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A New Integral Transform Called "Saxena & Gupta Transform" and Relation between New Transform and other Integral Transforms

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Abstract- We investigate a new integral transform called Saxena and Gupta transform, in this paper. Some important properties of this transform are also investigated. Also discussed relation between Saxena & Gupta transform and other integral transform like Laplace transform, Elzaki transform, Sumudu transform, Mahgoub transform. Integral transform of some elementary functions are given in table form in different section.

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I. Introduction

Recently, Integral transformations have played an important role in many fields of science and engineering [1,10,14,15,], especially mathematical physics [5], engineering mathematics [16,19], Cryptography [11], image processing [18], mathematical electrodynamics [5] and, a few others, because they have been successfully used in solving many problems in those fields. Integral transforms are one of the most effective tools for solving problems in physics and engineering to obtain a solution for a given differential equation or integral equations by means of inverse transformation. The importance of an integral transforms is that they provide powerful operational methods for solving initial value problems and initial- boundary value problems for linear differential and integral equations.

In view of many interesting properties which make visualization easier, we introduced a new integral transform, termed as "Saxena and Gupta" Transformation. Many of these transforms have been introduced which were extensively used and applied on theory and applications, such as Laplace [6,12,13,17], Fourier [15], Sumudu [4,12], Elzaki [3,8,9], Aboodh [2,19]. Among these, the most widely used is the Laplace transform. Here, a new integral transform is proposed to avoid the complexity of previous transforms.

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Ref

11.Higgins, W. E. and D. C. Munson, (1988)."A Hankel transform approach to tomographic image reconstruction", IEEE Trans. Med. Imaging, vol. 7, pp 59-72.

Definition of New Transform

This transform of a function $f(t)$ defined for all real numbers $t \geq 0$ is the function

$$f(\upsilon) = Z[f(t)] = \frac{1}{\upsilon}\int_0^\infty f(\upsilon t)e^{-t}\,dt \qquad \dots (1.1)$$

The above integral convergent.

Definition of other transform

a) *Aboodh transform:* The Aboodh transform[2,19] A of a function $f(t)$ for t is greater than or equal to zero given by

$$A[f(t)] = \frac{1}{\upsilon}\int_0^\infty f(t)e^{-\upsilon t}\,dt \qquad (0 \leq k_1 \leq t \leq k_2) \qquad \dots (1.2)$$

b) *Laplace transform:* The Laplace transform [6],[12],[13],[17] of a function $f(t)$ for t is greater than zero given by

$$L[f(t)] = \int_0^\infty f(t)e^{-\upsilon t}\,dt \qquad \dots (1.3)$$

c) *Sumudu transform:* The Sumudu transform S [4],[12] of a function $f(t)$for t is greater than or equal to zero given by

$$S[f(t)] = \int_0^\infty f(\upsilon t)e^{-t}\,dt \qquad \dots (1.4)$$

d) *Elzaki transform:* The Elzaki transform E [3],[8],[9] of a function $f(t)$for t is greater than or equal to zero given by

$$E[f(t)] = \upsilon\int_0^\infty f(t)e^{-t/\upsilon}\,dt \qquad \dots (1.5)$$

e) *Mahgoub transform:* The Mahgoub (Laplace-Carson)M [1] transform of the function $f(t), t \geq 0$ is given by

$$M[f(t)] = \upsilon\int_0^\infty f(t)e^{-\upsilon t}\,dt \qquad \dots (1.6)$$

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9. Elzaki, Tarig. M.,(2011)."The New Integral Transform 'ELzaki Transform", Global Journal of Pure and Applied Mathematics, Vol. 7, No. 1, pp 57-64.

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II. Properties of Saxena and Gupta Transform

i) *Linearty property*

Let $Z\{f(t)\}$ denote the New transform of the real function f. Let f, g be a function such that $Z\{f\}$and $Z\{g\}$ exist.

$$Z\{af(t)+bg(t)\}=aZ\{f(t)\}+bZ\{g(t)$$

Proof: Let $a, b \in c$ or R be constant

$$Z\{af(t)+bg(t)\}=\frac{1}{u}\int_0^\infty e^{-ut}\ \{af(\upsilon t)+bg(\upsilon t)\}dt$$

$$Z\{af(t)+bg(t)\}=\frac{a}{u}\int_0^\infty e^{-ut}\,f(\upsilon t)dt+\frac{b}{u}\int_0^\infty e^{-ut}\,g(\upsilon t)dt$$

$$=aZ\{f(t)\}+bZ\{g(t) \qquad \ldots(2.1)$$

ii) *Shifting property*

If the function $\bar{f}(u)=Z\{f\}$ is the transform of the function $f(t)$ then

$$Z\{e^{at}f\}=\frac{1}{(1-a\upsilon)}\bar{f}\left(\frac{\upsilon}{1-a\upsilon}\right) \qquad \ldots(2.2)$$

Proof: $Z\{e^{at}f\}=\frac{1}{\upsilon}\int_0^\infty e^{-t}\,e^{a\upsilon t}f(\upsilon t)dt$

$$=\frac{1}{\upsilon}e^{-(1-a\upsilon)t}f(\upsilon t)dt \qquad \ldots(2.3)$$

Let $(1-a\upsilon)t=x, t=\frac{x}{1-a\upsilon}, dt=\frac{dx}{1-a\upsilon}$ and put these values in eq.(2.3)

$$Z\{e^{at}f\}=\frac{1}{u}\int_0^\infty e^{-x}\,f\left(\frac{\upsilon x}{1-a\upsilon}\right)\frac{dx}{1-a\upsilon}$$

$$Z\{e^{at}f\} = \frac{1}{\upsilon(1-a\upsilon)}\int_0^{\infty} e^{-x} f\left(\frac{\upsilon x}{1-a\upsilon}\right) dx$$

$$Z\{e^{at}f\} = \frac{1}{(1-a\upsilon)}\bar{f}\left(\frac{\upsilon}{1-a\upsilon}\right)$$

iii) *The convolution of two function*

Definition: Assume that f and g are peicewise continuous function, or one of them is dirac's generalized function [7]. The convolution of f and g is a function denoted $f * g$ by and given by the following expression

$$(f * g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau \qquad \text{... (2.4)}$$

For every piecewise continuous function f, g *and* h the following properties hold.

1. Commutativity $f * g = g * f$
2. Associativity $f * (g * h) = (f * g) * h$
3. Distributivity $f * (g + h) = f * g + f * h$
4. Neutral element $f * 0 = 0$
5. Identity element $f * 1 = f$

Theorem 1: If the function f and g have well defined transform $Z\{f\}$ and $Z\{g\}$ then

$$Z\{f * g\} = \upsilon^2 Z\{f\}Z\{g\} \qquad \text{... (2.5)}$$

Proof: $Z\{f * g\} = \frac{1}{\upsilon}\int_0^{\infty} e^{-t}\,(f * g)(\upsilon t)dt$

$$= \frac{1}{\upsilon}\int_0^{\infty} e^{-t}\left[\int_0^{\infty} f(\tau)g(\upsilon t-\tau)d\tau\right]dt \qquad \text{... (2.6)}$$

Ref

7. Dirac, paul (1930), "The principal of Quantum Mechanics" (1st ed.), Oxford university press.

Let λ= υt, dλ = υdt put these values in eq.(2.6) then

$$Z\{ f * g\} = \frac{1}{\upsilon}\int_0^{\infty} e^{-\lambda/\upsilon}\left[\int_0^{\lambda} f(\tau)g(\lambda-\tau)d\tau\right]d\lambda/\upsilon$$

Notes

$$= \frac{1}{\upsilon^2}\int_0^{\infty}\int_0^{\lambda} e^{-\frac{\lambda}{\upsilon}} f(\tau)g(\lambda-\tau)d\tau d\lambda \qquad ...(2.7)$$

Put γ= λ -τ, dγ = dλ in eq. (2.7) since the variable τ is constant hence when we integrate with respect to τ, we get

$$Z\{ f * g\} = \frac{1}{\upsilon^2}\int_0^{\infty}\int_0^{\infty} e^{-\frac{\gamma+\tau}{\upsilon}} f(\tau)g(\gamma)d\tau d\gamma$$

$$= \frac{1}{\upsilon^2}\int_0^{\infty} e^{-\frac{\gamma}{\upsilon}} g(\gamma)d\gamma . \int_0^{\infty} e^{-\frac{\tau}{\upsilon}} f(\tau)d\tau \qquad ...(2.8)$$

$$let\ x = \frac{\gamma}{\upsilon}\ and\ y = \frac{\tau}{\upsilon}\ then\ we\ have\ dx = \frac{d\gamma}{\upsilon}\ and\ dy = \frac{d\tau}{\upsilon}$$

$$= \frac{1}{\upsilon^2}\left[\int_0^{\infty} e^{-x}\,\upsilon\, g(\upsilon x)dx\right]\left[\int_0^{\infty} e^{-y}\upsilon\, f(\upsilon y)dy\right]$$

$$Z\{f * g\} = \int_0^{\infty} e^{-x}\, g(\upsilon x)dx . \int_0^{\infty} e^{-y}\, f(\upsilon y)dy$$

$$Z\{f * g\} = \upsilon^2 Z\{f\}.Z\{g\}$$

iv) *The New Transform of Derivative and Integral*

Derivative property:

If the function $Z\{F\}$and $Z\{F'\}$ are well defined then

First derivative

$$Z\{f'(t)\} = \frac{1}{\upsilon}\int_0^{\infty} e^{-t} f'(\upsilon t)dt$$

$$= \frac{1}{\upsilon}\left[\frac{-f(0)}{\upsilon} + z\{f(t)\}\right]$$

$$Z\{F'(t)\} = \frac{F(\upsilon)}{\upsilon} - \frac{f(0)}{\upsilon^2} \qquad \dots (2.9)$$

Second derivative $Z\{F''(t)\} = \frac{1}{\upsilon}Z\{f'(t)\} - \frac{f\prime(0)}{\upsilon^2}$

$$= \frac{1}{\upsilon}\left[\frac{F(\upsilon)}{\upsilon} - \frac{f(0)}{\upsilon^2}\right] - \frac{f\prime(0)}{\upsilon^2}$$

$$Z\{F''(t)\} = \frac{F(\upsilon)}{\upsilon^2} - \frac{f(0)}{\upsilon^3} - \frac{f'(0)}{\upsilon^2} \qquad \dots (2.10)$$

Nth derivative

$$Z\{f^n(t)\} = \frac{F(\upsilon)}{\upsilon^n} - \frac{f(0)}{\upsilon^{n+1}} - \frac{f'(0)}{\upsilon^n} - \frac{f''(0)}{\upsilon^{n-1}}$$

$$Z\{F^n(t)\} = \frac{F(\upsilon)}{\upsilon^n} - \sum_{k=0}^{n-1}\frac{f^k(0)}{\upsilon^{n+1-k}} \qquad \dots (2.11)$$

v) *The New transform of Integral*

Theorem 2: If the following $Z[f(t)]$ is well defined then

$$Z\left[\int_0^t f(\tau)d\tau\right] = \upsilon^2 Z\{f\}$$

Proof: Suppose that g(t-τ)=1 then

$$Z\left[\int_0^t f(\tau)d\tau\right] = Z\left[\int_0^t g(t-\tau)f(\tau)d\tau\right] = Z[g * f]$$

$$= \upsilon^2 Z[f]Z[g]$$

Notes

$$= \upsilon^2 Z[f]$$

vi) *The inverse of New transform*

Definition: Let the function $\bar{f}(\upsilon)$=$\bar{Z}${F} is the transform of the function $f(t)$, then $f(t)$ is called the inverse transform $\bar{f}(\upsilon)$ and we will write it as

$$F(t) = Z^{-1}\{\bar{f}(\upsilon)\}$$

Note: The inverse transform has the linear combination property

vii) *Solution of some elementary function*

1. when $f(t) = 1$ then

$$\frac{1}{\upsilon}\int_0^\infty e^{-t} f(\upsilon t)dt = \frac{1}{v}\int_0^\infty e^{-t}.1dt = 1/\upsilon[-e^{-t}]_0^\infty = 1/\upsilon$$

2. when $f(t) = t$ then

$$\frac{1}{\upsilon}\int_0^\infty e^{-t} f(\upsilon t)dt = \frac{1}{\upsilon}\int_0^\infty e^{-t}.\upsilon t\, dt =$$

$$= 1/\upsilon[(-t-1)e^{-t}]_0^\infty = \frac{1}{\upsilon}$$

3. when $f(t) = t^2$ then

$$\frac{1}{\upsilon}\int_0^\infty e^{-t} f(\upsilon t)dt = \frac{1}{\upsilon}\int_0^\infty e^{-t}.\upsilon^2\, t^2 dt = \frac{1}{\upsilon}[(-t^2-2t-2)e^{-t}]_0^\infty = \frac{2}{\upsilon}$$

4. when $f(t) = t^n$ then

$$\frac{1}{\upsilon}\int_0^\infty e^{-t} f(vt)dt = \frac{1}{\upsilon}\int_0^\infty e^{-t}.\upsilon^n t^n dt = v^{n-1}\Gamma(\text{n+1})$$

5. when $f(t) = e^{at}$ then

$$\frac{1}{\upsilon}\int_0^\infty e^{-t} f(\upsilon t)dt = \frac{1}{\upsilon}\int_0^\infty e^{-t}.e^{\upsilon at}dt$$

$$= \frac{1}{\upsilon}\left[\frac{e^{(a\upsilon-1)t}}{a\upsilon-1}\right]_0^\infty = \frac{1}{\upsilon\,(1-a\upsilon)}$$

6. when $f(t) = sinat$ then

$$\frac{1}{\upsilon}\int_0^\infty e^{-t}f(\upsilon t)dt = \frac{1}{\upsilon}\int_0^\infty e^{-t}.sina\upsilon t\; dt$$

$$= 1/\upsilon\left[-\frac{e^{-t}.(\sin(a\upsilon t) + a\upsilon cos(a\upsilon t))}{a^2\upsilon^2+1}\right]_0^\infty$$

$$= \frac{a}{a^2\upsilon^2+1}$$

7. when $f(t) = cosat$ then

$$\frac{1}{\upsilon}\int_0^\infty e^{-t}f(\upsilon t)dt = \frac{1}{\upsilon}\int_0^\infty e^{-t}.cosa\upsilon t\; dt$$

$$= \frac{1}{\upsilon}\left[\frac{e^{-t}(a\upsilon sin(a\upsilon t) - \cos(a\upsilon t))}{a^2\upsilon^2+1}\right]_0^\infty$$

$$= \frac{1}{\upsilon(a^2\upsilon^2+1)}$$

8. when$f(t) = sinhat$ then

$$\frac{1}{\upsilon}\int_0^\infty e^{-t}f(\upsilon t)dt = \frac{1}{\upsilon}\int_0^\infty e^{-t}.sinha\upsilon t\; dt$$

$$= \frac{1}{\upsilon}\int_0^\infty \left(\frac{e^{a\upsilon t} - e^{-a\upsilon t}}{2}\right)e^{-t}dt$$

$$= \frac{1}{2\upsilon}\left[\int_0^\infty e^{t(a\upsilon-1)}dt - \int_0^\infty e^{-t(a\upsilon+1)}dt\right]$$

Notes

$$= \frac{1}{2v}\left[\frac{e^{t}}{a\upsilon - 1} + \frac{e^{t}}{av + 1}\right]_0^{\infty} = \frac{a}{1 - a^2\upsilon^2}$$

9. when $f(t) = coshat$ then

Notes

$$\frac{1}{\upsilon}\int_0^{\infty} e^{-t} f(\upsilon t)dt = \frac{1}{\upsilon}\int_0^{\infty} e^{-t}.\, cosha\upsilon t\, dt$$

$$= \frac{1}{\upsilon}\int_0^{\infty}\left(\frac{e^{a\upsilon t} + e^{-avt}}{2}\right)e^{-t}dt$$

$$= \frac{1}{2\upsilon}\left[\int_0^{\infty} e^{t(a\upsilon-1)}dt + \int_0^{\infty} e^{-t(a\upsilon+1)}dt\right]$$

$$= \frac{1}{2\upsilon}\left[\frac{e^{t}}{a\upsilon - 1} - \frac{e^{t}}{a\upsilon + 1}\right]_0^{\infty}$$

$$= \frac{1}{\upsilon(1 - a^2\upsilon^2)}$$

viii) *New transform of some elementary function*

Table 1

S.NO.	Function $f(t)$	New transform $Z[f(t)]$
1	1	$\frac{1}{v}$
2	t	1
3	t^2	2v
4	t^n	$u^{n-1}\Gamma(\mathrm{n}+1)$
5	e^{at}	$\frac{1}{v\,(1-av)}$
6	$sinat$	$\frac{a}{1+a^2v^2}$
7	$cosat$	$\frac{1}{v\,(1+a^2v^2\,)}$
8	$sinhat$	$\frac{a}{1-a^2v^2}$
9	$coshat$	$\frac{1}{v\,(1-a^2v^2\,)}$

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Notes

ix) *Compare between New transform and other transform*

Table 2: *Sumudu transform and New transform*

S.NO.	Function $f(t)$	Sumudu transform $S[f(t)]$	New transform $Z[f(t)]$
1	1	1	$\frac{1}{\upsilon}$
2	t	υ	1
3	t^2	$2!/\upsilon^2$	2υ
4	t^n	$n!/\upsilon^n$	$\frac{\upsilon^{n-1}}{\Gamma(n+1)}$
5	e^{at}	$\frac{1}{(1-a\upsilon)}$	$\frac{1}{\upsilon(1-a\upsilon)}$
6	$sinat$	$\frac{a\upsilon}{1+a^2\upsilon^2}$	$\frac{a}{1+a^2\upsilon^2}$
7	$cosat$	$\frac{1}{(1+a^2\upsilon^2)}$	$\frac{1}{\upsilon(1+a^2\upsilon^2)}$
8	$sinhat$	$\frac{a\upsilon}{1-a^2\upsilon^2}$	$\frac{a}{1-a^2\upsilon^2}$
9	$coshat$	$\frac{1}{(1-a^2\upsilon^2)}$	$\frac{1}{\upsilon(1-a^2\upsilon^2)}$

Table 3: *Aboodh transform and New transform*

S.NO.	Function $f(t)$	Aboodh transform $A[f(t)]$	New transform $Z[f(t)]$
1	1	$\frac{1}{\upsilon^2}$	$\frac{1}{\upsilon}$
2	t	$\frac{1}{\upsilon^3}$	1
3	t^2	$\frac{2!}{\upsilon^4}$	2υ
4	t^n	$\frac{n!}{\upsilon^{n+2}}$	$\frac{\upsilon^{n-1}}{\Gamma(n+1)}$
5	e^{at}	$\frac{1}{\upsilon(\upsilon-a)}$	$\frac{1}{\upsilon(1-a\upsilon)}$
6	$sinat$	$\frac{a}{\upsilon(a^2+\upsilon^2)}$	$\frac{a}{1+a^2\upsilon^2}$
7	$cosat$	$\frac{1}{(a^2+\upsilon^2)}$	$\frac{1}{\upsilon(1+a^2\upsilon^2)}$

8	$sinhat$	$\frac{a}{\upsilon(v^2-a^2)}$	$\frac{a}{1-a^2v^2}$
9	$coshat$	$\frac{1}{(v^2-a^2)}$	$\frac{1}{v(1-a^2v^2)}$

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Table 4: *Laplace transform and New transform*

S.NO.	Function $f(t)$	Laplace transform $L[f(t)]$	New transform $Z[f(t)]$
1	1	$\frac{1}{\upsilon}$	$\frac{1}{v}$
2	t	$\frac{1}{\upsilon^2}$	1
3	t^2	$\frac{2!}{\upsilon^3}$	2υ
4	t^n	$\frac{n!}{s^{n+1}}$	$\frac{v^{n-1}}{\Gamma(n+1)}$
5	e^{at}	$\frac{1}{s-a}$	$\frac{1}{v(1-av)}$
6	$sinat$	$\frac{a}{a^2+s^2}$	$\frac{a}{1+a^2v^2}$
7	$cosat$	$\frac{s}{a^2+s^2}$	$\frac{1}{v(1+a^2v^2)}$
8	$sinhat$	$\frac{a}{s^2-a^2}$	$\frac{a}{1-a^2v^2}$
9	$coshat$	$\frac{s}{s^2-a^2}$	$\frac{1}{v(1-a^2v^2)}$

Table 5: *Mahgoub and New transform*

S.NO.	Function $f(t)$	Mahgoub transform $M[f(t)]$	New transform $Z[f(t)]$
1	1	1	$\frac{1}{v}$
2	t	$\frac{1}{v}$	1
3	t^2	$\frac{2!}{v^2}$	2υ

Notes

4	t^n	$\frac{n!}{v^n}$	$\frac{v^{n-1}}{\Gamma(n+1)}$
5	e^{at}	$\frac{\upsilon}{(\upsilon - a)}$	$\frac{1}{v(1-av)}$
6	$sinat$	$\frac{a\upsilon}{a^2+v^2}$	$\frac{a}{1+a^2v^2}$
7	$cosat$	$\frac{v^2}{(a^2+v^2)}$	$\frac{1}{v(1+a^2v^2)}$
8	$sinhat$	$\frac{a\upsilon}{v^2-a^2}$	$\frac{a}{1-a^2v^2}$
9	$coshat$	$\frac{v^2}{(v^2-a^2)}$	$\frac{1}{v(1-a^2v^2)}$

Table 6: Elzaki and New transform

S.NO.	Function $f(t)$	Elzaki transform $E[f(t)]$	New transform $Z[f(t)]$
1	1	v^2	$\frac{1}{v}$
2	t	v^3	1
3	t^2	$2!\ v^4$	2υ
4	t^n	$n!\ v^{n+2}$	$\frac{v^{n-1}}{\Gamma(n+1)}$
5	e^{at}	$\frac{v^2}{(1-a\upsilon)}$	$\frac{1}{v(1-av)}$
6	$sinat$	$\frac{av^3}{1+a^2v^2}$	$\frac{a}{1+a^2v^2}$
7	$cosat$	$\frac{v^2}{(1+a^2v^2)}$	$\frac{1}{v(1+a^2v^2)}$
8	$sinhat$	$\frac{av^3}{1-a^2v^2}$	$\frac{a}{1-a^2v^2}$
9	$coshat$	$\frac{v^2}{1-a^2v^2}$	$\frac{1}{v(1-a^2v^2)}$

Conclusion: In this paper, we introduce a new integral transform. After that we compare some integral transforms with this new integral transform. It has shows that the new integral transform cover those exiting transforms such as Laplace, Elzaki and Sumudu transform for different values. Also gave properties of new transform and derivatives. Researcher can use this new integral transform for solving ODE, integral equations and fractional differential and integral equations.

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