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Intuitionistic Fuzzy Structures

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Keywords: *ideal, filter, i-convergence, intuitionistic fuzzy normed spaces.*

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Intuitionistic Fuzzy Structures

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Abstract- In this article we introduce to certain class of intuitionistic fuzzy structures.

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I. Introduction

The fuzzy theory has emerged as the most active area of research in many branches of science and engineering. Among various developments of the theory of fuzzy sets[35] a progressive development has been made to find the fuzzy analogues of the classical set theory. In fact the fuzzy theory has become an area of active research for the last 50 years. It has a wide range of applications in the field of science and engineering, e.g. application of fuzzy topology in quantum particle physics that arises in string and $e^{(\infty)}$-theory of El-Naschie[7-11], electronic engineering, chaos control, computer programming, electrical engineering, nonlinear dynamical system, population dynamics and biological engineering etc.

In [35], Zadeh introduced the theory of fuzzy sets. Atanassov[1-2] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Later on the theory of intuitionistic fuzzy sets caught the interest of various mathematicians round the globe and a huge literature in this direction has been produced in forms of books and research papers published in famous journals round the globe. By reviewing the literature, one can reach them easily, (e. g., see, [1-6], [12], [13], [15],[26], [30] and [31] and the references there in.)

Sequences play an important role in various fields of Real Analysis, Complex Analysis, Functional Analysis and Topology. These are very useful tools in demonstrating abstract concepts through constructing examples and counter examples. The topic "Sequence spaces" is very broad in its own sense as one can study from various point of views. In [22] Kostyrko, Salat and Wilczynski introduced and studied the concept of I-Convergence and later on there has been much progress and development in the study in this direction. (e. g., see, [12], [14-19], [32] and [34] and the references there in.)

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In many branches of science and engineering we often come across with different type of sequences and certainly there are situations of inexactness where the idea of ordinary convergence does not work. So to deal with such situations we have to introduce new measures and tools which are suitable to the said situation. Here we give the preliminaries.

II. Preliminaries

Notes

Now we quote the following definitions which will be needed in the sequel

Definition 2.1. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is said to be continuous t-norm if it satisfies the following conditions;

(a) * is associative and commutative.

(b) * is continuous.

(c) a*1=a for all a∈[0,1]

(d) a*c≤b*d whenever a≤b and c≤d for each a,b,c,d∈[0,1].

For example, $a * b = a \cdot b$ is a continuous t-norm.

Definition 2.2. A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is said to be continuous t-conorm if it satisfies the following conditions;

(a) $\diamond$ is associative and commutative.

(b) $\diamond$ is continuous.

(c) a$\diamond$0=a for all a∈[0,1]

(d) a$\diamond$c≤b$\diamond$d whenever a≤b and c≤d for each a,b,c,d∈[0,1].

For example, a$\diamond$b=min$\{a + b, 1\}$ is a continuous t-conorm.

Definition 2.3. Let * be a continuous t-norm and $\diamond$ be a continuous t-conorm and X be a linear space over the field(R or C). If μ and ν are fuzzy sets on X×(0,∞) satisfying the following conditions, the five- tuple (X,μ,ν,*,$\diamond$) is said to be an intuitionistic fuzzy normed space(IFNS) and (μ,ν) is called an intuitionistic fuzzy norm. For every x, y ∈ X and s, t>0,

(a)μ(x,t)+ν(x,t)≤ 1,

(b)μ(x,t)>0,

(c)μ(x,t)=1 iff x=0,

(d)μ(ax,t)=μ(x, $\frac{t}{|a|}$) for each a ≠0,

(e)μ(x,t)*μ(y,s)≤μ(x+y,t+s)

(f)μ(x,·) : (0,∞)→[0,1] is continuous,

(g) $\lim_{t\to\infty} \mu(x,t) = 1$ and $\lim_{t\to 0} \mu(x,t) = 0$,

(h)ν(x,t)< 1,

Notes

(i)ν(x,t)=0 iff x=0,

(j)ν(ax,t)=ν(x, $\frac{t}{|a|}$) for each a $\neq$0,

(k)ν(x,t)$\diamond\nu$(y,s)$\geq\nu$(x+y,t+s)

(l)ν(x,$\cdot$) : (0,∞)$\rightarrow$[0,1] is continuous,

(m)$\lim_{t\to\infty}\nu(x,t)=1$ and $\lim_{t\to 0}\nu(x,t)=0$,

(n)a*a=a, a$\diamond$a=a for all a$\in$[0,1].

Definition 2.4. Let (X, μ,ν,*,$\diamond$) be IFNS and (x_n) be a sequence in X. Sequence (x_n) is said to be convergent to L$\in$ X with respect to the intuitionistic fuzzy norm (μ,ν) if for every $\epsilon>0$ and $t>0$, there exists a positive integer n_0 such that $\mu(x_n$-L,t)$>1-\epsilon$ and $\nu(x_n$-L,t)$<\epsilon$ whenever $n>n_0$. In this case we write (μ,ν)- limx_n=L as $n\rightarrow\infty$.

Definition 2.5. If X be a non- empty set, then a family of set $I\subset P(X)(P(X)$ denoting the power set of X) is called an ideal in X if and only if

(a) $\phi\in I$;

(b) For each $A,B\in I$, we have $A\cup B\in I$;

(c) For each $A\in I$ and $B\subset A$ we have $B\in I$.

Deffinition 2.6. If X be a non- empty set. A non- empty family of sets $F\subset P(X)(P(X)$ denoting the power set of X) is called a filter on X if and only if

(a) $\phi\notin F$;

(b) For each $A,B\in F$, we have $A\cap B\in F$;

(c) For each $A\in F$ and $A\subset B$ we have $B\in F$.

Definition 2.7. Let $I\subset P(N)$ be a non trivial ideal and (X, μ,ν,*,$\diamond$) be an IFNS. A sequence $x=(x_n)$ of elements in X is said to be I- convergent to L$\in$ X with respect to the intuitionistic fuzzy norm (μ,ν) if for every $\epsilon>0$ and $t>0$, The set

$$\{n\in N:\mu(x_n-L,t)\geq 1-\epsilon \text{ or } \nu(x_n-L,t)\leq\epsilon\}\in I$$

In this case L is called the I-limit of the sequence (x_n) with respect to the intuitionistic fuzzy norm (μ,ν) and we write $I_{(\mu,\nu)}$ - limx_n=L.

Definition 2.8. A convergence field of I-convergence is a set

$$F(I)=\{x=(x_n)\in\ell_\infty:\text{there exists } I-\lim x\in R\}.$$

The convergence field $F(I)$ is a closed linear subspace of ℓ_∞ with respect to the supremum norm, $F(I)=\ell_\infty\cap c^I$.

As an insight, while working in the direction of fuzzy theory and Ideal Convergence the author has the following observations

Let N, R and C be the sets of all natural, real and complex numbers respectively. We write

$$\omega = \{x = (x_k) : x_k \in R \text{ or } C\},$$

the space of all real or complex sequences.
Listed below are few complementary structures which are intuitionistic fuzzy in nature.

$c_0 = \{x \in \omega : \lim_k |x_k| = 0\}$, the space of null sequences.

$c = \{x \in \omega : \lim_k x_k = l, \quad \text{for some} \quad l \in C\}$, the space of convergent sequences.

$\ell_\infty = \{x \in \omega : \sup_k |x_k| < \infty\}$, the space of bounded sequences.

Recently Kostyrko, Šalát and Wilczyński[22], Šalát Tripathy and Ziman[32] introduced and studied the following sequence spaces

$c_0^I = \{(x_k) \in \omega : \{k \in N : |x_k| \geq \epsilon\} \in I\}$,

$c^I = \{(x_k) \in \omega : \{k \in N : |x_k - L| \geq \epsilon\} \in I, \text{for some L} \in C \}$,

$\ell_\infty^I = \{(x_k) \in \omega : \{k \in N : |x_k| \geq M\} \in I, \text{ for each fixed M>0}\}$.

Ruckle [27-29] used the idea of a modulus function f to construct the sequence space

$$X(f) = \{x = (x_k) : \sum_{k=1}^{\infty} f(|x_k|) < \infty\}$$

Khan and Ebadullah[16] introduced the following sequence spaces

$c_0^I(f) = \{(x_k) \in \omega : I - \lim f(|x_k|) = 0\}$;

$c^I(f) = \{(x_k) \in \omega : I - \lim f(|x_k|) = L \text{ for some L} \in C \}$;

$\ell_\infty^I(f) = \{(x_k) \in \omega : \sup_k f(|x_k|) < \infty\}$.

Lindenstrauss and Tzafriri[23] used the idea of Orlicz functions to construct the sequence space

$$\ell_M = \{x \in \omega : \sum_{k=1}^{\infty} M(\tfrac{|x_k|}{\rho}) < \infty, \text{for some } \rho > 0\}$$

Tripathy and Hazarika[34] introduced the following sequence spaces

$c_0^I(M) = \{x = (x_k) \in \omega : I - \lim M(\frac{|x_k|}{\rho}) = 0 \text{ for some } \rho > 0\}$;

$c^I(M) = \{x = (x_k) \in \omega : I - \lim M(\frac{|x_k - L|}{\rho}) = 0 \text{ for some L} \in C \text{ and } \rho > 0\}$;

$\ell_\infty^I(M) = \{x = (x_k) \in \omega : \sup_k M(\frac{|x_k|}{\rho}) < \infty \text{ for some } \rho > 0\}$;

Ref

32. T.Šalát, B.C.Tripathy, M.Ziman, On some properties of I-convergence. *Tatra Mt. Math. Publ.*,(28) (2004), 279-286.

Kolk[20-21] gave an extension of $X(f)$ by considering a sequence of modulii $F = (f_k)$ and defined the sequence space

$$X(F) = \{x = (x_k) : (f_k(|x_k|)) \in X\}.$$

Khan, Suantai and Ebadullah[17] introduced the following sequence spaces

$c_0^I(F) = \{(x_k) \in \omega : I - \lim f_k(|x_k|) = 0\};$

$c^I(F) = \{(x_k) \in \omega : I - \lim f_k(|x_k|) = L$ for some L$\in C$ };

$\ell_\infty^I(F) = \{(x_k) \in \omega : \sup_k f_k(|x_k|) < \infty\}.$

The σ-convergent sequences

$V_\sigma = \{x = (x_k) : \sum_{m=1}^{\infty} t_{m,k}(x) = L$ uniformly in k, L $= \sigma - \lim x\}$,
where $m \geq 0, k > 0$

Khan and Ebadullah[19] introduced the following sequence spaces

$V_{0\sigma}^I(m,\epsilon) = \{(x_k) \in \ell_\infty : (\forall m)(\exists \epsilon > 0)\{k \in N : |t_{m,k}(x)| \geq \epsilon\} \in I\}$

$V_\sigma^I(m,\epsilon) = \{(x_k) \in \ell_\infty : (\forall m)(\exists \epsilon > 0)\{k \in N : |t_{m,k}(x) - L| \geq \epsilon\} \in I,$ for some L$\in C$ }

Mursaleen [24-25] defined the sequence space BV_σ, the space of all sequences of σ-bounded variation

$$BV_\sigma = \{x \in \ell_\infty : \sum_m |\phi_{m,k}(x)| < \infty, \text{uniformly in k}\}.$$

Khan, Ebadullah and Suantai [18] introduced the following sequence space

$$BV_\sigma^I = \{(x_k) \in \ell_\infty : \{k \in N : |\phi_{m,k}(x) - L| \geq \epsilon\} \in I, \text{for some L} \in C \}$$

Şengönül[33] introduced the Zweier sequence spaces $\mathcal{Z}_0$ and $\mathcal{Z}$ as follows

$$\mathcal{Z}_0 = \{x = (x_k) \in \omega : Z^p x \in c_0\}.$$

$$\mathcal{Z} = \{x = (x_k) \in \omega : Z^p x \in c\}$$

Khan, Ebadullah and Yasmeen [14] introduced the following classes of sequence spaces

$\mathcal{Z}_0^I = \{x = (x_k) \in \omega : I - \lim Z^p x = 0\};$

$\mathcal{Z}^I = \{x = (x_k) \in \omega : I - \lim Z^p x = L$ for some L$\in C$ };

$\mathcal{Z}_\infty^I = \{x = (x_k) \in \omega : \sup_k |Z^p x| < \infty\}.$

Ref

17. V. A.Khan, S. Suantai, and K. Ebadullah. On Some I-Convergent Sequence Spaces Defined By a Sequence of Moduli, Journal of Nonlinear Analysis and Optimization,3(2) (2012), 145-152.

III. Main Results

The approach is to construct and study new intuitionistic fuzzy I-convergent sequenence spaces

$$c^I_{(\mu,\nu)} = \{\{k \in N : \mu(x_k - L, t) \leq 1 - \epsilon \text{ or } \nu(x_k - L, t) \geq \epsilon\} \in I\},$$

$$c^I_{0(\mu,\nu)} = \{\{k \in N : \mu(x_k, t) \leq 1 - \epsilon \text{ or } \nu(x_k, t) \geq \epsilon\} \in I\},$$

$$V^I_{\sigma(\mu,\nu)} = \{\{k \in N : \mu(t_{m,k}(x) - L, t) \leq 1 - \epsilon \text{ or } \nu(t_{m,k}(x) - L, t) \geq \epsilon\} \in I\},$$

$$V^I_{0\sigma(\mu,\nu)} = \{\{k \in N : \mu(t_{m,k}(x), t) \leq 1 - \epsilon \text{ or } \nu(t_{m,k}(x), t) \geq \epsilon\} \in I\},$$

$$BV^I_{\sigma(\mu,\nu)} = \{\{k \in N : \mu(\phi_{m,k}(x) - L, t) \leq 1 - \epsilon \text{ or } \nu(\phi_{m,k}(x) - L, t) \geq \epsilon\} \in I\},$$

$$\mathcal{Z}^I_{(\mu,\nu)} = \{\{k \in N : \mu(x^/_k - L, t) \leq 1 - \epsilon \text{ or } \nu(x^/_k - L, t) \geq \epsilon\} \in I\},$$

$$\mathcal{Z}^I_{0\,(\mu,\nu)} = \{\{k \in N : \mu(x^/_k, t) \leq 1 - \epsilon \text{ or } \nu(x^/_k, t) \geq \epsilon\} \in I\}.$$

using the (μ,ν) intuitionistic fuzzy norm.

IV. Conclusion

We can study the algebraic, topological and elementary properties of intuitionistic fuzzy I-convergent sequenence spaces $c^I_{(\mu,\nu)}$, $c^I_{0(\mu,\nu)}$, $V^I_{\sigma(\mu,\nu)}$, $V^I_{0\sigma(\mu,\nu)}$, $BV^I_{\sigma(\mu,\nu)}$, $Z^I_{(\mu,\nu)}$,$Z^I_{0(\mu,\nu)}$.

Further, as a future research directions, the sequence spaces $c^I_{(\mu,\nu)}$, $c^I_{0(\mu,\nu)}$, $V^I_{\sigma(\mu,\nu)}$, $V^I_{0\sigma(\mu,\nu)}$, $BV^I_{\sigma(\mu,\nu)}$, $Z^I_{(\mu,\nu)}$,$Z^I_{0(\mu,\nu)}$. can be studied using the Lacunary, Modulus function, Orlicz function, Sequence of moduli, Musielak-Orlicz function and Fibonacci sequences.

The proposed new intuitionistic fuzzy I-convergent sequence spaces are quite pathological from algebraic and topological point of view and one can study it in the direction of double sequences also.

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