Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 23 Issue 3 Version 1.0 Year 2023
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Plane Wave Propagation and Fundamental Solution for Nonlocal Homogenous Isotropic Thermoelastic Media with Diffusion

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GJSFR-F Classification: DDC Code: 684.08 LCC Code: TT180

PLANEWAVEPROPAGATIONANDFUNDAMENTALSOLUTIONFORNONLOCALHOMOGENOUSISOTROPICTHERMOELASTICMEDIAWITHDIFFUSION

Strictly as per the compliance and regulations of:

Plane Wave Propagation and Fundamental Solution for Nonlocal Homogenous Isotropic Thermoelastic Media with Diffusion

Krishan Kumar [α], Deepa Gupta [σ], Sangeeta Malik [ρ], Raj Kumar Sharma [Ѡ] & Ankush Antil [¥]

Abstract- In the present problem, we study plane wave propagation and establish fundamental solution in the theory of nonlocal homogenous isotropic thermoelastic media with diffusion. We observe that there exists a set of three coupled waves namely longitudinal wave(P), thermal wave(T) and mass diffusion wave(MD) and one uncoupled transverse wave(SV) with different phase velocities. The effects of nonlocal parameter and diffusion on phase velocity, attenuation coefficient, penetration depth and specific loss have been studied numerically and presented graphically with respect to angular frequency. It is observed that characteristics of all the waves are influenced by the diffusion and nonlocal parameter. Fundamental solution of differential equations of motion in case of steady oscillations has been investigated and basic properties have also been discussed. Particular case of interest is also deduced from the present work and compared with the established result. The analysis of fundamental solution is very useful to investigate various problems of nonlocal thermoelastic solid with diffusion. The graphical analysis of current study is also very beneficial in order to investigate the different fields of geophysics, aerospace and electronics like seismology, manufacturing of aircrafts, volcanology, telecommunication etc.

Keywords: *nonlocal, diffusion, fundamental solution, steady oscillations.*

I. Introduction

It is well known that linear theory of elasticity describes the effective properties of various materials like steel, wood and concrete etc. But this theory is unable to explore the nano mechanical applications like nano structure vibrations, nano device stability etc. The theory of nonlocal elasticity is of great importance in determinig the properties of nano structure and wave propagation. The nonlocal theory of elasticity takes account of remote action between atoms because in nonlocal elasticity, stresses at a point not only depend on strain at that point but also on all points of the body. Eringen[1-3]elaborated the concept of nonlocality to elasicity and proposed the theory of nonlocal elasticity. Eringen and Edelen[4] obtained constitutive equations for the nonlinear theory. Gurtin[5] gave linear thermoelastic model to investigate the stresses produced to temperature field and distribution of temperature

Ref

4. Eringen A.C., Edelen G.B., On nonlocal elasticity. International Journal of engineering sciences.1972; 10: 233-248.

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due to action of internal forces. Nowacki[6-7] constructed asymptotic solution of boundary value problems of three dimensional micropolar theory of elasticity with free field of rotations and displacements. Green and Naghdi[8-9] introduced a new thermodynamical theory which uses a general entropy balance and discussed thermoelastic behaviour without energy dissipation. Kupradze et.al.[10] discussed three dimensional problem of the mathematical theory of elasticity and thermoelasticity. Kumar and Kumar[11] studied plane wave propagation in nonlocal micropolar thermoelastic material with voids. Kaur and Singh[12] studied propagation of plane wave in a nonlocal magneto-thermoelastic semiconductor solid with rotation and identified four types of reflected coupled longitudinal waves.

Diffusion is the spontaneous movement of anything generally from a region of higher concentration to that of lower concentration and thermal diffusion makes use of heat transfer. The thermoelastic diffusiion in elastic solids is due to coupling of mass diffusion field of temperature and that of strain in addition to mass and heat exchange with environment. Auoadi[13-16] derived equation of motion and constitutive equations for a generalized thermoelastic diffusion with one relaxation time and obtained variation principle for the governing equations. He proved uniqueness theorem for these equations by using Laplace transform. Free vibration of a thermoelastic diffusive cylinder was investigated by Sharma et al.[17]. Hörmander[18-19] contained analysed the partial differential operators which are very useful in order to find fundamental solution in the thermoelastic diffusion solid. To examine boundary value problem of thermoelasticity, it is mandatory to evaluate the fundamental solution of the system of partial differential equation and to discuss their basic properties. Fudamental solution in the classical theory of coupled thermoelasticity was firstly studied by Hetnarski[20-21]. Svanadze[22-25] obtained fundamental solution of equations of steady oscillations in different types of thermoelastic solids. Scarpetta[26], Ciarletta et al.[27], Svanadze et al.[28] found fundamental solution in the theory of micropolar elasticity. Fundamental solution in the theory of thermoelastic diffusion is established by Kumar and Kansal[29-30]. Many problems related to plane wave propagation and fundamental solution have been studied by some of other researchers like Sharma and Kumar[31], Kumar[32], Kumar et.al.[33], Kumar and Devi[34], Biswas[35], Kumar and Batra[36], Biswas[37-38], Kumar et al.[39], Poonam et al.[40], Kumar and Batra[41]. However, from the best of author's knowledge, no study has been done for investigating the combining effect of nonlocal and diffusion on fundamental solution of homogenous isotropic thermoelastic solid. In current problem, we have discussed plane wave propagation and established the fundamental solution of differential equations in case of steady oscillations in terms of elementary functions for nonlocal homogenous isotropic thermoelastic solid with diffusion. Some basic properties and special case are also discussed.

Ref

10. Kupradze V.D., Gegelia T.G., Basheleishvili MO, Burchuladze TV, Three dimensional problems of the mathematical theory of elasticity and thermoelasticity. North-Holand pub. company: amsterdam New-York.; 1979.

Notes

II. Basic Equations

In three dimensional Euclidean space E^3, let $\mathbf{X} = (x, y, z)$ be a point, t represents the time variable and $\mathbf{D}_x \equiv (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$. Following Eringen [1-3], the constitutive relations for nonlocal generalised thermoelastic solid with diffusion are given by

$$(1 - \varepsilon^2 \nabla^2)\, \sigma_{ij} = \sigma'_{ij} = 2\mu e_{ij} + [\lambda\, e_{kk} - \beta_1 T - \beta_2 C]\, \delta_{ij} \tag{1}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{2}$$

Using constitutive relations, equation of motion for nonlocal homogenous isotropic thermoelastic solid with diffusion is

$$\mu\, u_{i,jj} + (\lambda + \mu)\, u_{j,ij} - \beta_1 T_{,i} - \beta_2 C_{,i} = \rho(1 - \varepsilon^2 \nabla^2)\ddot{u}_i \tag{3}$$

Equations of heat conduction and mass diffusion for nonlocal homogenous isotropic thermoelastic solid with diffusion are given by

$$\rho\, C_E(\dot{T} + \tau_0 \ddot{T}) + \beta_1 T_0(\dot{e}_{kk} + \tau_0 \ddot{e}_{kk}) + a^* T_0(\dot{C} + \tau_0 \ddot{C}) = K T_{,ii} \tag{4}$$

$$D^* \beta_2\, e_{kk,ii} + D^* a^* T_{,ii} + (1 - \varepsilon^2 \nabla^2)(\dot{C} + \tau \ddot{C}) = D^* b^* C_{,ii} \tag{5}$$

where $\mathbf{u} = (u_1,\, u_2,\, u_3)$ is the displacement vector; σ_{ij} are the stress components and e_{ij} are components of strain tensor; e_{kk} is dilatation; σ'_{ij} corresponds to the local thermoelastic solid with diffusion; T is the temperature change measured from the absolute temperature T_0; C_E denotes specific heat at constant strain; K is the thermal conductivity; τ_0 is the relaxation time parameter and τ is the relaxation time of diffusion; C is the concentration; D^* is the thermoelastic diffusion constant; a^* and b^* respectively measures the thermo-diffusion effects and diffusive effects; ρ is mass density; β_1, β_2 are material coefficients with $\beta_1 = (3\lambda + 2\mu)\, \alpha_t$, $\beta_2 = (3\lambda + 2\mu)\, \alpha_c$; λ and μ are Lame's constants; α_t the coefficient of linear thermal expansion and α_c is the coefficient of linear diffusion expansion; ∇^2 denotes the Laplacian operator; $\varepsilon = e_0 a$ is the nonlocal parameter; e_0 corresponds to the material constant; a denotes the characteristic length; δ_{ij} is kronecker delta. In the above equations, superposed dot represents the derivative with respect to time and ',' in the subscript denotes the partial derivatives with respect to x, y, z for i, $j = 1, 2, 3$ respectively.

For two-dimensional problem, we will suppose that all quantities related to the medium are functions of cartesian coordinates x, z ($i.e. \frac{\partial}{\partial y} \equiv 0$) and time t and are independent of y. Displacement vector is considered as

$$\mathbf{u} = (u_1,\, 0,\, u_3) \tag{6}$$

We define the following dimensionless quantities

$$x' = \frac{\omega_1 x}{c_1},\ z' = \frac{\omega_1 z}{c_1},\ u_1' = \frac{\rho\omega_1 c_1}{\beta_1 T_0}u_1,\ u_3' = \frac{\rho\omega_1 c_1}{\beta_1 T_0}u_3,$$

$$t' = \omega_1 t,\ T' = \frac{T}{T_0},\ C' = \frac{\beta_2}{\beta_1 T_0}C,\ \tau_0' = \omega_1\tau_0,\ \tau' = \omega_1\tau \tag{7}$$

where $\omega_1 = \frac{\rho C_E c_1^2}{K},\ \ c_1 = \sqrt{\frac{\lambda+2\mu}{\rho}}$

Now using equation (7) in equations (3), (4), (5) and suppressing the primes, we obtain

$$\alpha_1\,\nabla^2\mathbf{u} + \alpha_2\,grad\,div\,\mathbf{u} - grad\,T - grad\,C = (1-\varepsilon_1^2\,\nabla^2)\,\ddot{\mathbf{u}} \tag{8}$$

$$\tau_t^0\,(\dot{T} + \alpha_3\,div\,\dot{\mathbf{u}} + \alpha_4\,\dot{C}) = \nabla^2 T \tag{9}$$

$$\alpha_5\,\nabla^2\,div\,\mathbf{u} + \alpha_6\,\nabla^2 T - \alpha_7\,\nabla^2 C + (1-\varepsilon_1^2\,\nabla^2)\,\tau_c^0\,\dot{C} = 0 \tag{10}$$

where

$$\alpha_1 = \frac{\lambda+\mu}{\lambda+2\mu},\ \alpha_2 = \frac{\mu}{\lambda+2\mu},\ \alpha_3 = \frac{\beta_1^2 T_0}{\rho K\omega_1},\ \alpha_4 = \frac{a^*\beta_1 T_0 c_1^2}{K\omega_1\beta_2},\ \alpha_5 = \frac{D^*\beta_2^2\omega_1}{\rho c_1^4},$$

$$\alpha_6 = \frac{D^* a^*\omega_1\beta_2}{\beta_1 c_1^2},\ \alpha_7 = \frac{D^* b^*\omega_1}{c_1^2}\ \varepsilon_1^2 = \frac{\varepsilon^2\omega_1^2}{c_1^2},\ \tau_t^0 = 1+\tau_0\,\omega_1\frac{\partial}{\partial t},\ \tau_c^0 = 1+\tau\,\omega_1\frac{\partial}{\partial t}$$

The displacement vector $\mathbf{u}$ is related to the potential functions $\phi_1(x,z,t)$ and $\phi_2(x,z,t)$ as

$$u_1 = \frac{\partial\phi_1}{\partial x} + \frac{\partial\phi_2}{\partial z},\qquad u_3 = \frac{\partial\phi_1}{\partial z} - \frac{\partial\phi_2}{\partial x} \tag{11}$$

Using equation (11) in equations (8)-(10), we obtain

$$(\alpha_1+\alpha_2)\nabla^2\phi_1 - T - C = (1-\varepsilon_1^2\nabla^2)\ddot{\phi_1} \tag{12}$$

$$\alpha_1\nabla^2\phi_2 = (1-\varepsilon_1^2\nabla^2)\ddot{\phi_2} \tag{13}$$

$$(\frac{\partial}{\partial t} + \tau_0\,\omega_1\frac{\partial^2}{\partial t^2})\,(T + \alpha_4\,C + \alpha_3\,\nabla^2\phi_1) = \nabla^2 T \tag{14}$$

$$\alpha_5\,\nabla^4\phi_1 + \alpha_6\nabla^2 T - \alpha_7\,\nabla^2 C + (1-\varepsilon_1^2\nabla^2)\,(\frac{\partial}{\partial t} + \tau\,\omega_1\frac{\partial^2}{\partial t^2})\,C = 0 \tag{15}$$

Equations (12), (14) and (15) show that ϕ_1, T and C are coupled and ϕ_2 remains decoupled.

III. Plane Wave

Notes

We consider a plane wave propagating in a nonlocal homogenous isotropic thermoelastic media with diffusion and assume the solution of the form

$$(\phi_1,\, \phi_2,\, T,\, C) = (\bar{\phi_1},\, \bar{\phi_2},\, \bar{T},\, \bar{C})\; exp\{ik(\mathbf{n}.\mathbf{r} - ct)\} \tag{16}$$

where $\omega = kc$ is the frequency, c is the wave velocity, k is the wave number, $\bar{\phi_1}$, $\bar{\phi_2}$, $\bar{T}$, $\bar{C}$ are undetermined amplitudes that depend on the time and co-ordinates $\mathbf{r} = (x, 0, z)$, $\mathbf{n}$ is the unit vector.

Using equation (16) in equations (12)-(15), we obtain

$$(B_1 k^2 + \omega^2)\bar{\phi_1} = \bar{T} + \bar{C} \tag{17}$$

$$(B_2 k^2 - \omega^2)\bar{\phi_2} = 0 \tag{18}$$

$$(k^2 - B_3)\bar{T} - \alpha_4 B_3 \bar{C} + \alpha_3 B_3 k^2 \bar{\phi_1} = 0 \tag{19}$$

$$\alpha_5 k^4 \bar{\phi_1} - \alpha_6 k^2 \bar{T} + (B_4 k^2 - B_5)\bar{C} = 0 \tag{20}$$

where

$$B_1 = \varepsilon_1^2 \omega^2 - 1, \quad B_2 = \alpha_1 - \varepsilon_1^2 \omega^2, \quad B_3 = i\omega + \tau_0\, \omega_1\, \omega^2$$

$$B_4 = \alpha_7 - \varepsilon_1^2 (i\omega) - \varepsilon_1^2 \tau\, \omega_1\, \omega^2, \quad B_5 = i\omega + \tau\, \omega_1\, \omega^2$$

Solving equations (17), (19) and (20) for $\bar{\phi_1}$, $\bar{T}$, $\bar{C}$ we obtain a cubic equation in k^2 as

$$F_1 k^6 + G_1 k^4 + H_1 k^2 + J_1 = 0 \tag{21}$$

where

$$F_1 = B_4 B_1 + \alpha_5, \quad J_1 = B_3 B_5 \omega^2$$

$$G_1 = \alpha_3\alpha_6 B_3 + B_4\omega^2 - B_1 B_3 B_4 + \alpha_3 B_3 B_4 - B_5 B_1 + \alpha_4\alpha_5 B_3 - \alpha_4\alpha_6 B_1 B_3 - \alpha_5 B_3$$

$$H_1 = -B_4 B_3 \omega^2 - B_5 \omega^2 + B_5 B_3 B_1 - \alpha_3 B_3 B_5 - \alpha_4 \alpha_6 B_3 \omega^2$$

Solving equation (21), we obtain six values of k in which three values k_1, k_2, k_3 correspond to positive z-direction and the other three values of k correspond to negative z-direction. Corresponding to k_1, k_2 and k_3 there exist three coupled waves, namely, longitudinal wave(P), thermal wave(T) and mass diffusion wave(MD).

The expressions for the phase velocity, attenuation coefficients, penetration depth and specific loss of above waves are evaluated as

Phase Velocity: The phase velocities v_1, v_2 and v_3 of P-wave, T-wave, and MD-wave, respectively, are given by

$$v_j = \frac{\omega}{|Re(k_j)|} \quad j = 1, 2, 3 \tag{22}$$

Attenuation Coefficients: The attenuation coefficients Q_1, Q_2 and Q_3 of P-wave, T-wave and MD-wave, respectively, can be written as

$$Q_j = Im(k_j) \quad j = 1, 2, 3 \tag{23}$$

Penetration Depth: The penetration depth D_1, D_2 and D_3 of P-wave, T-wave and MD-wave, respectively, is defined as

$$D_j = \frac{1}{|Im(k_j)|} \quad j = 1, 2, 3 \tag{24}$$

Specific Loss: The Specific Loss L_1, L_2 and L_3 of P-wave, T-wave and MD-wave, respectively, are given by

$$L_j = 4\pi|\frac{Re(k_j)}{Im(k_j)}| \quad j = 1, 2, 3 \tag{25}$$

Solving equation(18), we obtain two values of k in which one value k_4 corresponds to positive z-direction representing transverse wave(SV) and other value of k corresponds to negative z-direction. The phase velocity of transverse wave is given by $v_4 = \sqrt{B_2}$

IV. Steady Oscillations

Assume that displacement vector, temperature change and concentration are functions as

$$(\mathbf{u}(x,z,t), T(x,z,t), C(x,z,t)) = Re[(\mathbf{u}^*(x,z,t), T^*(x,z,t), C^*(x,z,t))e^{-i\omega t}] \tag{26}$$

Using equation (26) in equations (8),(9),(10), we obtain following system of equations of steady oscillations

$$[(\alpha_1 - \omega^2\varepsilon_1^2)\nabla^2 + \omega^2]\mathbf{u}^* + \alpha_2\, grad\, div\, \mathbf{u}^* - grad\, T^* - grad\, C^* = 0 \tag{27}$$

$$-\tau_t^{01}[\alpha_3\, div\, \mathbf{u}^* + \alpha_4\, C^*] + (\nabla^2 - \tau_t^{01})T^* = 0 \tag{28}$$

$$\alpha_5\, \nabla^2\, div\, \mathbf{u}^* + \alpha_6\, \nabla^2 T^* + [\tau_c^{01} - (\alpha_7 + \varepsilon_1^2\, \tau_c^{01})\, \nabla^2]\, C^* = 0 \tag{29}$$

Notes

Ref

18. Hörmander L., Linear partial differential operators. Berlin: Springerverlag; 1963.

where

$$\tau_t^{01} = -i\omega(1 - i\omega\tau_0), \quad \tau_c^{01} = -i\omega(1 - i\omega\tau)$$

We define matrix differential operator

$$\mathbf{B}(\mathbf{D}_x) = \left[B_{mn}(\mathbf{D}_x)\right]_{4\times 4} \tag{30}$$

where

$$B_{m1}(\mathbf{D}_x) = [(\alpha_1 - \omega^2\varepsilon_1^2)\nabla^2 + \omega^2]\delta_{m1} + \alpha_2 \frac{\partial^2}{\partial x\,\partial x^*}$$

$$B_{m2}(\mathbf{D}_x) = [(\alpha_1 - \omega^2\varepsilon_1^2)\nabla^2 + \omega^2]\delta_{m2} + \alpha_2 \frac{\partial^2}{\partial z\,\partial x^*}$$

$$B_{m3}(\mathbf{D}_x) = B_{m4}(\mathbf{D}_x) = -\frac{\partial}{\partial x^*}, \quad B_{3n}(\mathbf{D}_x) = -\tau_t^{01}\,\alpha_3 \frac{\partial}{\partial x^*}$$

$$B_{4n}(\mathbf{D}_x) = \alpha_5\,\nabla^2 \frac{\partial}{\partial x^*}, \quad B_{33}(\mathbf{D}_x) = \nabla^2 - \tau_t^{01}, \quad B_{34}(\mathbf{D}_x) = \tau_t^{01}\,\alpha_4,$$

$$B_{43}(\mathbf{D}_x) = \alpha_2\,\nabla^2, \quad B_{44}(\mathbf{D}_x) = -\alpha_1\,\nabla^2 + (1 - \varepsilon_1^2\,\nabla^2)\tau_c^{01}; \quad m, n = 1, 2$$

For $m = n = 1$, $x^* = x$ and for $m = n = 2$, $x^* = z$; δ_{mn} is kronecker's delta.
The system of equations (27)-(29) can be written as

$$\mathbf{B}(\mathbf{D}_x)\mathbf{V}(\mathbf{X}) = 0 \tag{31}$$

where $\mathbf{V} = (u_1^*,\, u_3^*,\, T^*,\, C^*)$ ia a four component vector function.
Assume that

$$-(\alpha_1 + \alpha_2 - \omega^2\,\varepsilon_1^2)(\alpha_1 - \omega^2\varepsilon_1^2)\,(\alpha_1 + \varepsilon_1^2\tau_c^{01}) \neq 0 \tag{32}$$

If condition (32) is satisfied then $\mathbf{B}$ is an elliptic differential operator (Hörmander [18]).

Definition. The fundamental solution of system of equations (27)-(29) is the matrix $\mathbf{A}(\mathbf{X}) = \left[A_{ij}(\mathbf{X})\right]_{4\times 4}$ satisfying the condition

$$\mathbf{B}(\mathbf{D}_x)\mathbf{A}(\mathbf{X}) = \delta(\mathbf{X})\mathbf{I}(\mathbf{X}) \tag{33}$$

where δ is Dirac delta, $\mathbf{I} = \left[\delta_{ij}\right]_{4\times 4}$ is the unit matrix.
We now construct $\mathbf{A}(\mathbf{X})$ in terms of elementary functions.

V. Fundamental Solution of System of Equations of Steady Oscillations

We consider the system of equations

$$[(\alpha_1-\omega^2\varepsilon_1^2)\nabla^2+\omega^2]\mathbf{u}^*+\alpha_2\, grad\, div\, \mathbf{u}^*-\tau_t^{01}\,\alpha_3\, grad\, T^*+\alpha_5\,\nabla^2\, grad\, C^*=\mathbf{J'} \tag{34}$$

$$-div\,\mathbf{u}^*+(\nabla^2-\tau_t^{01})T^*+\alpha_2\nabla^2C^*=L \tag{35}$$

$$-div\,\mathbf{u}^*-\tau_t^{01}\alpha_4\,T^*+[-\alpha_7\,\nabla^2+(1-\varepsilon_1^2\,\nabla^2)\tau_c^{01}]C^*=M \tag{36}$$

where $\mathbf{J'}$ is a vector function on E^3 and L, M are scalar functions on E^3. The system of equations (34)-(36) may be written in the following form

$$\mathbf{B}^{tr}(\mathbf{D}_x)\mathbf{V}(\mathbf{X})=\mathbf{G}(\mathbf{X}) \tag{37}$$

where $\mathbf{B^{tr}}$ is the transpose of matrix $\mathbf{B}$ and $\mathbf{G}=(\mathbf{J'},\ L,\ M)$
Applying operator div to the equation (34), we get

$$[(1-\omega^2\varepsilon_1^2)\nabla^2+\omega^2]\, div\,\mathbf{u}^*-\tau_t^{01}\,\alpha_3\,\nabla^2T^*+\alpha_5\,\nabla^4\,C^*=div\,\mathbf{J'} \tag{38}$$

$$-div\,\mathbf{u}^*+(\nabla^2-\tau_t^{01})T^*+\alpha_2\nabla^2C^*=L \tag{39}$$

$$-div\,\mathbf{u}^*-\tau_t^{01}\alpha_4\,T^*+[-\alpha_7\,\nabla^2+(1-\varepsilon_1^2\,\nabla^2)\tau_c^{01}]C^*=M \tag{40}$$

equations (38)-(40) may be expressed as

$$D(\nabla^2)\,\mathbf{P}=\mathbf{Q} \tag{41}$$

where $\mathbf{P}=(div\,\mathbf{u}^*,\ T^*,\ C^*)$, $\mathbf{Q}=(div\,\mathbf{J'},\ L,\ M)=(d_1,\ d_2,\ d_3)$ and

$$\mathbf{D}(\nabla^2)=[D_{mn}]_{3\times3}$$

$$=\begin{bmatrix}(1-\omega^2\varepsilon_1^2)\nabla^2+\omega^2 & -\tau_t^{01}\,\alpha_3\,\nabla^2 & \alpha_5\,\nabla^4\\ -1 & \nabla^2-\tau_t^{01} & \alpha_2\nabla^2\\ -1 & -\tau_t^{01}\alpha_4 & -\alpha_7\,\nabla^2+(1-\varepsilon_1^2\,\nabla^2)\tau_c^{01}\end{bmatrix}_{3\times3} \tag{42}$$

equations (38)-(40) may be expressed as

$$\Gamma_1(\nabla^2)\,\mathbf{P}=\sigma \tag{43}$$

Notes

Notes

where

$$\sigma = (\sigma_1,\, \sigma_2,\, \sigma_3); \quad \sigma_n = e_1^* \sum_{m=1}^{3} D_{mn}^*\, d_m$$

$$\Gamma_1(\nabla^2) = e_1^*\, det\, \mathbf{D}(\nabla^2), \;\; e_1^* = -\frac{1}{(1-\omega^2\varepsilon_1^2)\,(\alpha_7+\varepsilon_1^2\,\tau_c^{01})} \tag{44}$$

and D_{mn}^* is the cofactor of elements D_{mn} of matrix $\mathbf{D}$
From equations (42) and (44), we have

$$\Gamma_1(\nabla^2) = \prod_{m=1}^{3} (\nabla^2 + \Lambda_m^2)$$

where $\Lambda_m^2,\; m = 1, 2, 3$ are roots of equation $\Gamma_1\,(-r) = 0$ (with respect to r)
Applying $\Gamma_1(\nabla^2)$ to the equation (27) and using equation (43), we obtain

$$\Gamma_1(\nabla^2)\,[(\alpha_1 - \omega^2\varepsilon_1^2)\nabla^2 + \omega^2]\mathbf{u}^* = -\alpha_2\, grad\,\sigma_1 + grad\,\sigma_2 + grad\,\sigma_3 \tag{45}$$

This equation may also be written as

$$\Gamma_1(\nabla^2)\,\Gamma_2(\nabla^2)\,\mathbf{u}^* = \sigma' \tag{46}$$

where

$$\Gamma_2(\nabla^2) = (\alpha_1 - \omega^2\varepsilon_1^2)\nabla^2 + \omega^2 \tag{47}$$

and

$$\sigma' = -\alpha_2\, grad\,\sigma_1 + grad\,\sigma_2 + grad\,\sigma_3 \tag{48}$$

It can be written as

$$\Gamma_2\,(\nabla^2) = (\nabla^2 + \Lambda_4^2) \tag{49}$$

where Λ_4^2 is a root of equation $\Gamma_2\,(-r) = 0$ (with respect to r)
From equations (43) and (46), we can write

$$\boldsymbol{\Theta}(\nabla^2)\,\mathbf{V}(\mathbf{X}) = \hat{\sigma}(\mathbf{X}) \tag{50}$$

where $\hat{\sigma}(\mathbf{X}) = (\sigma',\, \sigma_2,\, \sigma_3)$ and $\boldsymbol{\Theta}(\nabla^2) = \left[\Theta_{gh}(\nabla^2)\right]_{4\times4}$

$$\Theta_{mm}(\nabla^2) = \Gamma_1(\nabla^2)\,\Gamma_2(\nabla^2)$$

$$\Theta_{33} = \Theta_{44} = \Gamma_1(\nabla^2)$$

$$\Theta_{gh}(\nabla^2) = 0$$

$$m = 1, 2; \quad g, h = 1, 2, 3, 4; \quad g \neq h$$

Equations (41),(44) and (48) can also be written as

$$\sigma' = c_{11}(\nabla^2)\, grad\, div\, \mathbf{J}' + c_{21}(\nabla^2)\, grad\, L + c_{31}(\nabla^2)\, grad\, M \tag{51}$$

$$\sigma_2 = c_{12}(\nabla^2)\, div\, \mathbf{J}' + c_{22}(\nabla^2)\, L + c_{32}(\nabla^2)\, M \tag{52}$$

$$\sigma_3 = c_{13}(\nabla^2)\, div\, \mathbf{J}' + c_{23}(\nabla^2)\, L + c_{33}(\nabla^2)\, M \tag{53}$$

where

$$c_{11}(\nabla^2) = -\alpha_2\, e_1^*\, D_{11}^* + e_1^*\, D_{12}^* + e_1^*\, D_{13}^*, \quad c_{12} = e_1^*\, D_{12}^*, \quad c_{13} = e_1^*\, D_{13}^*,$$

$$c_{21}(\nabla^2) = -\alpha_2\, e_1^*\, D_{21}^* + e_1^*\, D_{22}^* + e_1^*\, D_{23}^*, \quad c_{22} = e_1^*\, D_{22}^*, \quad c_{23} = e_1^*\, D_{23}^*,$$

$$c_{31}(\nabla^2) = -\alpha_2\, e_1^*\, D_{31}^* + e_1^*\, D_{32}^* + e_1^*\, D_{33}^*, \quad c_{32} = e_1^*\, D_{32}^*, \quad c_{33} = e_1^*\, D_{33}^*$$

From equations (51)-(53), we get

$$\hat{\sigma}(\mathbf{X}) = \mathbf{H}^{tr}\,(\mathbf{D}_x)\,\mathbf{G}(\mathbf{X}) \tag{54}$$

where

$$\mathbf{H} = \left[H_{gh}\right]_{4X4}$$

$$H_{m1}(\mathbf{D}_x) = c_{11}(\nabla^2)\,\frac{\partial^2}{\partial x\,\partial x^*}, \quad H_{m2}(\mathbf{D}_x) = c_{11}(\nabla^2)\,\frac{\partial^2}{\partial z\,\partial x^*}, \quad H_{m3}(\mathbf{D}_x) = c_{21}(\nabla^2)\,\frac{\partial}{\partial x^*},$$

$$H_{m4}(\mathbf{D}_x) = c_{31}(\nabla^2)\,\frac{\partial}{\partial x^*}, \quad H_{3n}(\mathbf{D}_x) = c_{12}(\nabla^2)\,\frac{\partial}{\partial x^*}, \quad H_{4n}(\mathbf{D}_x) = c_{13}(\nabla^2)\,\frac{\partial}{\partial x^*},$$

$$H_{33}(\mathbf{D}_x) = c_{22}(\nabla^2), \quad H_{34}(\mathbf{D}_x) = c_{32}(\nabla^2), \quad H_{43}(\mathbf{D}_x) = c_{43}(\nabla^2),$$

$$H_{44}(\mathbf{D}_x) = c_{44}(\nabla^2); \qquad m, n = 1, 2 \tag{55}$$

Notes

Notes

For $m = n = 1, \;\; x^* = x$ and for $m = n = 2, \;\; x^* = z$
From equations (37), (46) and (50), we obtain

$$\mathbf{\Theta}\,\mathbf{V} = \mathbf{H}^{tr}\,\mathbf{B}^{tr}\,\mathbf{V}$$

Above equation can be rewritten as

$$\mathbf{H}^{tr}\,\mathbf{B}^{tr} = \mathbf{\Theta}$$

Therefore, we have

$$\mathbf{B}(\mathbf{D}_x)\,\mathbf{H}(\mathbf{D}_x) = \mathbf{\Theta}(\nabla^2) \tag{56}$$

we assume that

$$\lambda_m^2 \neq \lambda_n^2 \neq 0; \quad m, n = 1, 2, 3, 4; \quad m \neq n$$

We now define

$$\mathbf{W}(\mathbf{X}) = \left[W_{rs}(\mathbf{X})\right]_{4\times 4}$$

$$W_{mm}(\mathbf{X}) = \sum_{n=1}^{4} q_{1n}\xi_n(\mathbf{X}), \;\; W_{33}(\mathbf{X}) = W_{44}(\mathbf{X}) = \sum_{n=1}^{3} q_{2n}\xi_n(\mathbf{X}), \;\; W_{uv}(\mathbf{X}) = 0$$

$m = 1, 2; \quad u, v = 1, 2, 3, 4; \quad u \neq v$

where

$$\xi_n(\mathbf{X}) = -\frac{1}{4\,\Pi\,|\mathbf{X}|}\,\exp\left(i\Lambda_n|\mathbf{X}|\right), \quad n = 1, 2, 3, 4$$

$$q_{1l} = \prod_{m=1,\; m\neq l}^{4} (\Lambda_m^2 - \Lambda_l^2)^{-1}, \quad l = 1, 2, 3, 4$$

$$q_{2u} = \prod_{m=1,\; m\neq u}^{3} (\Lambda_m^2 - \Lambda_u^2)^{-1}, \quad u = 1, 2, 3 \tag{57}$$

Now, we prove the following Lemma:

Lemma: The matrix $\mathbf{W}$ defined above is the fundamental matrix of operator $\mathbf{\Theta}(\nabla^2)$, that is

$$\mathbf{\Theta}(\nabla^2)\,\mathbf{W}(\mathbf{X}) = \delta(\mathbf{X})\,\mathbf{I}(\mathbf{X}) \tag{58}$$

Proof: To prove the Lemma, it is sufcient to show that

$$\Gamma_1(\nabla^2)\,\Gamma_2(\nabla^2)\,W_{11}(\mathbf{X}) = \delta(\mathbf{X})$$

$$\Gamma_1(\nabla^2)\,W_{33}(\mathbf{X}) = \delta(\mathbf{X}) \tag{59}$$

Consider

$$q_{21} + q_{22} + q_{23} = \frac{-g_1 + g_2 - g_3}{g_4}$$

where

$$g_1 = (\Lambda_2^2 - \Lambda_3^2), \quad g_2 = (\Lambda_1^2 - \Lambda_3^2), \quad g_3 = (\Lambda_1^2 - \Lambda_2^2),$$

$$g_4 = (\Lambda_1^2 - \Lambda_2^2)\,(\Lambda_1^2 - \Lambda_3^2)(\Lambda_2^2 - \Lambda_3^2)$$

Solving above relations, we get

$$q_{21} + q_{22} + q_{23} = 0 \tag{60}$$

Similarly, from equation (57) we can also find out

$$q_{22}\,(\Lambda_1^2 - \Lambda_2^2) + q_{23}\,(\Lambda_1^2 - \Lambda_3^2) = 0 \tag{61}$$

$$q_{23}(\Lambda_1^2 - \Lambda_3^2)\,(\Lambda_2^2 - \Lambda_3^2) = 1 \tag{62}$$

Also, we have

$$(\nabla^2 + \Lambda_m^2)\,\xi_n(\mathbf{X}) = \delta(\mathbf{X}) + (\Lambda_m^2 - \Lambda_n^2)\,\xi_n(\mathbf{X}), \qquad m, n = 1, 2, 3 \tag{63}$$

Now consider

$\Gamma_1(\nabla^2)\,W_{33}(\mathbf{X})$

$$= (\nabla^2 + \Lambda_1^2)\,(\nabla^2 + \Lambda_2^2)\,(\nabla^2 + \Lambda_3^2)\sum_{n=1}^{3} q_{2n}\,\xi_n(\mathbf{X})$$

$$= (\nabla^2 + \Lambda_2^2)\,(\nabla^2 + \Lambda_3^2)\sum_{n=1}^{3} q_{2n}\,[\delta(\mathbf{X}) + (\Lambda_1^2 - \Lambda_n^2)\,\xi_n(\mathbf{X})]$$

$$= (\nabla^2 + \Lambda_2^2)\,(\nabla^2 + \Lambda_3^2)\,[\,\delta(\mathbf{X})\sum_{n=1}^{3} q_{2n} + \sum_{n=2}^{3} q_{2n}\,(\Lambda_1^2 - \Lambda_n^2)\,\xi_n(\mathbf{X})]$$

Notes

$$= (\nabla^2 + \Lambda_2^2)\,(\nabla^2 + \Lambda_3^2)\,\sum_{n=2}^{3} q_{2n}\,(\Lambda_1^2 - \Lambda_n^2)\,\xi_n(\mathbf{X})$$

$$= (\nabla^2 + \Lambda_3^2)\sum_{n=2}^{3} q_{2n}\,(\Lambda_1^2 - \Lambda_n^2)\,[\delta(\mathbf{X}) + (\Lambda_2^2 - \Lambda_n^2)\,\xi_n(\mathbf{X})]$$

Notes

$$= (\nabla^2 + \Lambda_3^2)\sum_{n=3}^{3} q_{2n}\,(\Lambda_1^2 - \Lambda_n^2)\,(\Lambda_2^2 - \Lambda_n^2)\,\xi_n(\mathbf{X})$$

$$= (\nabla^2 + \Lambda_3^2)\,q_{23}\,(\Lambda_1^2 - \Lambda_3^2)\,(\Lambda_2^2 - \Lambda_3^2)\,\xi_3(\mathbf{X})$$

$$= (\nabla^2 + \Lambda_3^2)\,\xi_3(\mathbf{X})$$

$$= \delta(\mathbf{X})$$

Similarly, equation $(59)_1$ can be proved
Now, Define matrix

$$\mathbf{A}(\mathbf{X}) = \mathbf{H}(\mathbf{D}_x)\,\mathbf{W}(\mathbf{X}) \tag{64}$$

Using equations (56), (58) and (64), we obtain

$$\mathbf{B}(\mathbf{D}_x)\,\mathbf{A}(\mathbf{X}) = \mathbf{B}(\mathbf{D}_x)\,\mathbf{H}(\mathbf{D}_x)\,\mathbf{W}(\mathbf{X}) = \boldsymbol{\Theta}(\nabla^2)\,\mathbf{W}(\mathbf{X}) = \delta(\mathbf{X})\,\mathbf{I}(\mathbf{X}) \tag{65}$$

Therefore, $\mathbf{A}(\mathbf{X})$ is solution of equation (33).
Hence, we have proved the following Theorem:

Theorem: The matrix $\mathbf{A}(\mathbf{X})$ defined by the equation (64) is the fundamental solution of system of equations (27)-(29).

VI. Basic Properties of the Matrix A(X)

Property 1. Every column of the matrix $\mathbf{A}(\mathbf{X})$ is the solution of equations (27)-(29) for all points $\mathbf{X}\,\epsilon\,E^3$ except the origin.

Property 2. The matrix $\mathbf{A}(\mathbf{X})$ can be written as

$$\mathbf{A} = [A_{rs}]_{4\times 4}$$

$$\mathbf{A}_{pq}(\mathbf{X}) = \mathbf{H}_{pq}(\mathbf{D}_x)\,W_{11}(\mathbf{X}),$$

$$\mathbf{A}_{pm}(\mathbf{X}) = \mathbf{H}_{pm}(\mathbf{D}_x)\,W_{33}(\mathbf{X}),$$

$$p = 1, 2, 3, 4; \quad q = 1, 2; \quad m = 3, 4.$$

VII. Special Case

If we neglect nonlocal parameter ($\varepsilon = 0$) in equations (27)-(29), we obtain the system of equations of steady state oscillations for homogenous isotropic generalized thermoelastic solid with diffusion as:

$$[\alpha_1 \nabla^2 + \omega^2]\mathbf{u}^* + \alpha_2\, grad\, div\, \mathbf{u}^* - grad\, T^* - grad\, C^* = 0 \tag{66}$$

$$-\tau_t^{01}[\alpha_3\, div\, \mathbf{u}^* + \alpha_4\, C^*] + (\nabla^2 - \tau_t^{01})T^* = 0 \tag{67}$$

$$\alpha_5\, \nabla^2\, div\, \mathbf{u}^* + \alpha_6\, \nabla^2 T^* + [\tau_c^{01} - \alpha_7\, \nabla^2]\, C^* = 0 \tag{68}$$

The fundamental solution of above system of equations is similar as obtained by Kumar and Kansal [29].

VIII. Numerical Results and Discussion

For numerical calculations, values of relevant parameters for homogenous isotropic generalized thermoelastic solid with diffusion have been swiped from Sharma et al. [17] given in Table 1.

Phase velocity, attenuation coefficient, penetration depth and specific loss are computed numerically by using software MATLAB. Variation of Phase velocity, attenuation coefficient, penetration depth and specific loss with respect to angular frequency of P-wave, T-wave, MD-wave and SV-wave are shown graphically in four types of elastic solids:

Table 1: Numerical values of parametres

Notation	value	Notation	value
λ	$7.76 \times 10^{10}\, Nm^{-2}$	α_t	$1.78 \times 10^{-5}\, K^{-1}$
μ	$3.86 \times 10^{10}\, Nm^{-2}$	α_c	$2.65 \times 10^{-4}\, m^3 Kg^{-1}$
K	$386\, Jm^{-1}S^{-1}K^{-1}$	e_0	0.38
C_E	$383.1\, JKg^{-1}K^{-1}$	a	0.5431×10^{-9}
ρ	$8.954 \times 10^3\, Kgm^{-3}$	T_0	$293\, K$
a^*	$1.2 \times 10^4\, m^2 KS^2$	τ_0	$0.5\, S$
b^*	$0.9 \times 10^6\, Kgm^5 S^2$	τ	$0.5\, S$
D^*	$0.88 \times 10^{-8}\, KgSm^{-3}$		

1. Nonlocal thermoelastic solid with diffusion
2. Local thermoelastic solid with diffusion
3. Nonlocal thermoelastic solid without diffusion
4. Local thermoelastic solid without diffusion

In all graphs, solid line and dashed line represent the impact of local and nonlocal parameter on the variation of phase velocity, attenuation coefficient, penetration depth and specific loss in thermoelastic solid without diffusion respectively whereas dash-dotted and dotted line represent variation in local and nonlocal thermoelastic solid with diffusion respectively.

Ref

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Notes

Phase velocity: Figures 1-3 represent the impact of nonlocal and diffusion parameter on the phase velocities v_1, v_2 and v_3 of P-wave, T-wave, and MD-wave respectively with respect to angular frequency ω. From figures 1-2, it is clear that behaviour of phase velocity of P-wave and T-wave with respect to angular frequency is same with difference in magnitude values. The phase velocities v_1 and v_2 decreases monotonically, reaches to minimum value at $\omega = 10^0$ for all types of thermoelastic solids. The values of v_1 and v_2 for local solid are more than that of nonlocal solid. From physical point of view, the stresses produced in nonlocal medium is weak due to impact of nano-structured particles and change in stress causes the change in wave characteristics accordingly. Also phase velocity has lower value in elastic solid with diffusion in comparison to elastic solid without diffusion. Thermoelastic waves exhibit different dispersion characteristics in thermodiffusive solid which in turn influence the Phase velocity, attenuation coefficient, penetration depth and specific loss. Figure 3 shows that phase velocity v_3 of

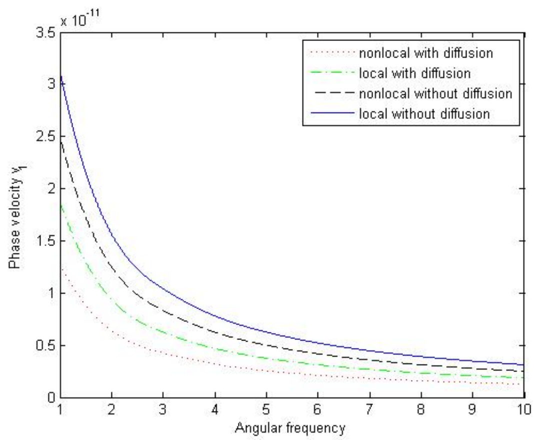

Figure 1: Variation of phase velocity of P-wave

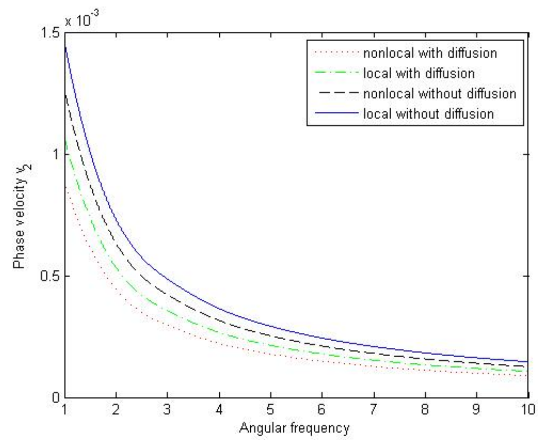

Figure 2: Variation of phase velocity of T-wave

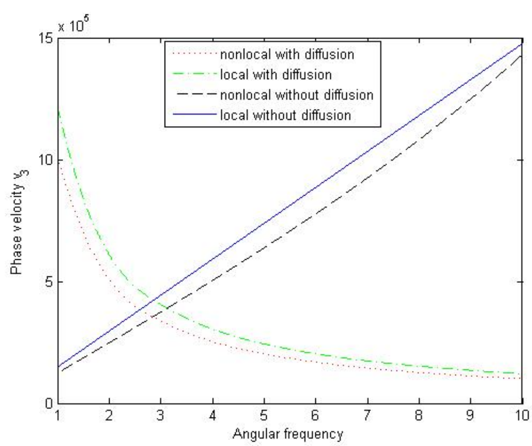

Figure 3: Variation of phase velocity of MD-wave

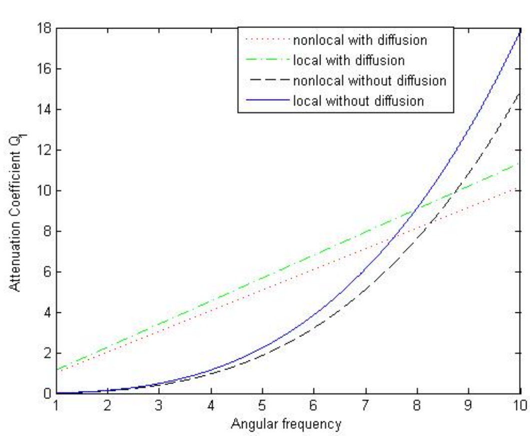

Figure 4: Variation of attenuation coefficient of P-wave

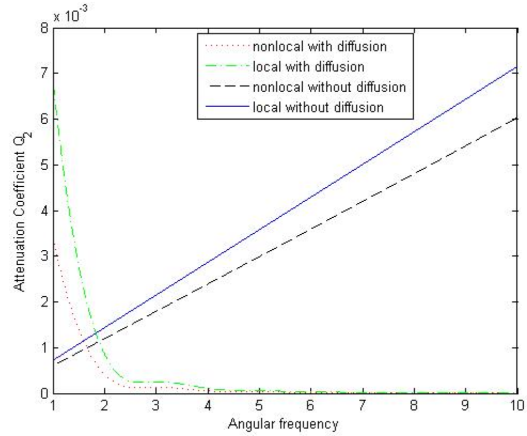

Figure 5: Variation of attenuation coefficient of T-wave

MD-wave decreases monotonically reaches to minimum value at $\omega = 10^0$ for thermoelastic solid with diffusion whereas it increases linearly, reaches to maximum value at $\omega = 10^0$ for elastic solid without diffusion.

Attenuation Coefficients: Figures 4-6 represent the impact of nonlocal and diffusion parameter on the attenuation coefficients Q_1, Q_2 and Q_3 of P-wave, T-wave, and MD-wave respectively with respect to angular frequency ω. It has been observed from the figure 4 that attenuation coefficient Q_1 of P-wave increases parabolically for $1^0 \leq \omega \leq 10^0$ in elastic solid without diffusion. For thermoelastic solid with diffusion, Q_1 increases linearly in local as well as nonlocal solid. Figure 5 shows that attenuation coefficient Q_2 of T-wave increases linearly with increase in ω in thermoelastic solid without diffusion whereas in thermodiffusive solid, it decrease sharply for $1^0 \leq \omega \leq 2.5^0$. From figure 6, it is clear that attenuation coefficient Q_3 of MD-wave increases linearly in all types of solids with difference in magnitude. The value of Q_3 is lower in diffusive solid in comparison to solid without diffusion. Also, figures 4-6 show that attenuation coefficients has smaller value in nonlocal solid in comparison to local elastic solid due to nonlocal parameter.

Penetration Depth: Figures 7-9 represent the effect of diffusion and nonlocal parameter on the penetration depth D_1, D_2 and D_3 of P-wave, T-wave,

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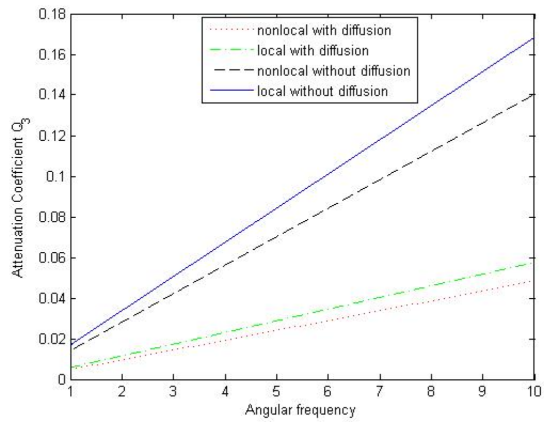

Figure 6: Variation of attenuation coefcient of MD-wave

and MD-wave respectively with respect to angular frequency ω. Figure 7 depicts the variation of penetration depth D_1 of P-wave with respect to angular frequency ω. In thermoelastic solid without diffusion, the penetration depth D_1 decreases sharply for $1^0 \leq \omega \leq 2.5^0$ and then slowly for $\omega \geq 2.5^0$. The value of D_1 decreases monotonically, reaches to minimum value at $\omega = 10^0$ in diffusive elastic solid. It has been observed from the figure 8 that penetration depth D_2 of T-wave increases in diffusive solid and decreases in solid without diffusion with increase in angular frequency ω having greater value in local solid in comparison to nonlocal elastic solid. From figure 9, it is clear that penetration depth D_3 of MD-wave decreases monotonically in all types of solids with difference in magnitude. Also, the value of D_3 is lower in diffusive solid in comparison to solid without diffusion.

Specific Loss: Figures 10-12 show the effect of diffusion and nonlocal parameter on the specific loss L_1, L_2 and L_3 of P-wave, T-wave, and MD-wave respectively with respect to angular frequency ω. From figures 10 and 12, it has been observed that behaviour of specific loss L_1 of P-wave is similar to behaviour of specific loss L_3 of MD-wave with difference in magnitude. Specific loss L_1 and L_3 decreases monotonically, reaches to minimum value at $\omega = 10^0$ in thermoelastic solid without diffusion whereas it increases linearly, reaches to maximum value at $\omega = 10^0$ in thermodiffusive solid. Figure 11 represents the variation of specific loss L_2 of T-wave with respect to an-

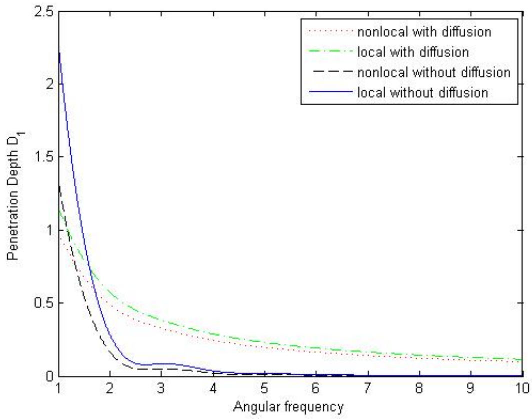

Figure 7: Variation of penetration depth of P-wave

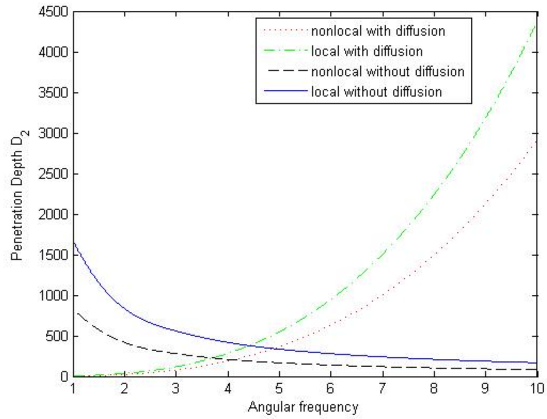

Figure 8: Variation of penetration depth of T-wave

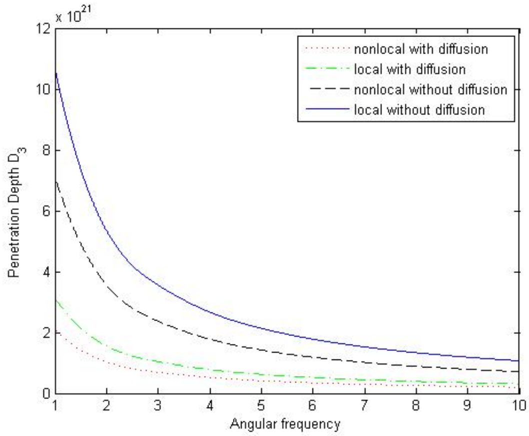

Figure 9: Variation of penetration depth of MD-wave

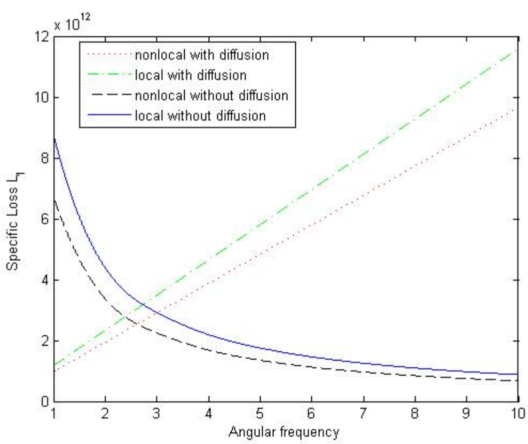

Figure 10: Variation of specific loss of P-wave

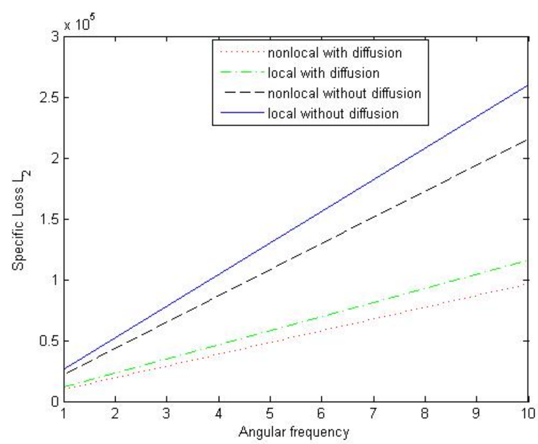

Figure 11: Variation of specific loss of T-wave

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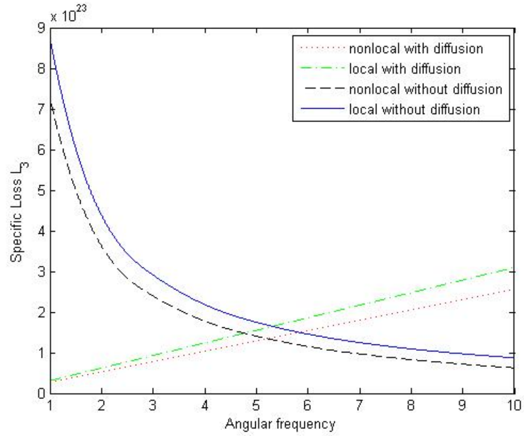

Figure 12: Variation of specific loss of MD-wave

gular frequency ω. In all types of solids, L_2 increases linearly, having lower value in diffusive solid as comparison to solid without diffusion. Also from figures 10-12, it is clear that specific loss has greater value in local solid as comparison to nonlocal elastic solid.

IX. Conclusion

We have examined the effects of nonlocal parameter on the propagation of plane wave in nonlocal homogenous isotropic thermoelastic diffusion.
The major consequences of current problems are:
(1)There exist three coupled waves namely P-wave, T-wave, MD-wave and one transverse wave(SV) propagating with different phase velocities. Furthermore phase velocity, attenuation coefficients, penetration depth and specific loss with respect to angular frequency are studied graphically.
(2) It has been found that characteristics of all the waves are affected by diffusion and nonlocal parameter of the medium.
(3) The fundamental solution of system of differential equations for steady oscillations has been constructed.
(4) The analysis of fundamental solution $\mathbf{M}(\mathbf{X})$ of the system of equations (27)-(29) are helpful to investigate three dimensional problems of nonlocal homogenous isotropic elastic solid with diffusion.
(5) The graphical analysis of present work is very helpful in order to investigate the various fields of aerospace, electronics and geophysics like volcanology, telecommunication etc.

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