}{}^3(\bar{y}) = n_A{}^3\left(\alpha(\bar{y})\right) \leq n_A{}^3\left(\alpha(y)\right) = n_{\alpha^{-1}(A)}{}^3(y)$$

Converse part is obviously true to be discussed in Theorem 3.10.

Theorem 3.10: Let $\alpha : G \to G^1$ be an extension homomorphism. If $\alpha^{-1}(A)$ is an extension fermatean uncertainty subgroup of (G, ρ) then A is also an extension fermatean uncertainty group of (G^1, ρ).

Notes

Global Journal of Science Frontier Research (F) Volume XXII Issue V Version I Year 2022

Theorem 3.11: Let $\alpha : G \to G^1$ be a map. If A is an extension fermatean uncertainty subgroup of (G,ρ) then $\alpha(A)=\left(y, \alpha_{\sup}\left(m^3{}_A\right), \alpha_{\inf}\left(n^3{}_A\right)\right)$ is also an extension fermatean uncertainty subgroup of (G^1,ρ).

Proof: Let $\alpha : G \to G^1$ be a map and for any $x_1{}^n, y_1{}^n \in G$ then

$$\left\{y_i / y_i \in \alpha^{-1}\left(\rho\left(x_1{}^n\right)\right)\right\} \subset \left\{\rho\left(y_1{}^n\right) \in G / x_1 \in \alpha^{-1}(x_1), y_2 \in \alpha^{-1}(x_2), \ldots, y_n \in \alpha^{-1}(x_n)\right\}$$

We have $\alpha_{\sup}\left(m^3{}_A\right)\left(\rho\left(x_1{}^n\right)\right) = \sup\left\{m^3{}_A\left(y_1{}^n\right) / y_i \in \alpha^{-1}\left(\rho\left(x_1{}^n\right)\right)\right\}$

$$\geq \sup\left\{m^3{}_A\left(\rho\left(y_1{}^n\right)\right) / y_1 \in \alpha^{-1}(x_1), y_2 \in \alpha^{-1}(x_2), \ldots, y_n \in \alpha^{-1}(x_n)\right\}$$

$$\geq \sup\left\{T\left\{m^3{}_A(y_1), m^3{}_A(y_2), \ldots m^3{}_A(y_n) / y_1 \in \alpha^{-1}(x_1), y_2 \in \alpha^{-1}(x_2), \ldots, y_n \in \alpha^{-1}(x_n)\right\}\right\}$$

$$= T\left\{\sup\left\{m^3{}_A(y_1) / y_1 \in \alpha^{-1}(x_1)\right\}, \sup\left\{m^3{}_A(y_2) / y_2 \in \alpha^{-1}(x_2)\right\}, \ldots, \sup\left\{m^3{}_A(y_n) / y_n \in \alpha^{-1}(x_n)\right\}\right\}$$

$$= T\left\{\alpha_{\sup}\left(m^3{}_A(x_1)\right), \alpha_{\sup}\left(m^3{}_A(x_2)\right), \ldots, \alpha_{\sup}\left(m^3{}_A(x_n)\right)\right\}$$

$$\alpha_{\sup}\left(n^3{}_A\right)\left(\rho\left(x_1{}^n\right)\right) = \inf\left\{n^3{}_A\left(y_1{}^n\right) / y_i \in \alpha^{-1}\left(\rho\left(x_1{}^n\right)\right)\right\}$$

$$\leq \inf\left\{n^3{}_A\left(\rho\left(y_1{}^n\right)\right) / y_1 \in \alpha^{-1}(x_1), y_2 \in \alpha^{-1}(x_2), \ldots, y_n \in \alpha^{-1}(x_n)\right\}$$

$$\leq \inf\left\{S\left\{n^3{}_A(y_1), n^3{}_A(y_2), \ldots n^3{}_A(y_n) / y_1 \in \alpha^{-1}(x_1), y_2 \in \alpha^{-1}(x_2), \ldots, y_n \in \alpha^{-1}(x_n)\right\}\right\}$$

$$= S\left\{\inf\left\{n^3{}_A(y_1) / y_1 \in \alpha^{-1}(x_1)\right\}, \inf\left\{n^3{}_A(y_2) / y_2 \in \alpha^{-1}(x_2)\right\}, \ldots, \inf\left\{n^3{}_A(y_n) / y_n \in \alpha^{-1}(x_n)\right\}\right\}$$

$$= S\left\{\alpha_{\inf}\left(n^3{}_A(x_1)\right), \alpha_{\inf}\left(n^3{}_A(x_2)\right), \ldots, \alpha_{\inf}\left(n^3{}_A(x_n)\right)\right\}$$

$$\alpha_{\sup}\left(m^3{}_A\right)(y) = \sup\left\{m^3{}_A(y) / y \in \alpha^{-1}(\rho(x))\right\}$$

$$\geq \sup\left\{m^3{}_A(y) / y \in \alpha^{-1}(\rho(x))\right\}$$

$$= \alpha_{\sup}\left(m^3{}_A\right)(y)$$

$$\alpha_{\sup}\left(n^3{}_A\right)(y) = \inf\left\{n^3{}_A(y) / y \in \alpha^{-1}(\rho(x))\right\}$$

$$\leq \inf\left\{n^3{}_A(y) / y \in \alpha^{-1}(\rho(x))\right\}$$

$$= \alpha_{\sup}\left(n^3{}_A\right)(y)$$

Hence the Theorem is proved.

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Corollary 3.12: A fermatean uncertainty collection A on $(G,\bullet)$ is a fermatean uncertainty subgroup iff it satisfies the following relations for any $x, y \in G$ given by

(i) $m^3{}_A(xy) \geq T\{m^3{}_A(x),\ m^3{}_A(y)\}$ and $n^3{}_A(xy) \leq S\{n^3{}_A(x),\ n^3{}_A(y)\}$

(ii) $m^3{}_A(x) \geq T\{m^3{}_A(y),\ m^3{}_A(xy)\}$ and $n^3{}_A(x) \leq S\{n^3{}_A(y),\ n^3{}_A(xy)\}$

(iii) $m^3{}_A(y) \geq T\{m^3{}_A(x),\ m^3{}_A(xy)\}$ and $n^3{}_A(y) \leq S\{n^3{}_A(x),\ n^3{}_A(xy)\}$

Theorem 3.13: Let A be a collection of an extension fermatean uncertainty subgroup (G,ρ). For any $x, y \in G$, there exist $a \in G$ such that $m^3{}_A(a) \geq m^3{}_A(x)$ and $n^3{}_A(a) \leq n^3{}_A(x)$ then A is an extension fermatean uncertainty subgroup of $ret_\alpha(G,\rho)$.

Proof: For each $a, x, y \in G$, we have the relation

$$m^3{}_A(xy) = m^3{}_A\left(\rho\left(x, a^{(n-2)}, y\right)\right)$$

$$\geq T\{m^3{}_A(x),\ m^3{}_A(a), m^3{}_A(y)\}$$

$$= T\{m^3{}_A(x),\ m^3{}_A(y)\}$$

Also, we write
$$n^3{}_A(xy) = n^3{}_A\left(\rho\left(x, a^{(n-2)}, y\right)\right)$$

$$\leq S\{n^3{}_A(x),\ n^3{}_A(a), n^3{}_A(y)\}$$

$$= S\{n^3{}_A(x),\ n^3{}_A(y)\}$$

$$m^3{}_A(x^{-1}) = m^3{}_A\left(\rho\left(a, x^{(n-3)}, \bar{x}, \bar{a}\right)\right) \geq T\{m^3{}_A(x), m^3{}_A(\bar{x}), m^3{}_A(a), m^3{}_A(\bar{a})\}$$

$$= m^3{}_A(x)$$

$$n^3{}_A(x^{-1}) = n^3{}_A\left(\rho\left(a, x^{(n-3)}, \bar{x}, \bar{a}\right)\right) \leq S\{n^3{}_A(x), n^3{}_A(\bar{x}), n^3{}_A(a), n^3{}_A(\bar{a})\}$$

$$= n^3{}_A(x)$$

In theorem 3.13, for that we have $m^3{}_A(a) \geq m^3{}_A(x)$ *and* $n^3{}_A(a) \leq n^3{}_A(x)$.

This completes the proof. Followed by the numerical example is discussed here.

Example 3.14: Define a map: $\rho: Z_4{}^3 \to Z_4$ on (Z_4,ρ) defined by $\rho(y_1,y_2,y_3) = S(y_1,y_2,y_3)$. Clearly, the ternary subgroup (Z_4,ρ) obtained from Z_4 under this map. The fermatean uncertainty collection A is given by

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$$m^3{}_A(y) = \begin{cases} 1, & if \ \ y=0 \\ 0.3, & if \ \ y=1,2,3 \end{cases}$$

$$n^3{}_A(y) = \begin{cases} 0, & if \ \ y=0 \\ 0.8, & if \ \ y=1,2,3 \end{cases}$$

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Clearly, we say that A is also a fermatean uncertainty ternary subgroup of (Z_4, ρ). For $ret_1(Z_4, \rho)$, we have the algebraic operations are

$m^3{}_A(0 \bullet 0) = m^3{}_A(\rho(0, 1, 0)) = m^3{}_A(1) = 0.3$ is not greater than $T\{m^3{}_A(0), m^3{}_A(0)\} = 1$.

Also, $n^3{}_A(0 \bullet 0) = n^3{}_A(\rho(0, 1, 0)) = n^3{}_A(1) = 0.8$ is not less than $S\{n^3{}_A(0), n^3{}_A(0)\} = 0$.

Hence, for that we have $m^3{}_A(a) \geq m^3{}_A(x)$ *and* $n^3{}_A(a) \leq n^3{}_A(x)$.

Theorem 3.15: Let A be an extension fermatean uncertainty subgroup of $ret_\alpha(G, \rho)$. For any $x, a \in G$ satisfies $m^3{}_A(a) \geq m^3{}_A(x)$ and $n^3{}_A(a) \leq n^3{}_A(x)$ then A is an extension fermatean uncertainty subgroup under (G, ρ).

Proof: From Theorem 2.1: for any extension subgroup of the form given in Eqn. (2), we have

$$(G, \bullet) = ret_\alpha(G, \rho), \ \phi(x) = \rho\left(a, y, y^{(n-2)}\right) \text{ and } b = \rho\left(\bar{a}, \ldots, \bar{a}\right) \text{ then}$$

$$m^3{}_A(\phi(y)) = m^3{}_A\left(\rho\left(\bar{a}, y, y^{(n-2)}\right)\right)$$

$$\geq T\left\{m^3{}_A(\bar{a}), m^3{}_A(y), m^3{}_A(a)\right\} = m^3{}_A(y),$$

$$m^3{}_A\left(\phi^2(y)\right) = m^3{}_A\left(\rho\left(\bar{a}, \phi(y), y^{(n-2)}\right)\right)$$

$$\geq T\left\{m^3{}_A(\bar{a}), m^3{}_A(\phi(y)), m^3{}_A(a)\right\} = m^3{}_A(\phi(y))$$

$$\geq m^3{}_A(y)$$

In general, for any $y \in G$, $k \in N$ then the relation $m^3{}_A\left(\phi^k(y)\right) \geq m^3{}_A(y)$ is true.

Similarly,
$$n^3{}_A(\phi(y)) = n^3{}_A\left(\rho\left(\bar{a}, y, y^{(n-2)}\right)\right)$$

$$\leq S\left\{n^3{}_A(\bar{a}), n^3{}_A(y), n^3{}_A(a)\right\} = n^3{}_A(y),$$

$$n^3{}_A\left(\phi^2(y)\right) = n^3{}_A\left(\rho\left(\bar{a}, \phi(y), y^{(n-2)}\right)\right)$$

$$\leq S\left\{n^3{}_A(\bar{a}), n^3{}_A(\phi(y)), n^3{}_A(a)\right\} = n^3{}_A(\phi(y))$$

$$\leq n^3{}_A(y).$$

In general, for any $y \in G$, $k \in N$ then the relation $n^3{}_A\left(\phi^k(y)\right) \leq n^3{}_A(y)$ is true.

For every $x \in G$, the following relations are obviously true. We have,

$$m^3{}_A(b) = m^3{}_A\left(\rho\left(\bar{a},\ldots,\bar{a}\right)\right) \geq m^3{}_A\left(\bar{a}\right) \geq m^3{}_A(y)$$

$$n^3{}_A(b) = n^3{}_A\left(\rho\left(\bar{a},\ldots,\bar{a}\right)\right) \leq n^3{}_A\left(\bar{a}\right) \leq n^3{}_A(y)$$

thus,

$$m^3{}_A\left(\rho\left(y_1{}^n\right)\right) = m^3{}_A\left(y_1 \circ \phi(y_2) \circ \phi^2(y_3) \circ \cdots \circ \phi^{n-2}(y_n) \circ b\right)$$

$$\geq T\left\{m^3{}_A(\phi(y_1)), m^3{}_A(\phi(y_2)), m^3{}_A\left(\phi^2(y_3)\right), \ldots, m^3{}_A\left(\phi^{n-1}(y_n)\right), m^3{}_A(b)\right\}$$

$$\geq T\left\{m^3{}_A(y_1), m^3{}_A(y_2), m^3{}_A(y_3), \cdots, m^3{}_A(y_n), m^3{}_A(b)\right\}$$

$$\geq T\left\{m^3{}_A(y_1), m^3{}_A(y_2), m^3{}_A(y_3), \ldots, m^3{}_A(y_n)\right\} \text{ and}$$

$$n^3{}_A\left(\rho\left(y_1{}^n\right)\right) = n^3{}_A\left(y_1 \circ \phi(y_2) \circ \phi^2(y_3) \circ \cdots \circ \phi^{n-2}(y_n) \circ b\right)$$

$$\leq S\left\{n^3{}_A(\phi(y_1)), n^3{}_A(\phi(y_2)), n^3{}_A\left(\phi^2(y_3)\right), \ldots, n^3{}_A\left(\phi^{n-1}(y_n)\right), n^3{}_A(b)\right\}$$

$$\leq S\left\{n^3{}_A(y_1), n^3{}_A(y_2), n^3{}_A(y_3), \ldots, n^3{}_A(y_n), n^3{}_A(b)\right\}$$

$$\leq S\left\{n^3{}_A(y_1), n^3{}_A(y_2), n^3{}_A(y_3), \ldots, n^3{}_A(y_n)\right\}$$

Finally, we have,

$$\bar{y} = \left(\phi(y) \circ \phi^2(y) \circ \cdots \circ \phi^{n-2}(y) \circ b\right)^{-1}$$

Thus

$$m^3{}_A(y) = m^3{}_A\left(\phi(y) \circ \phi^2(y) \circ \cdots \circ \phi^{n-2}(y) \circ b\right)^{-1}$$

$$\geq m^3{}_A\left(\phi(y) \circ \phi^2(y) \circ \cdots \circ \phi^{n-2}(y) \circ b\right)$$

$$\geq T\left\{m^3{}_A(\phi(y)) \circ m^3{}_A\left(\phi^2(y)\right) \circ \cdots \circ m^3{}_A\left(\phi^{n-2}(y)\right) \circ m^3{}_A(b)\right\}$$

$$\geq T\left\{m^3{}_A(y) \circ m^3{}_A(b)\right\} = m^3{}_A(y) \quad \text{and}$$

$$n^3{}_A(y) = n^3{}_A\left(\phi(y) \circ \phi^2(y) \circ \cdots \circ \phi^{n-2}(y) \circ b\right)^{-1}$$

$$\leq n^3{}_A\left(\phi(y) \circ \phi^2(y) \circ \cdots \circ \phi^{n-2}(y) \circ b\right)$$

$$\leq S\left\{n^3{}_A(\phi(y)) \circ n^3{}_A\left(\phi^2(y)\right) \circ \cdots \circ n^3{}_A\left(\phi^{n-2}(y)\right) \circ n^3{}_A(b)\right\}$$

$$\leq S\left\{n^3{}_A(y) \circ n^3{}_A(b)\right\} = n^3{}_A(y)$$

This result is derived.

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Corollary 3.16: Let (G,ρ) be a ternary group. For any fermatean uncertainty subgroup $ret_\alpha(G,\rho)$ is also a fermatean uncertainty ternary subgroup under (G,ρ).

Proof: Let $\bar{a}$ be an intermediate element of $ret_\alpha(G,\rho)$. For each $y \in G$, the following inequalities are $m^3{}_A(a) \geq m^3{}_A(\bar{y})$ and $n^3{}_A(a) \leq n^3{}_A(\bar{y})$.

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then we have $m^3{}_A(\bar{a}) \geq m^3{}_A(y)$ and $n^3{}_A(\bar{a}) \leq n^3{}_A(y)$

But in ternary group $\bar{\bar{a}} = a$ (involution operator). For any $a, y \in G$ we have

$$m^3{}_A(a) = m^3{}_A(\bar{\bar{a}}) \geq m^3{}_A(\bar{a}) \geq m^3{}_A(y) \text{ and}$$

$$n^3{}_A(a) = n^3{}_A(\bar{\bar{a}}) \leq n^3{}_A(\bar{a}) \leq n^3{}_A(y)$$

So, $$m^3{}_A(a) = m^3{}_A(\bar{\bar{a}}) \geq m^3{}_A(y) \text{ and}$$

$$n^3{}_A(a) = n^3{}_A(\bar{\bar{a}}) \leq n^3{}_A(y)$$

The above relations are satisfying the Theorem 3.15. Followed by the numerical example is discussed here.

Example 3.17: Define a map: $\rho: Z_{12}{}^3 \to Z_{12}$ under the additive group (Z_{12},ρ) defined by $\rho(y_1, y_2, y_3) = S(y_1, y_2, y_3)$. Let A be a fermatean uncertainty subgroup of $ret_1(G,\rho)$ induced by subgroups $S_1 = \{11\}$, $S_2 = \{5,11\}$ and $S_3 = \{1,3,5,7,9,11\}$.

Define the fermatean uncertainty collection A as follows

$$m^3{}_A(y) = \begin{cases} 0.7, & if\ \ y=11 \\ 0.5, & if\ \ y=5 \\ 0.3, & if\ \ y=1,3,7,9 \\ 0.1, & if\ \ y \in S_3 \end{cases}$$

$$n^3{}_A(y) = \begin{cases} 0.2, & if\ \ y=11 \\ 0.4, & if\ \ y=5 \\ 0.6, & if\ \ y=1,3,7,9 \\ 0.8, & if\ \ y \in S_3 \end{cases}$$

Then, $m^3{}_A(5) = m^3{}_A(7) = 0.3$ is not greater than are equal to $0.5 = m^3{}_A(5)$

$n^3{}_A(5) = m^3{}_A(7) = 0.6$ is not less than are equal to $0.4 = n^3{}_A(5)$

Hence, the collection A is not a fermatean uncertainty ternary subgroup.

Remarks 3.18: From Example 3.17, the following results are true.

(i) The fermatean uncertainty subgroups of group $ret_{\alpha}(G,\rho)$ are not an extension fermatean uncertainty subgroups.

(ii) In Theorem 3.15: the conditions $m^3{}_A(a) \geq m^3{}_A(y)$ and $n^3{}_A(a) \leq n^3{}_A(y)$ cannot be neglected. Also, in the Example 3.17: we have $m^3{}_A(a)=0.3$ is not greater than are equal to $0.5 = m^3{}_A(5)$ and $n^3{}_A(a)= 0.6$ is not less than are equal to $0.4=n^3{}_A(5)$.

(iii) The relations $m^3{}_A(a) \geq m^3{}_A(y)$ and $n^3{}_A(a) \leq n^3{}_A(y)$ cannot be replaced by $m^3{}_A(\bar{a}) \geq m^3{}_A(y)$ and $n^3{}_A(\bar{a}) \leq n^3{}_A(y)$, where $\bar{a}$ is an identity element of $ret_{\alpha}(G,\rho)$. In Example 3.17, for any $y \in Z_{12}, T=11$ then $m^3{}_A(11) \geq m^3{}_A(y)$ and $n^3{}_A(11) \leq n^3{}_A(y)$.

Theorem 3.19: An extension subgroup (G,ρ) of b-derived group $(G, \bullet)$. For any collection A in an extension fermatean uncertainty subgroup under $(G, \bullet)$ such that $m_A(b) \geq m_A(y)$ and $n_A(b) \leq n_A(y)$, $\forall\ y \in G$ is also an extension fermatean uncertainty subgroup under (G,ρ).

Proof: According to the extension principle rules (EFUSG1) and (EFUSG1) defined in Definition 3.1: to prove the extension principle rules (EFUSG3) and (EFUSG4) under the extension group (G,ρ), the b-exempted from an extension fermatean uncertainty subgroup $(G, \bullet)$ gives

$$Y = \left(y^{(n-2)} \circ b\right)^{-1}$$

Followed by for each $y \in G$,

$$m^3{}_A(y) = m^3{}_A\left(y^{(n-2)} \bullet b\right)^{-1} \geq m^3{}_A\left(y^{(n-2)} \circ b\right)$$

$$\geq T\left\{m^3{}_A\left(y^{(n-2)},\ m^3{}_A(b)\right)\right\} = m^3{}_A(y)$$

$$n^3{}_A(y) = n^3{}_A\left(y^{(n-2)} \bullet b\right)^{-1} \geq n^3{}_A\left(y^{(n-2)} \circ b\right)$$

$$\leq S\left\{n^3{}_A\left(y^{(n-2)},\ n^3{}_A(b)\right)\right\} = n^3{}_A(y)$$

This completes the proof of the result.

Corollary 3.20: For any fermatean uncertainty group of a group $(G, \bullet)$ is an extension fermatean uncertainty subgroup of $(G,\ \rho)$ obtained by $(G, \bullet)$.

Proof: An extension group $(G,\ \rho)$ is obtained from $(G, \bullet)$ by substituting an element $b = e$ in Theorem 3.19 gives the relations $m_A(e) \geq m_A(y), n_A(e) \leq n_A(y)$, $\forall\ y \in G$. Therefore,

$m^3{}_A(5) = m^3{}_A(7) = 0.3$ is not greater than are equal to $0.5 = m^3{}_A(5)$

$n^3{}_A(5) = n^3{}_A(7) = 0.6$ is not less than are equal to $0.4 = n^3{}_A(5)$

Hence, the collection A is not a fermatean uncertainty ternary subgroup of (G,ρ).

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IV. Conclusions

We explained different aspects of cubic fermatean uncertainty soft ideal structures and fermatean uncertainty multi-group over multi-homomorphism. An extension of this fermatean uncertainty study using soft subgroup structures with norm. Moreover, we discussed the important mathematical proofs as Theorems, corollaries and suitable examples. In future, we extended this research into field of picture uncertainty collection and spherical uncertainty collections.

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Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 22 Issue 5 Version 1.0 Year 2022
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

The Effect of Vibration on the Sediment of the Bases of Turbine Units

By Sh. Altynbekov
South Kazakhstan State Pedagogical University

Abstract- The paper presents a mathematical model of the problem of assessing the effect of vibration on the sediment of the bases of turbine units of a built array of hereditarily aging heterogeneous two-phase soil. The constructed model takes into account the distribution of the vibration wave in the soil mass, the initial porosity coefficient is assumed to be a variable value depending on spatial coordinates. For the initial conditions of the problem, solutions of the Laplace equation in the corresponding boundary conditions are accepted. The solution of the boundary value problem is found by methods of applied mathematics and mathematical physics. The sediment of the bases of turbine units was determined by the method of V. A. Florin. Preliminary calculations confirmed the possibility of a significant impact of vibration on the sediment of the bases of turbine units.

Keywords: *vibration, sediment of turbine unit bases, reliability, iteration and approximation methods.*

GJSFR-F Classification: *DDC Code: 330.028 LCC Code: HB139*

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The Effect of Vibration on the Sediment of the Bases of Turbine Units

Sh. Altynbekov

Abstract- The paper presents a mathematical model of the problem of assessing the effect of vibration on the sediment of the bases of turbine units of a built array of hereditarily aging heterogeneous two-phase soil. The constructed model takes into account the distribution of the vibration wave in the soil mass, the initial porosity coefficient is assumed to be a variable value depending on spatial coordinates. For the initial conditions of the problem, solutions of the Laplace equation in the corresponding boundary conditions are accepted. The solution of the boundary value problem is found by methods of applied mathematics and mathematical physics. The sediment of the bases of turbine units was determined by the method of V. A. Florin. Preliminary calculations confirmed the possibility of a significant impact of vibration on the sediment of the bases of turbine units.

Keywords: *vibration, sediment of turbine unit bases, reliability, iteration and approximation methods.*

I. Введение

Одним из стихийных бедствий в большой степени зависит от вибрационного состояния турбоагрегатов атомной электростанции, от связующих веществ в составе грунта и от распределения вибрационной волны в грунтовом массиве. В зависимости от них вибрационное воздействие на характер деформации плотно уплотненных грунтов может быть малосущественным, существенным, разрушительным. Этому вопросу при различных постановках посвящена множество работ. Проведение первых детальных исследований процесса разжижения песчаных грунтов, их консолидации и динамического уплотнения взрывным методом началось под руководством проф. П.Л. Иванова (Иванов, 1983). Эти исследования базировались на теории консолидации, созданной проф. В.А.Флориным (Флорин 1961). За рубежом, в особенности в США и Японии, ведутся масштабные исследования явления разжижения грунтов оснований и сооружений (Seed 1982, Idriss 2008, Boulanger 2017, Ishihara 2016, Towhata 2015, Kokusho 2015). Данные исследования проводилась в США проф. Seed H. B. и были продолжены его учениками проф. I. M. Idrissипроф. R. W. Boulanger (Idriss 2008, Boulanger 2017). Многолетние исследования проф. K. Ishihara (Япония) были обобщены ученым в монографии «Поведение грунтов при землетресениях¸ (СПб., 2006, пер. на рус. яз.) (Ishihara2006).М. Ю. Абелев (Абелев 1983) описал методы проектирования, строительства и эксплуатации промышленных и гражданских сооружений на слабых водо-насыщенных глинистых грунтах. В работе рассмотрены методы оценки свойств указанных грунтов, а также методы проектирования и

Author: *South Kazakhstan State Pedagogical University. e-mail: sh.altynbekov@mail.ru*

устройства различных видов искусственных оснований и пределы применимости различных методов уплотнения грунтов в зависимости от свойства оснований конструкции сооружений. А. Л. Гольдин, С. Р. Месчян, Г.Ф. Рустамян (Гольдин, Месчян, Рустамян1985) решили плоской задачи консолидации водо-насыщенноголинистого грунта с учетом вибрационной ползучести скелета, при пренебрежение сжимаемости твердых частиц и поровой воды и изменяемость коэффициента фильтрации. Для решения этой задачи использовано обычное уравнение плоской задачи фильтрационной консолидации. При выводе общего уравнения вибрационной консолидации грунтового основания с учетом вибрационной ползучести скелета его мера представлена в виде произведения меры статические ползучести и функции амплитуды колебания. С. Я. Кушнир (Кушнир 1988) исследовала консолидационные явления в торфяных грунтах при динамических воздействиях. Л.Б. Маслов (Маслов 2012) рассмотрел теоретические вопросы расчета вынужденных гармонических колебаний пористых структур в насыщенных жидкостью. Показано, что с увеличением частоты возбуждения эффект инерционного взаимодействия фаз пороупругогоматериала становится весьма существенным, особенно для амплитуд давления жидкости в порах. В статье О.П. Минаева (Минаев 2018) приведены результаты теоретического обоснования на примере разработанной аналитической расчетной модели и полевых испытанийметода глубинного виброуплотнения оснований. Ш.Алтынбеков (Алтынбеков2016, 2018) сформулировал многопараметрической математической модели задачи вибрационной консолидации соленых и несоленых грунтов. Иследовал вопросы существования и единственности. Обосновал методы решения. Задача сводится к конечно – разностной краевой задаче, для которой исследована погрешность локально – одномерной схемы. Даны априорные оценки и ее решения. Приведены результаты предварительных расчетов. В статьях Гейдт (2018a и 2018 b)решение уравнения консолидации получено без каких-либо дополнительных допущений кроме учета начальных услови. Решение показывает, что после выключения вибрационного воздействия отсутствует какой - либо остаточный эффект влияния вибрационного воздействия. Е. С. Соболев (Соболев 2014), А.З. Тер-Мартиросян (Тер-Мартиросян 2010).Е.Л. Усошина (Усошина 2016) внесли определенные вклады в повышения надежности работы оснований зданий и сооружений.

Анализ существующих работ привел к следующему выводу: характер влияния вибрационных воздействий на уплотнение грунта является очень сложным и недостаточно исследованным. Пути исследователи столкнулись с целым рядом серьезных проблем, связанных главным образом с не изученностью динамического воздействия на процесс консолидации связных грунтов.

Фундамент турбоагрегатов атомной электростанций представляют собой сложную конструкцию, включающую нижнюю фундаментную плиту,систему колонн-стоек и систему верхних ригелей. Жесткие крепления вала-провода системы «турбина-генератор¸в подшипниках допускают весьма малое смещение опор подшипников и таким образом накладывают жесткие ограничения на прогибы нижних фундаментных плит. Современные технические условия ограничивают эти прогибы величинами

Notes

Global Journal of Science Frontier Research (F) Volume XXII Issue V Version I Year 2022

1/10000 длины фундаментной плиты для турбоагрегатов мощностью до 300 мВт. В случае строительства атомных электростанции наглинистых водо-насыщенных основаниях в результате процессов консолидации и ползучести скелета грунта имеет место нарастание прогибов фундаментов во времени, которое необходимо прогнозировать для оценки надежности турбоагрегатов. Надежность турбоагрегатов в значительной мереопределяется их вибрационным состоянием и характером уплотнения грунтовых оснований. При неравномерной осадке грунтовых оснований возникает неуравновешенная центробежная сила, что не желательно на практике. При столь малых величинах допускаемых прогибов возникает необходимость в разработке совершенных методов расчета, в которых с наибольшей полнотой учитывались бы реальные свойства глинистых грунтов. До настоящего времени во всех существующих исследованиях начальный коэффициент пористости считался постоянной величиной. В действительности он не является постоянной величиной, он зависит от пространственных координат. Это утверждение особенно верно, когда изучаемые объекты являются неоднородной пористой средой. Не исследовано влияние переменность начального коэффициента пористости на характер уплотнения грунтов. В расчетах недостаточно изучены влияния параметров неоднородности, степени физической нелинейности, краевых условий, переменности коэффициентов фильтрации, бокового давления, мгновенного уплотнения и начального коэффициента пористости на характер вибрационной консолидации грунтов.Как известно, воздействия вибрации приводит к дополнительным остаточным осадкам. Грунт испытывает вибрационный эффект и со временем в зависимости от физико – механических свойств, амплитудой и частотой колебании могут плотно уплотнятся. Влияние вибрации на деформации плотных уплотненных грунтов изучено недостаточно полно. В работах (Гольдин, Месчян, Рустамян 1985), (Алтынбеков 2016, 2018) при учете вибрационной ползучести скелета грунта, функция, характеризующая деформации ползучести представлены в виде произведение меры статической ползучести и функции амплитуды колебаний. Возможность такой постановки вопроса ограничено. Распределение волн вибрации в грунтовых основаниях остается в тени.В работе все эти вопросы находит свои ответы. С целью теоретического исследования данных вопросов рассмотрим следующую задачу.

II. Постановка Задачи.

Рассмотрим уплотнение земляной среды, находящееся под действием распределенной нагрузки с интенсивностью q. Чтобы изучить этот процесс, позвольте:

- Грунт состоит из твердой и жидкой фаз.
- Модуль деформации грунта и коэффициент бокового давления в процессе уплотнения изменяются по глубине соответственно с законами:

$$E(x_3) = E_0(\alpha_1 + \alpha_2 e^{-\alpha_3 x_3})^{-1},$$

$$a(x_3) = \frac{1}{E(x_3)} = a_0(\alpha_1 + \alpha_2 e^{-\alpha_3 x_3}), \quad (1)$$

$$\xi(x_3) = \xi_0 e^{-\alpha_4 x_3} \tag{2}$$

где $E_0, a_0, \xi_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ - опытные данные,

- Уравнение состояния скелета грунта представлено в виде

$$\varepsilon(t) = \varepsilon(\tau_1) - \frac{1}{1 + (n-1)\xi(x)} \cdot$$

$$\cdot \left\{a(x_3)\theta(t) - \int_{\tau_1}^{t} \theta(t) K\big(t, \tau, x, \theta(t)\big) d\tau\right\} \tag{3}$$

$$K\big(t, \tau, x, \theta(t)\big) = \eta(x) \frac{f(\tau, \theta(\tau))}{\theta(\tau)} * \frac{\partial C(t, \tau, \theta(\tau))}{\partial \tau}, \tag{4}$$

Суммаглавных нормальных напряжений (гипотеза Флорин1961) определена в виде

$$\theta(x, t) = n\gamma\big(H_0(x) - H(x, t)\big) \tag{5}$$

Функция $C\big(t, \tau, \theta(\tau)\big)$ в (4), характеризующая деформации вибрационной ползучести скелета грунта, аппроксимирована выражением

$$C\big(t, \tau, \theta(\tau)\big) = \varphi(\tau, \theta(\tau))(t - \tau)^{\widetilde{m}} U(x, t), \widetilde{m} > 1, \tag{6}$$

Функция$\eta(x)$, характеризующая коэффициент деформации ползучести принята в виде

$$\eta(x) = \alpha_5 + \alpha_6 e^{-\alpha_7 x_3} \tag{7}$$

Функция$f(\tau, \theta(\tau))$, входящая в (4), представлена в следующем виде:

$$f\big(\tau, \theta(\tau)\big) = \beta_1(\tau)\theta(\tau) + \beta_2(\tau)\theta^m(\tau), m > 0 \tag{8}$$

В качестве функции старения предложены:

$$\varphi\big(\tau, \theta(\tau)\big) = C_0 + \frac{A_k}{\tau^k + B_k \theta(\tau)} \tag{9}$$

$$\beta_1(\tau) = \beta_{10} + \frac{\beta_{11}}{\tau^k + \beta_{12}}, \beta_2(\tau) = \beta_{20} + \frac{\beta_{21}}{\tau^k + \beta_{22}}, k > 0 \tag{10}$$

Начальный коэффициент включен в (3) представлена в следующем виде:

$$\varepsilon(x_1) = \alpha_8 ch(\alpha_q x_l) \cdot \alpha_{10} ch(\alpha_{11} x_2) \cdot \alpha_{12} e^{-\alpha_{13} x_3}, \tag{11}$$

$$0 < \alpha_8 < 1,\ 0 < \alpha_9 < 1,\ 0 < \alpha_{10} < 1,\ 0 < \alpha_{11} < 1,$$

$$0 < \alpha_{12} < 1,\ 0 < \alpha_{13} < 1$$

Notes

Notes

Здесь $\varepsilon(t)$- коэффициент пористости; $\varepsilon(\tau_1)$ - начальный коэффициент пористости; τ - возраст скелета грунта; n =1,2,3 в зависимости от мерности рассматриваемой задачи,γ- удельный вес воды; $H_0(x)$,H(x.t)- функции напоров; x=(x_1, x_2, x_3)- пространственные координаты; C_0– предельное значение меры ползучести;A_k, B_k – некоторые параметры, зависящие от свойства и условий старения среды; $\alpha_5, \alpha_6, \alpha_7, \alpha_8, \alpha_9, \alpha_{10}, \alpha_{11}, \alpha_{12}, \alpha_{13}, \widetilde{m}, m, k, \beta_{10}, \beta_{11}, \beta_{12}, \beta_{20}, \beta_{21}, \beta_{22}$ - опытные данные.

- Коэффициент фильтрации линейно зависит от коэффициента пористости

$$K_s = K_{0s} + K_{1s}\varepsilon(x,t), s = 1,2,3, \tag{12}$$

где $K_{0s} > 0, K_{1s} > 0$ - опытные данные.

- Уплотнение грунта подчинена модели (Терцаги1933)– (Флорина 1948)
- На поверхности земной породы происходит переменный водо-обмен с окружающей средой.

Тогда математическая модель вибрационной консолидации неоднородного грунта в области $\Omega_\infty = G \times (t, \infty)$ описывается в следующей краевой задачи

$$\frac{\partial H}{\partial t} = C_v(x_3)\sum_{s=0}^{3}\frac{\partial}{\partial x_s}\left((K_s(x,\tau,t,H_0,U,H)\frac{\partial H}{\partial x_s}\right) +$$

$$+C_1(x,\tau,t,H_0,U,H) = L(H),\ x,t \in \Omega_\infty = G \times (t,\infty), \tag{13}$$

$$G = (-l_1 \le x_1 \le l_1, -l_2 \le x_2 \le l_2, 0 \le x_3 \le h)$$

$$H|_{t=\tau_1} = H_0(x),\ \ x \in G, \tag{14}$$

$$\chi_1^{(1)}\frac{\partial H}{\partial x_1} - \chi_1^{(2)}H|_{x_1=-l_1} = \psi_1(x_2,x_3,t),\ \chi_1^{(3)}\frac{\partial H}{\partial x_1} + \chi_1^{(4)}H|_{x_1=l_1} = \psi_2(x_2,x_3,t), \tag{15}$$

$$\chi_2^{(1)}\frac{\partial H}{\partial x_2} - \chi_2^{(2)}H|_{x_1=-l_2} = \psi_3(x_1,x_3,t),\ \ \chi_2^{(3)}\frac{\partial H}{\partial x_2} + \chi_2^{(4)}H|_{x_1=l_2} = \psi_4(x_1,x_3,t), \tag{16}$$

$$\chi_3^{(1)}\frac{\partial H}{\partial x_3} - \chi_3^{(2)}H|_{x_3=0} = \psi_5(x_1,x_2,t),\ \ \chi_3^{(3)}\frac{\partial H}{\partial x_3} + \chi_3^{(4)}H|_{x_3=h} = \psi_6(x_1,x_2,t), \tag{17}$$

Здесь

$$C_v(x_3) = \frac{(1+\varepsilon_{cp})(1+2\xi_0 e^{-\alpha_4 x_3})}{3a_0\gamma(\alpha_1+\alpha_2 e^{-\alpha_3 x_3})}, \tag{18}$$

$\chi_n^{(\alpha)}$ и $\chi_n^{(\alpha+1)}$ (α=1,2,3; n = 1,2,3) – коэффициенты водоотдачи, удовлетворяющие условиям: $\chi_n^{(\alpha)} \ge 0,\ \chi_n^{(\alpha+1)} \ge 0,\ \left(\chi_n^{(\alpha)}\right)^2 + \left(\chi_n^{(\alpha+1)}\right)^2 \ne 0$ для любого$x \in G$; $\psi(x,t)$– напор водоносного горизонта, прилегающего к рассматриваемой территории; виды функций $C_1(x,\tau,t,H_0,U,H)$, $K_s(x,\tau,t,H_0,U,H)(s = 1,2,3)$ в (13) определяются зависимостями (1), (2), (3), (4), (5),(6),(7),(8),(9),(10),(11)и (12). Эти функции непрерывны и ограниченны.

Следует отметить, что функция $H_0(x)$ в (5) и (14), то есть

$$H_0(x) = \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty} D_{0i_1i_2}\left(cos\frac{v_{i_1}}{\sqrt{K_{11}}}x_1 + B_{1i_1}\, sin\frac{v_{i_1}}{\sqrt{K_{11}}}x_1\right)\times$$

$$\times\left(cos\frac{\rho_{i_2}}{\sqrt{K_{12}}}x_2 + B_{2i_2}\, sin\frac{\rho_{i_2}}{\sqrt{K_{12}}}x_2\right)\cdot$$

$$\left(ch\sqrt{\frac{v_{i_1}^2+\rho_{i_2}^2}{K_{13}}}x_3 + B_{3i_1i_2}sh\sqrt{\frac{v_{i_1}^2+\rho_{i_2}^2}{K_{13}}}x_3\right) \qquad (19)$$

решение задачи

$$K_{01}\frac{\partial^2 H_0}{\partial x_1^2} + K_{02}\frac{\partial^2 H_0}{\partial x_1^2} + K_{03}\frac{\partial^2 H_0}{\partial x_1^2} = 0, x \in G$$

$$\chi_1^{(1)}\frac{\partial H_0}{\partial x_1} - \chi_1^{(2)}H_0|_{x_1=-\ell_1} = 0,\quad \chi_1^{(3)}\frac{\partial H_0}{\partial x_1} + \chi_1^{(4)}H_0|_{x_1=\ell_1} = 0,$$

$$\chi_2^{(1)}\frac{\partial H_0}{\partial x_2} - \chi_2^{(2)}H_0|_{x_2=-\ell_2} = 0,\quad \chi_2^{(3)}\frac{\partial H_0}{\partial x_2} + \chi_2^{(4)}H_0|_{x_2=\ell_2} = 0,$$

$$\chi_3^{(1)}\frac{\partial H_0}{\partial x_3} - \chi_3^{(2)}H_0|_{x_3=0} = 0,$$

$$\chi_3^{(3)}\frac{\partial H_0}{\partial x_3} + \chi_3^{(4)}H_0|_{x_3=h} = \frac{q}{\gamma},\quad |x_1| \le a, |x_2| \le b,$$

$$\chi_3^{(3)}\frac{\partial H_0}{\partial x_3} + \chi_3^{(4)}H_0|_{x_3=h} = 0,$$

где D_{1ij}, B_{1i}, B_{2j} и F_{ij} – известные коэффициенты, определяемые в процессе решения задачи,μ_{1i}, μ_{2i} – положительные корни уравнения вида

$$ctg\mu = \frac{\chi_s^{(1)}\chi_s^{(3)}\frac{\mu^2}{4l_s^2} - \chi_s^{(2)}\chi_s^{(4)}}{(\chi_s^{(1)}\chi_s^{(4)} + \chi_s^{(2)}\chi_s^{(3)})\frac{\mu}{4l_s}},\ s = 1,2; \qquad (20)$$

Функция $U(x,t)$ в (6)

$$U(x,t) = \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty}\sum_{i_3=1}^{\infty}\left(\tilde{A}_{i_1i_2i_3}cos\pi\tilde{\lambda}_{i_1i_2i_3}a_0t + \tilde{B}_{i_1i_2i_3}sin\pi\tilde{\lambda}_{i_1i_2i_3}a_0t\right)\times$$

Notes

$$\times\left(cos\frac{\tilde{\nu}_{i_1}}{\sqrt{K_1}}x_1+\tilde{B}_{1i_1}sin\frac{\tilde{\nu}_{i_1}}{\sqrt{K_2}}x_1\right)\times\left(cos\frac{\tilde{\rho}_{i_2}}{\sqrt{K_2}}x_2+\tilde{B}_{2i_2}sin\frac{\tilde{\rho}_{i_2}}{\sqrt{K_2}}x_2\right)\tilde{\nabla}_{\nu_{i_1i_2}}\left(\frac{2\tilde{\lambda}_{i_1i_2i_3}}{\tilde{\alpha}\sqrt{K_3}}e^{-\frac{\tilde{\alpha}}{2}x_3}\right) \quad (21)$$

решение задачи

$$\frac{\partial^2 U}{\partial t^2}=a_0^2e^{-\tilde{\alpha}x_3}\left(K_1\frac{\partial^2 U}{\partial x_1^2}+K_2\frac{\partial^2 U}{\partial x_2^2}+K_3\frac{\partial^2 U}{\partial x_3^2}\right)$$

$$U(x,0)=\tilde{\varphi}(x),\ \ \frac{\partial U\,(x,0)}{\partial t}=\tilde{\psi}(x)$$

$$\tilde{\chi}_1^{(1)}\frac{\partial U}{\partial x_1}-\tilde{\chi}_1^{(2)}U|_{x_1=-\ell_1}=0,\tilde{\chi}_1^{(3)}\frac{\partial U}{\partial x_1}+\tilde{\chi}_1^{(4)}U|_{x_1=\ell_1}=0,$$

$$\tilde{\chi}_2^{(1)}\frac{\partial U}{\partial x_2}-\tilde{\chi}_1^{(2)}U|_{x_2=-\ell_2}=0,\tilde{\chi}_2^{(3)}\frac{\partial U}{\partial x_2}+\tilde{\chi}_2^{(4)}U|_{x_2=\ell_2}=0,$$

$$\tilde{\chi}_3^{(1)}\frac{\partial U}{\partial x_3}-\tilde{\chi}_3^{(2)}U|_{x_3=0}=0,\tilde{\chi}_3^{(3)}\frac{\partial U}{\partial x_3}+\tilde{\chi}_3^{(4)}U|_{x_3=h}=0,$$

Здесь: $\tilde{A}_{i_1i_2i_3},\tilde{B}_{i_1i_2i_3},\tilde{B}_{1i_1},\tilde{B}_{2i_2}$- известные коэффициенты,определяемые из начальных и граничных условий; $\tilde{\nabla}_{\nu_{i_1i_2}}(x_3)$ - функция состоящая из комбинации функций Бесселя первого и второго рода индекса $\nu_{i_1i_2}$; $\tilde{\lambda}_{i_1i_2i_3}$- положительные корни уравнения, составленного из комбинаций этих функций; $\tilde{\nu}_{i_1},\tilde{\rho}_{i_2}$- положительные корни уравнения вида (20); K_s $(s=1,2,3)$– коэффициенты, характеризующие сопротивление пористой среды движущихся волн; $\tilde{\chi}_n^{\alpha}$ и $\tilde{\chi}_n^{(\alpha+1)}$ $(\alpha=1,2,3;n=1,2,3)$– коэффициенты волно отдачи, удовлетворяющиеусловиям: $\tilde{\chi}_n^{(\alpha)}\geq 0$, $\tilde{\chi}_n^{(\alpha+1)}\geq 0,(\tilde{\chi}_n^{(\alpha)})^2+(\tilde{\chi}_n^{(\alpha+1)})^2\neq 0$для любого $x\in G$;уравнения

$$K_3Z''(x_3)+\left(\tilde{\lambda}^2e^{-\tilde{\alpha}x_3}-(\tilde{\nu}^2+\tilde{\rho}^2)\right)Z(x_3)=0$$

последовательным введением новых переменных (Коренев 1960)

$$y=-\frac{\tilde{\alpha}}{2h}x_3+\frac{1}{2}ln\frac{4h^2\tilde{\lambda}^2}{\tilde{\alpha}^2K_3}\ \text{и}\ z=e^y$$

приведено к уравнению Бесселя (Коренев 1971),общее решение которого известно; ортогональность функций $\left\{\tilde{\nabla}_{\nu_{i_1i_2}}(x_3)\right\}$с весом $e^{-\tilde{\alpha}x_3}$ очевидно и не требуется вдоказательстве.

III. Существование И Единственность Решения

Теорема 3.1.

Пусть $C_v(x_3)$, $C_1(x,\tau,t,H_0,U,H)$,$K_s(x,\tau,t,H_0,U,H)(s=1,2,3)$– *положительные функции класса* $C^2(x\in G,0\leq\tau_1\leq t<T<\infty)\cap C(Q_\infty)$, *функция* $H(x,t)$ *удовлетворяет*

Notes

уравнению (12) в Q_∞, *начальному условию (13) и граничным условиям (14), (15), (16) и* $L(H) \geq 0(L(H) \leq 0)$ *в* Q_∞, *функция* $H_0(x)$ *содержится в области определения оператора Лапласа. Тогда задача (13), (14), (15), (16), (17) имеет единственное решение. Это решение непрерывно зависит от начальных и граничных данных, параметров коэффициентов фильтрации, мгновенного уплотнения и бокового давления, а также свободного элемента.*

Доказательствотеоремы 3.1 приводитсяпо той же схеме, что и доказательство теоремыприведенной в работах В. С. Владимирова 26, В. Я. Арсенина 27, Ш. Алтынбекова 28,29,30. В доказательстве учтены: свойства параметров уравнение состояние среды (4); гипотеза В.А.Флорина (5); условия гладкости и условие согласованности; принципы максимума и минимума; теоремы С. Банаха и разложения по собственным функциям; признаки сходимости мажорируемых рядов. В ходе доказательства подтверждено физически очевидный факт, что давление в поровой жидкости движется только из мест с высоким давлением в место с более низким давлением.

IV. Методы Решения Задачи.

Существует многочисленные методы расчетов фильтрации жидкости 31,32,33,34,35. и фильтрационной консолидации грунтов 15,16,36,37,38,39,40,41. Здесь предпочтение отдается итерационному методу, методу введения новой неизвестной функции, методу преобразования неоднородных граничных условий в однородные, методу Фурье, методу аппроксимации, методу введения новых переменных и теоремеоразложенийпо собственным функциям.

Представим итерационный метод в виде теоремы.

Теорема 4.1 (метод итерации).

Пусть выполняются условия теоремы3.1. $\mathrm{H}(\mathrm{x},\mathrm{t})$ – решение задачи (13), (14), (15), (16), (17) и $\mathrm{H}_\mathrm{k}(\mathrm{x},\mathrm{t})$(k=1,2,3,...) – решение дифференциального уравнения

$$\frac{\partial H_k}{\partial t} = C_v(x_3) \cdot \left(K_{11}\frac{\partial^2 H_k}{\partial x_1^2} + K_{12}\frac{\partial^2 H_k}{\partial x_2^2} + K_{13}\frac{\partial^2 H_k}{\partial x_3^2}\right) + \Phi_{k-1}(x,t),$$

$$k=1,2,3,\ldots, \qquad (22)$$

удовлетворяющие начальным (14) и граничным условиям (15), (16), (17), и $\mathrm{H} > \mathrm{H}_1$. Тогда последовательность $\{\mathrm{H}_\mathrm{k}\}$ (k=1,2,3,...) при $\mathrm{k} \to \infty$сходится к единственному решению $\mathrm{H}(\mathrm{x},\mathrm{t})$ задачи (13), (14), (15), (15), (17).

Функция$\Phi_{\mathrm{k}-1}(\mathrm{x},\mathrm{t})$, k=1,2,3,... в (21) определяется зависимостями (1), (2), (3), (4), (5), (6),(7),(8),(9),(10) и (11) и является непрерывным и ограниченным.

Доказательство.Рассуждения, аналогичные (Алтынбеков 2018) и использует принцип максимума и теоремы сравнения. Согласно теореме 3.1 задачи (13), (14), (15), (16), (17) и (21), (14), (15) , (16), (17) имеют единственные решения. В соответствии

Notes

с принципом максимума, гипотезы В.А.Флорина (5) и теоремы сравнения для этих решений, имеем

$$H_1 \leq H_3 \leq H_2 \leq H_5 \leq H_4 \leq \cdots \leq H_{2k+1} \leq H_{2k},$$

Notes

отсюда, перенумеровав их $H_1 = U_1$, $H_3 = U_2$, $H_2 = U_3, \ldots$, имеем $U_1 \leq U_2 \leq U_3 \leq \cdots \leq U_{2k-1} \leq U_{2k}$ или $U_{2k} \geq H_1 \leq H$.

Следовательно, последовательность $\{H_k\}$ при $k \to \infty$ сходится к решению $H(x,t)$ задачи (13), (14), (15), (16), (17). Теорема доказана.

Метод введения новой неизвестной функции. Метод преобразования неоднородных граничных условий в однородные.

Введем новую неизвестную функцию $W_k(x,t)$

$$H_k(x,t) = \psi(x,t) + W_k(x,t),\ k{=}1,2,3,\ldots, \tag{23}$$

Эта функция является отклонением от известной функции $\psi(x,t)$и будет определена как решение уравнения

$$\frac{\partial W_k}{\partial t} = C_v(x_3)\cdot\left(K_{11}\frac{\partial^2 W_k}{\partial x_1^2} + K_{12}\frac{\partial^2 W_k}{\partial x_2^2} + K_{13}\frac{\partial^2 W_k}{\partial x_3^2}\right) +$$

$$+\Phi_{1,k-1}(x,t,\Phi_{k-1}(x,t,)),\quad k{=}1,2,3,\ldots \tag{24}$$

с однородными граничными условиями

$$\chi_1^{(1)}\frac{\partial W_k}{\partial x_1} - \chi_1^{(2)}W_k|_{x_1=-l_1} = 0,\ \chi_1^{(3)}\frac{\partial W_k}{\partial x_1} + \chi_1^{(4)}W_k|_{x_1=l_1} = 0, \tag{25}$$

$$\chi_2^{(1)}\frac{\partial W_k}{\partial x_2} - \chi_2^{(2)}W_k|_{x_2=-l_2} = 0,\ \chi_2^{(3)}\frac{\partial W_k}{\partial x_2} + \chi_2^{(4)}W_k|_{x_2=l_2} = 0, \tag{26}$$

$$\chi_3^{(1)}\frac{\partial W_k}{\partial x_3} - \chi_3^{(2)}W_k|_{x_3=0} = 0,\ \chi_3^{(3)}\frac{\partial W_k}{\partial x_3} + \chi_3^{(4)}W_k|_{x_2=h} = 0, \tag{27}$$

и с начальным условием

$$W_k(x,\tau_1) = H_0(x) - \psi(x,\tau_1). \tag{28}$$

Представляя функцию $\psi(x,t)$ в виде

$$\psi(x,t) = \left(\alpha_1^{(1)}x_1 + \beta_1^{(1)}\right)\left(\chi_1^{(1)} + \psi_1(x_2,x_3,t)\right) + \left(\alpha_2^{(1)}x_1 + \beta_2^{(1)}\right)\times$$

$$\times\left(\chi_1^{(3)} + \psi_2(x_2,x_3,t)\right) + \left(\alpha_1^{(2)}x_2 + \beta_1^{(2)}\right)\times$$

$$\times(\chi_2^{(1)}+\psi_3(x_1,x_3,t))+\left(\alpha_2^{(2)}x_2+\beta_2^{(2)}\right)(\chi_2^{(3)}+\psi_4(x_1,x_3,t))+$$

$$+\left(\alpha_1^{(3)}x_3+\beta_1^{(3)}\right)(\chi_3^{(1)}+\psi_5(x_1,x_2,t))+\left(\alpha_2^{(3)}x_3+\beta_2^{(3)}\right)(\chi_3^{(3)}+\psi_6(x_1,x_2,t)) \quad (29)$$

требуем, она удовлетворяла условиям видов (15), (16) и (17).Тогда коэффициенты в (29)однозначно определяются

$$\alpha_1^{(1)}=\frac{\chi_1^{(4)}}{\chi_1^*},\quad \beta_1^{(1)}=\frac{\chi_1^{(3)}+\chi_1^{(4)}}{\chi_1^*},\quad \alpha_2^{(1)}=\frac{\chi_1^{(2)}}{\chi_1^*},\quad \beta_2^{(1)}=\frac{\chi_1^{(1)}+\chi_1^{(2)}}{\chi_1^*},$$

$$\alpha_1^{(2)}=\frac{\chi_2^{(4)}}{\chi_2^*},\quad \beta_1^{(2)}=\frac{\chi_2^{(3)}+\chi_2^{(4)}}{\chi_2^*},\quad \alpha_2^{(2)}=\frac{\chi_2^{(2)}}{\chi_2^*},\quad \beta_2^{(2)}=\frac{\chi_2^{(1)}+\chi_2^{(2)}}{\chi_2^*},$$

$$\alpha_1^{(3)}=\frac{\chi_3^{(4)}}{\chi_3^*},\quad \beta_1^{(3)}=\frac{\chi_3^{(3)}+\chi_3^{(4)}}{\chi_3^*},\quad \alpha_2^{(3)}=\frac{\chi_3^{(2)}}{\chi_3^*},\quad \beta_2^{(3)}=\frac{\chi_3^{(1)}}{\chi_3^*},$$

где

$$\chi_1^*=\chi_1^{(2)}\chi_1^{(3)}+2\chi_1^{(2)}\chi_1^{(4)}+\chi_1^{(1)}\chi_1^{(4)},$$

$$\chi_2^*=\chi_2^{(2)}\chi_2^{(3)}+2\chi_2^{(2)}\chi_2^{(4)}+\chi_2^{(1)}\chi_2^{(4)},$$

$$\chi_3^*=\chi_3^{(2)}\chi_3^{(3)}+\chi_3^{(2)}\chi_3^{(4)}+\chi_3^{(1)}\chi_3^{(4)}.$$

Метод Фурье. Метод аппроксимации. Метод введения новых переменных.
Пусть для начала:

$$\Phi_{1,k-1}(x,t,\Phi_{k-1}(x,t,))=0,\quad k=1,2,3,\ldots.$$

Тогда, согласно вышеуказанным методам, решение задачи (27), (28), (29), (30), (31) нетрудно представит так

$$W_k(x,t)=\sum_{i_1=1}^{\infty}\sum_{i_2=1}^{\infty}\sum_{i_3=1}^{\infty}D_{ki_1i_2i_3}\left(cos\frac{v_{i_1}}{\sqrt{K_{11}}}x_1+B_{1i_1}\,sin\frac{v_{i_1}}{\sqrt{K_{11}}}x_1\right)\times$$

$$\times\left(cos\frac{\rho_{i_2}}{\sqrt{K_{12}}}x_2+B_{2i_{12}}\,sin\frac{\rho_{i_2}}{\sqrt{K_{12}}}x_2\right)\times V_{vi_1i_2}\left(\frac{2\lambda_{i_1i_2i_3}}{\alpha\sqrt{K_{13}}}e^{-\frac{\alpha}{2}x_3}\right)\cdot e^{-\lambda_{i_1i_2i_3}t},\ k\in N \quad (30)$$

Здесь $D_{ki_1i_2i_3}$ – известные коэффициенты, определяемые в процессе решения задачи; $V_{vi_1i_2}(x_3)$– функция, состоящая из комбинации функций Бесселя первого и второго рода индекса $v_{i_1i_2},\lambda_{i_1i_2i_3}$;– положительные корни уравнения, состоящие из этих комбинаций.

Для решения задачи (24), (25), (26), (27), (28),здесь мы использовали метод аппроксимации. В соответствии с этимметодом функция $1+2\xi_0e^{-\alpha_4x_3}$ в (18) заменяется функцией $\tilde{\xi}(x_3)$:

Notes

$$\tilde{\xi}(x_3) = (1+2\xi_0)\,exp\left(\left(ln\frac{1+2\xi_0 e^{-\alpha_4 h}}{1+2\xi_0}\right)\cdot\frac{x_3}{h}\right),$$

функция $\alpha_1+\alpha_2 e^{-\alpha_3 x_3}$ в (18) заменяется функцией $\tilde{\alpha}(x_3)$:

$$\tilde{\alpha}(x_3) \approx (\alpha_1+\alpha_2)\,exp\left(\left(ln\frac{\alpha_1+\alpha_2 e^{-\alpha_3 h}}{\alpha_1+\alpha_2}\right)\cdot\frac{x_3}{h}\right), \text{т.е.}$$

$$1+2\xi_0 e^{-\alpha_4 x_3} \approx (1+2\xi_0)\,exp\left(\left(ln\frac{1+2\xi_0 e^{-\alpha_4 h}}{1+2\xi_0}\right)\cdot\frac{x_3}{h}\right), \tag{31}$$

$$\alpha_1+\alpha_2 e^{-\alpha_3 x_3} \approx (\alpha_1+\alpha_2)\,exp\left(\left(ln\frac{\alpha_1+\alpha_2 e^{-\alpha_3 h}}{\alpha_1+\alpha_2}\right)\cdot\frac{x_3}{h}\right). \tag{32}$$

Далее, принимая во внимание (31) и (32), функция $\frac{1+2\xi_0\,exp\,(-\alpha_4 x_3)}{\alpha_1+\alpha_2\,exp\,(-\alpha_3 x_3)}$ в (18) заменяется функцией $\tilde{C}_v(x_3)$:

$$\frac{1+2\xi_0 e^{-\alpha_4 x_3}}{\alpha_1+\alpha_2 e^{-\alpha_3 x_3}} \approx \frac{1+2\xi_0}{\alpha_1+\alpha_2}exp\left(\left(ln\frac{(1+2\xi_0 e^{-\alpha_4 h})(\alpha_1+\alpha_2)}{(1+2\xi_0)(\alpha_1+\alpha_2 e^{-\alpha_3 h})}\right)\cdot\frac{x_3}{h}\right). \tag{33}$$

Легко заметить, что при $x_3=0$ и $x_3=h$ аппроксимация (36) абсолютно точна, и при $\alpha_3,\alpha_4\to 0$ погрешность аппроксимации стремится к нулю. Она для малых значений α_3 и α_4 вполне допустимо в практических расчетах.

Далее, учитывая (33), последовательно вводим новых переменных

$$y = -\frac{\alpha}{2h}x_3 + \frac{1}{2}ln\frac{4h^2\lambda^2}{\alpha^2 K_{13}} \text{ и } z = e^y$$

Тогда дифференциальное уравнение

$$K_{13}Z''(x_3) + (\lambda^2 e^{-\alpha x_3} - (\nu^2+\rho^2)Z(x_3) = 0$$

легко сводится к уравнению Бесселя (Коренев 1971), общее решение которого известно. Здесь

$$\alpha = ln\frac{(1+2\xi_0 e^{-\alpha_4 h})(\alpha_1+\alpha_2)}{(1+2\xi_0)(\alpha_1+\alpha_2 e^{-\alpha_3 h})}/h.$$

Метод разложений в собственных функциях и решение задачи.

Пусть теперь :

$$\Phi_{1,k-1}(x,t,\Phi_{k-1}(x,t)) \neq 0,\ k=1,2,3,\dots .$$

Предполагаем, что непрерывная функция $\Phi_{1,k-1}(x,t,\Phi_{k-1}(x,t))$ имеет кусочно-непрерывную производную первого порядка по x, и для этой функци при всех $t \geq \tau_1 > 0$ выполняется все условия типа (25),(26),(27).Тогда решение задачи (24), (25), (26), (27), (29) можно представить так

Notes

$$W_k(x,t) = \sum_{i_1=1}^{\infty} \sum_{i_2=1}^{\infty} \sum_{i_3=1}^{\infty} T_{ki_1i_2i_3}(t) \left(cos \frac{\nu_{i_1}}{\sqrt{K_{11}}} x_1 + B_{1i_1} \, sin \frac{\nu_{i_1}}{\sqrt{K_{11}}} x_1 \right) \times$$

$$\times \left(\cos \frac{\rho_{i_2}}{\sqrt{K_{12}}} x_2 + B_{2i_{12}} \sin \frac{\rho_{i_2}}{\sqrt{K_{12}}} x_2 \right) . V_{\nu i_1 i_2} \left(\frac{2\lambda_{i_1i_2i_3}}{\alpha\sqrt{K_{13}}} e^{-\frac{\alpha}{2}x_3} \right), \qquad (34)$$

$$T_{ki_1i_2i_3}(t) = \left(\int \Phi^*_{1ki_1i_2i_3}(t) \cdot e^{\lambda^2_{i_1i_2i_3}t} dt + D_{ki_1i_2i_3} \right) e^{-\lambda^2_{i_1i_2i_3}t}.$$

Notes

Поставляя ряд (34) в (23) и учитывая (29), при $k \to \infty$ получим решение задачи (13), (14), (15), (16), (17).

Следствие.Если водопроницаемость грунта в направлении оси x_1 (оси x_2) пренебрежимо мала, то легко проверить, что собственные значения ν_{i_1} (ρ_{i_2}), характеризующие уровни пьезометрических напоров, равнынулю, а соответствующие им собственные функции, являющиеся волновыми функциями дифференциального оператора второго порядка, равны единице. В этом случае легко получить решение плоской краевой задачи из решения (37), и аналогично одномерной задачи.

V. Определение Осадок Оснований Турбоагрегатов.

Согласно методу, приведенному в работе (Флорин 1961) и полученных результатов (19),(23),(34), нетрудно определить осадок грунтового основания, вызванный нагрузкой q:

$$s_k(t) = \frac{3\gamma a_0}{1+\varepsilon_0} \int_0^h \frac{\alpha_1 + \alpha_2 e^{-\alpha_3 x_3}}{1 + 2\xi_0 e^{-\alpha_4 x_3}} (H_0(x) - H_k(x,t)) dx_3. \qquad (35)$$

Предварительные расчеты по формуле (35) показали:

- Нагрузка, приложенная на верхнюю поверхность слоя массива земляной среды, со временем передается к его скелету.
- При больших значениях параметра α_3и малых значениях параметров $\alpha_0, \alpha_1, \alpha_2$ уплотнение грунта не зависит от времени. Вибрационное уплотнение старого неоднородного грунта не зависит от времени, зависит от пространственных координат.
- С увеличением коэффициента бокового давления уменьшается осадок грунтовых оснований.
- Осадок слоя неоднородных грунтовых оснований с возрастанием параметровα_3и α_7увеличивается, а затем постепенно уменьшается.
- Возраст скелета достаточно заметно влияет на характер осадки грунтовых оснований. Это влияние может быть мало существенным только при$A_k, \beta_{10}, \beta_{11}, \beta_{20}, \beta_{21} \longrightarrow 0$
- В начальные моменты времени вибрационное воздействие на ползучесть скелета водо-насыщенного глинистого грунта наиболее ярко проявляется, а со временем становится мало заметным.
- Осадок неоднородных грунтовых оснований сильно зависит от типа граничных условий. В зависимости от них происходит обратный процесс уплотнения- набухания грунта.
- Вибрационное уплотнение сильно зависит от степени влажности грунта и вяжущих веществ в почве, параметров ползучести, статической нагрузки, амплитуды и частоты колебании. В зависимости от них, при применении модернизированного

вибрационного уплотнителя можно уменьшит разницу значений пористости по глубине уплотняемого основания и тем самым значительно улучшить равномерность уплотнения грунта. При этом можно достигнут значительное увеличение механических характеристик мелких и средних грунтов оснований.

VI. Заключение

Notes

Вибрационное воздействие на характер деформации плотно уплотненных грунтов зависит от вибрационного состояние турбоагрегата и от связующих веществ в составе грунта. В зависимости от них вибрационное воздействие на характер деформации плотно уплотненных грунтов может быть мало существенным, существенным, разрушительным. Всеми этими процессами можно управлять. Вопрос: как управлять – это вопрос завтрашнего дня. Работа посвящена одному из недостаточно изученных вопросов вибрационной консолидации неоднородных грунтов. Приводится новая математическая постановка задачи. Исследованы вопросы существования и единственности для нее. Обоснованы методы решения. Обоснованные методы аппроксимации и итерации являются новым вкладом в прикладную механику грунтов. Применение их позволяет изучить более сложные проблемы теории консолидации неоднородных грунтов.

Полученные результаты (19),(21),(29),(34) и формула расчета (35) дает нам возможность количественного и качественного анализа в оценке влияния вибрации на осадок оснований турбоагрегатов и сооружений.

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- Paper title should be in one column of font size 24.
- Author name in font size of 11 in one column.
- Abstract: font size 9 with the word "Abstract" in bold italics.
- Main text: font size 10 with two justified columns.
- Two columns with equal column width of 3.38 and spacing of 0.2.
- First character must be three lines drop-capped.
- The paragraph before spacing of 1 pt and after of 0 pt.
- Line spacing of 1 pt.
- Large images must be in one column.
- The names of first main headings (Heading 1) must be in Roman font, capital letters, and font size of 10.
- The names of second main headings (Heading 2) must not include numbers and must be in italics with a font size of 10.

Structure and Format of Manuscript

The recommended size of an original research paper is under 15,000 words and review papers under 7,000 words. Research articles should be less than 10,000 words. Research papers are usually longer than review papers. Review papers are reports of significant research (typically less than 7,000 words, including tables, figures, and references)

A research paper must include:

a) A title which should be relevant to the theme of the paper.
b) A summary, known as an abstract (less than 150 words), containing the major results and conclusions.
c) Up to 10 keywords that precisely identify the paper's subject, purpose, and focus.
d) An introduction, giving fundamental background objectives.
e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition, sources of information must be given, and numerical methods must be specified by reference.
f) Results which should be presented concisely by well-designed tables and figures.
g) Suitable statistical data should also be given.
h) All data must have been gathered with attention to numerical detail in the planning stage.

Design has been recognized to be essential to experiments for a considerable time, and the editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned unrefereed.

i) Discussion should cover implications and consequences and not just recapitulate the results; conclusions should also be summarized.
j) There should be brief acknowledgments.
k) There ought to be references in the conventional format. Global Journals recommends APA format.

Authors should carefully consider the preparation of papers to ensure that they communicate effectively. Papers are much more likely to be accepted if they are carefully designed and laid out, contain few or no errors, are summarizing, and follow instructions. They will also be published with much fewer delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and suggestions to improve brevity.

Format Structure

It is necessary that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

All manuscripts submitted to Global Journals should include:

Title

The title page must carry an informative title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) where the work was carried out.

Author details

The full postal address of any related author(s) must be specified.

Abstract

The abstract is the foundation of the research paper. It should be clear and concise and must contain the objective of the paper and inferences drawn. It is advised to not include big mathematical equations or complicated jargon.

Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Keywords

A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy: planning of a list of possible keywords and phrases to try.

Choice of the main keywords is the first tool of writing a research paper. Research paper writing is an art. Keyword search should be as strategic as possible.

One should start brainstorming lists of potential keywords before even beginning searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in a research paper?" Then consider synonyms for the important words.

It may take the discovery of only one important paper to steer in the right keyword direction because, in most databases, the keywords under which a research paper is abstracted are listed with the paper.

Numerical Methods

Numerical methods used should be transparent and, where appropriate, supported by references.

Abbreviations

Authors must list all the abbreviations used in the paper at the end of the paper or in a separate table before using them.

Formulas and equations

Authors are advised to submit any mathematical equation using either MathJax, KaTeX, or LaTeX, or in a very high-quality image.

Tables, Figures, and Figure Legends

Tables: Tables should be cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g., Table 4, a self-explanatory caption, and be on a separate sheet. Authors must submit tables in an editable format and not as images. References to these tables (if any) must be mentioned accurately.

Figures

Figures are supposed to be submitted as separate files. Always include a citation in the text for each figure using Arabic numbers, e.g., Fig. 4. Artwork must be submitted online in vector electronic form or by emailing it.

Preparation of Eletronic Figures for Publication

Although low-quality images are sufficient for review purposes, print publication requires high-quality images to prevent the final product being blurred or fuzzy. Submit (possibly by e-mail) EPS (line art) or TIFF (halftone/ photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Avoid using pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings). Please give the data for figures in black and white or submit a Color Work Agreement form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution at final image size ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs): >350 dpi; figures containing both halftone and line images: >650 dpi.

Color charges: Authors are advised to pay the full cost for the reproduction of their color artwork. Hence, please note that if there is color artwork in your manuscript when it is accepted for publication, we would require you to complete and return a Color Work Agreement form before your paper can be published. Also, you can email your editor to remove the color fee after acceptance of the paper.

Tips for Writing a Good Quality Science Frontier Research Paper

Techniques for writing a good quality Science Frontier Research paper:

1. Choosing the topic: In most cases, the topic is selected by the interests of the author, but it can also be suggested by the guides. You can have several topics, and then judge which you are most comfortable with. This may be done by asking several questions of yourself, like "Will I be able to carry out a search in this area? Will I find all necessary resources to accomplish the search? Will I be able to find all information in this field area?" If the answer to this type of question is "yes," then you ought to choose that topic. In most cases, you may have to conduct surveys and visit several places. Also, you might have to do a lot of work to find all the rises and falls of the various data on that subject. Sometimes, detailed information plays a vital role, instead of short information. Evaluators are human: The first thing to remember is that evaluators are also human beings. They are not only meant for rejecting a paper. They are here to evaluate your paper. So present your best aspect.

2. Think like evaluators: If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

3. Ask your guides: If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

4. Use of computer is recommended: As you are doing research in the field of science frontier then this point is quite obvious. Use right software: Always use good quality software packages. If you are not capable of judging good software, then you can lose the quality of your paper unknowingly. There are various programs available to help you which you can get through the internet.

5. Use the internet for help: An excellent start for your paper is using Google. It is a wondrous search engine, where you can have your doubts resolved. You may also read some answers for the frequent question of how to write your research paper or find a model research paper. You can download books from the internet. If you have all the required books, place importance on reading, selecting, and analyzing the specified information. Then sketch out your research paper. Use big pictures: You may use encyclopedias like Wikipedia to get pictures with the best resolution. At Global Journals, you should strictly follow here.

6. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

7. Revise what you wrote: When you write anything, always read it, summarize it, and then finalize it.

8. Make every effort: Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

9. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several unnecessary diagrams will degrade the quality of your paper by creating a hodgepodge. So always try to include diagrams which were made by you to improve the readability of your paper. Use of direct quotes: When you do research relevant to literature, history, or current affairs, then use of quotes becomes essential, but if the study is relevant to science, use of quotes is not preferable.

10. Use proper verb tense: Use proper verb tenses in your paper. Use past tense to present those events that have happened. Use present tense to indicate events that are going on. Use future tense to indicate events that will happen in the future. Use of wrong tenses will confuse the evaluator. Avoid sentences that are incomplete.

11. Pick a good study spot: Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

12. Know what you know: Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

13. Use good grammar: Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice.

Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

14. Arrangement of information: Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. Multitasking in research is not good: Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. Never copy others' work: Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. Go to seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. Refresh your mind after intervals: Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

20. Think technically: Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

21. Adding unnecessary information: Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

22. Report concluded results: Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

The introduction: This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear: Adhere to recommended page limits.

Mistakes to avoid:

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

Title page:

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

- o Single section and succinct.
- o An outline of the job done is always written in past tense.
- o Concentrate on shortening results—limit background information to a verdict or two.
- o Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

The following approach can create a valuable beginning:

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

Materials may be reported in part of a section or else they may be recognized along with your measures.

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- Give details of all of your remarks as much as possible, focusing on mechanisms.
- Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

The Administration Rules

Administration Rules to Be Strictly Followed before Submitting Your Research Paper to Global Journals Inc.

Please read the following rules and regulations carefully before submitting your research paper to Global Journals Inc. to avoid rejection.

Segment draft and final research paper: You have to strictly follow the template of a research paper, failing which your paper may get rejected. You are expected to write each part of the paper wholly on your own. The peer reviewers need to identify your own perspective of the concepts in your own terms. Please do not extract straight from any other source, and do not rephrase someone else's analysis. Do not allow anyone else to proofread your manuscript.

Written material: You may discuss this with your guides and key sources. Do not copy anyone else's paper, even if this is only imitation, otherwise it will be rejected on the grounds of plagiarism, which is illegal. Various methods to avoid plagiarism are strictly applied by us to every paper, and, if found guilty, you may be blacklisted, which could affect your career adversely. To guard yourself and others from possible illegal use, please do not permit anyone to use or even read your paper and file.

CRITERION FOR GRADING A RESEARCH PAPER (COMPILATION) BY GLOBAL JOURNALS

Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals.

Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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ISSN 9755896