Quark-Colorization of Cabibbo-Kobayashi-Maskawa Matrix CKM

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Abstract - This paper proposes an interesting representation $V^{CKM}(q_{RGB}, \Phi, \xi)$ of Cabibbo-Kobayashi-Maskawa Matrix CKM, which based on scalar products of quark color quantum numbers $q_R, q_G, q_B,$ or $q_{RGB}$ (00.1). This representation is called colorization of CKM in weak interaction. The colors of down-type quarks $q_w, (w=d, s, b)$ in the quarkcolor scalar products of CKM are "Color-Broken, $\xi \neq 0,$ which results in isospin $I_3(q_w)$ to be violated in weak interaction, further charges $Q^{CKM}_{dub}(\xi_w), \text{ to be a slight deviated from } \frac{1}{3} e$ of SM theoretical value. A short discussion of possible existence of higher-charges of quark $q$ is given in Epilogue.

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This paper proposes an interesting representation $V^{\text{CKM}}(q_{RGB}, \Phi, \zeta)$ of Cabibbo-Kobayashi-Maskawa Matrix CKM, which based on scalar products of quark color quantum numbers $q_R, q_G, q_B$, or $q_{RGB}$ (00.1). This representation is called colorization of CKM in weak interaction. The colors of down-type quarks $q_w, (w=d, s, b)$ in the quarkcolor scalar products of CKM are "Color-Broken, $\zeta \neq 0$", which results in isospin $I_3(q_w)$ to be violated in weak interaction, further charges $Q^{\text{CKM}}_{dsb}(\zeta_{rv})$, to be a slight deviated from $\frac{1}{3}e$ of SM theoretical value. A short discussion of possible existence of higher-charges of quark $q$ is given in Epilogue.

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0. Introduction

The three-colors R, G, B of quarks is really a curious and excellent concept in modern particle physics. In Standard Model SM, R, G, B are used to treat strong interaction quark classification and weak interaction flavor-transitions among particles in different generations.

In previous papers [1] when discussing SM, Colous Spectrum Diagram of Flavour CSDF is introduced by Spin Topological Space STS math frame [2], in which the concretization of color values \( q_R, q_G, q_B \) of each quark can be selected from the third components \( \pi_3(q) \) of one-sixth spin \( \vec{\pi}(q) \) below

\[
\pi_3(q) = \ldots, -\frac{29}{6}, -\frac{23}{6}, -\frac{17}{6}, -\frac{11}{6}, -\frac{5}{6}, -\frac{1}{6}, -\frac{7}{6}, -\frac{13}{6}, -\frac{19}{6}, -\frac{25}{6}, \ldots \subseteq q_{RGB} \equiv q_R, q_G, q_B
\]  

(0.1)

\[
\vec{\pi}(q) \times \vec{\pi}(q) = i\vec{\pi}(q)
\]  

(0.2)

To discuss hadronic constituents in strong interaction [3], colored quark, \( q(\chi, \alpha) = q(\chi) + q_\alpha \) is introduced (where quark spin \( q(\chi), \chi = \uparrow, \downarrow \) and quark color \( q_\alpha=q_{RGB} \)) \( (0.0.1) \). We will again make use of quark color \( q_\alpha \), turn to discuss weak interaction in this paper.

\( \cdot \) 1) One of the most distinguishing between weak interaction and strong interaction is the behavior of isospin of particle:

The third isospin component \( I_3 \) and total isospin \( I \) are conserved in strong interaction, but both \( I_3 \) and \( I \) are not invariant in weak interaction, that means flavor are not " pure ", there are flavor-transitions among particles in different generations. Now \( I_3 \) be violated, be broken. How to devise a beautiful math platform to demonstrate such kind process of physical values of \( I_3 \)?

Because these colors \( q_\alpha \) or \( q_R, q_G, q_B \) can offer an unified isospin \( I_3(q) \) representation [1] for all six quarks below, so we decide to use \( I_3(q) \) \( (0.0) \) to research weak interaction following

\[
I_3(q) = \frac{1}{3} (q_R + q_G + q_B) \equiv I_3(q_{RGB})
\]  

(0.0)

Or
\[ \overrightarrow{u} = (u_R, u_G, u_B) = \left( \frac{-5}{6}, \frac{1}{6}, \frac{-13}{6} \right), \quad I_3(u) = \frac{1}{3} \left( \frac{-5}{6} + \frac{1}{6} + \frac{-13}{6} \right) = \frac{-1}{2} \]  

\[ \overrightarrow{d} = (d_R, d_G, d_B) = \left( \frac{-11}{6}, \frac{-5}{6}, \frac{-27}{6} \right), \quad I_3(d) = \frac{1}{3} \left( \frac{-11}{6} + \frac{-5}{6} + \frac{-27}{6} \right) = \frac{-1}{2} \]  

\[ \overrightarrow{c} = (c_R, c_G, c_B) = \left( \frac{1}{6}, \frac{-7}{6}, \frac{-19}{6} \right), \quad I_3(c) = \frac{1}{3} \left( \frac{1}{6} + \frac{-7}{6} + \frac{-19}{6} \right) = \frac{3}{2} \]  

\[ \overrightarrow{s} = (s_R, s_G, s_B) = \left( \frac{-17}{6}, \frac{-11}{6}, \frac{-1}{6} \right), \quad I_3(s) = \frac{1}{3} \left( \frac{-17}{6} + \frac{-11}{6} + \frac{-1}{6} \right) = \frac{-3}{2} \]  

\[ \overrightarrow{t} = (t_R, t_G, t_B) = \left( \frac{-27}{6}, \frac{13}{6}, \frac{-25}{6} \right), \quad I_3(t) = \frac{1}{3} \left( \frac{-27}{6} + \frac{13}{6} + \frac{-25}{6} \right) = \frac{5}{2} \]  

\[ \overrightarrow{b} = (b_R, b_G, b_B) = \left( \frac{-23}{6}, \frac{-17}{6}, \frac{-5}{6} \right), \quad I_3(b) = \frac{1}{3} \left( \frac{-23}{6} + \frac{-17}{6} + \frac{-5}{6} \right) = \frac{-5}{2} \]  

\[ V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.975 & 0.224 & 0.004 \\ 0.224 & 0.974 & 0.042 \\ 0.009 & 0.041 & 0.999 \end{pmatrix} \]  

After that, CKM matrix is parameterized [5] to be written as a product of three rotation matrices, that called a smart Wolfenstein parametrization, one of its advantage is CP violation can be involved. (Ref [6],[7],...[12])

In this paper, CKM matrix is colorized by means of quark color \( q_R, q_G, q_B \). And scalar products of colors \( q_R, q_G, q_B \) and isospin broken \( I_3(\xi_{rw})_{CKM} \) are concerned about into Cabibbo-Kobayashi-Maskawa Matrix CKM.
Outline Flowchart for Isospin Violated in CKM Matrix

I. Quarkcolor Scalar Products in CKM Matrix

\[ V_{CKM} = V^{CKM}(q_{RGB}) \Rightarrow V(\vec{r} \cdot \vec{w}) \] (1)★ \[ \Rightarrow V(\vec{r}^\prime \cdot \vec{w}^\prime_r) \] (2)★ \[ \Rightarrow V(\vec{r}^\prime \cdot \vec{w}^\prime_r) \] (3)★

Here:

【1】
\[ \vec{r} \cdot \vec{w} = r_{RWR} + r_{GWG} + r_{BWB} \] (1.0)

【2】
\[ \vec{r}^\prime \cdot \vec{w}_r = r^\prime_{RWR} + r^\prime_{GWG} + r^\prime_{BWB} \] (2.0)

【3】
\[ \vec{r}^\prime \cdot \vec{w}_r = r^\prime_{RWR} + r^\prime_{GWG} + r^\prime_{BWB} \] (3.0)

Symbols (1)★, (2)★, (3)★ respectively stand for the quarkcolor scalar products (1.0), (2.0), (3.0) of CKM Matrix in different interaction regions shown below.
【1】 Strong interaction color representation of flavor, when $I_3(q)$ is conserved

$$\tilde{r} = \tilde{r}(q) = (r_R, r_G, r_B)$$  \hspace{1cm} (1.1)$$

$$\tilde{w} = \tilde{w}(q) = (w_R, w_G, w_B)$$  \hspace{1cm} (1.2)$$

【2】 Weak interaction color representation of flavor, when isospin $I_3(q)$ is conserved ($\xi = 0$).

$$\tilde{r} = \tilde{r}(q) + \frac{1}{6} \Phi_r$$  \hspace{1cm} (2.1)$$

$$\tilde{w}_r = \tilde{w}(q) + \frac{1}{6} \Phi_{rw} = (w_R, w_G, w_B) + \frac{1}{6} (\Phi_{rw}R, \Phi_{rw}G, \Phi_{rw}B)$$  \hspace{1cm} (2.2)$$

【3】 Weak interaction color representation of flavor, when isospin $I_3(q)$ is broken ($\xi \neq 0$)

$$\tilde{r} = \tilde{r}(q) + \frac{1}{6} \Phi_r$$  \hspace{1cm} (3.1)$$

$$\tilde{w}_r = \tilde{w}_r(\xi) = \tilde{w}(q) + \frac{1}{6} \Phi_{rw}(\xi)$$  \hspace{1cm} (3.2)$$

Where $r = u, c, t$ are up-type quarks, quark charge $Q_r = \frac{2}{3} e$ and $w = d, s, b$ are down-type quarks, quark charge $Q_w = \frac{1}{3} e$. It will be shown that in case【3】 , the charge $Q_w$ of down-type quark will be a slight deviated from $\frac{1}{3}$ due to isospin broken $I_3(q)$. Superscript " / " , that written on the top right of $r$ and $w$, stands for quark being in weak interaction region.
For clear logical route to quark-colorization of Cabibbo-Kobayashi-Maskawa Matrix CKM, in following an example (labelled by mark " $\downarrow $ ") of color scalar product $ \vec{u} \cdot \vec{d}^\downarrow $ is given, which (includes 2.1. $ \vec{u} \cdot \vec{d}^\downarrow $ and 2.2. $ \vec{u} \cdot \vec{d}^\uparrow $) arranged in the top left element $ V_{11} $ of CKM matrix.

2.1. In [1] Strong interaction color representation of flavor, (4) is color scalar product of $ u $ quark and $ d $ quark

$$\vec{u} \cdot \vec{d}^\downarrow = u_R d_R + u_G d_G + u_B d_B = \left( \frac{-5}{6} \right) \left( \frac{-11}{6} \right) + \left( \frac{+1}{6} \right) \left( \frac{-5}{6} \right) + \left( \frac{+13}{6} \right) \left( \frac{+7}{6} \right) = \frac{1}{36} \{ +55 - 5 + 91 \} = \frac{+141}{36} \downarrow \quad (4)$$

obtain isospin $ I_3(q_{RGB}) $

$$I_3(u) = \frac{1}{3} \left( \frac{-5}{6} + \frac{+1}{6} + \frac{+11}{6} \right) = \frac{-1}{2} \quad (0.1)$$

$$I_3(d) = \frac{1}{3} \left( \frac{-11}{6} + \frac{-5}{6} + \frac{+7}{6} \right) = \frac{+1}{2} \quad (0.2)$$

In this way, for CKM Matrix we get following

$$\text{Matrix (1) } \star \quad V_{\text{CKM}}(q_{RGB}) = \begin{pmatrix}
\vec{u} \cdot \vec{d}^\downarrow & \vec{u} \cdot \vec{s} & \vec{u} \cdot \vec{b} \\
\vec{c} \cdot \vec{d} & \vec{c} \cdot \vec{s} & \vec{c} \cdot \vec{b} \\
\vec{t} \cdot \vec{d} & \vec{t} \cdot \vec{s} & \vec{t} \cdot \vec{b}
\end{pmatrix} = \frac{1}{36} \begin{pmatrix}
+141 & +87 & +33 \\
+87 & -75 & -237 \\
+33 & -237 & -507
\end{pmatrix} \quad (5)$$

Matrix (5) always appears in strong interaction.
2.2. In [2] Weak interaction color representation of flavor, (10) is color scalar product of $u'$ quark and $d_u'$ quark

To research for the properties of quark color scalar product and quark isospin in Weak Interaction, so-called "weak interaction pairing" $\Phi$ of CSDF is introduced and $\Phi$ be attached to each of the six flavors $\uparrow$, $\uparrow'$, $\uparrow''$, $\downarrow$, $\downarrow'$, $\downarrow''$ of strong interaction.

(6),(7) are the concrete expressions of weak interaction pairing, for up-type quark $u$ and down-type quark $d$, by which, $\Phi_u$ and $\Phi_{ud}$ of express (2.1) and (2.2) are obtained.

$$\Phi_u = (\begin{pmatrix} -1 \\ 2 \\ 16 \\ 2 \\ -17 \\ 2 \end{pmatrix})$$

$$\Phi_{ud} = (\begin{pmatrix} 5 \\ 2 \\ 10 \\ 2 \\ -15 \\ 2 \end{pmatrix})$$

$$\uparrow' = \Phi_u + \frac{1}{6} \Phi_u = (\begin{pmatrix} -2 \\ 6 \\ 11 \\ 6 \\ -13 \\ 6 \end{pmatrix} + \frac{1}{6} (\begin{pmatrix} 1 \\ 2 \\ 16 \\ 2 \\ -17 \\ 2 \end{pmatrix}) = (\begin{pmatrix} -9 \\ 12 \\ 18 \\ 12 \\ 9 \\ 12 \end{pmatrix})$$

$$\downarrow' = \Phi_{ud} + \frac{1}{6} \Phi_{ud} = (\begin{pmatrix} -11 \\ 6 \\ 5 \\ 6 \\ -7 \\ 6 \end{pmatrix} + \frac{1}{6} (\begin{pmatrix} 5 \\ 2 \\ 10 \\ 2 \\ -15 \\ 2 \end{pmatrix}) = (\begin{pmatrix} -17 \\ 12 \\ 0 \\ 12 \\ -1 \\ 12 \end{pmatrix})$$

$$\uparrow' \cdot \downarrow' = (\Phi_u + \frac{1}{6} \Phi_u) \cdot (\Phi_{ud} + \frac{1}{6} \Phi_{ud}) = (\begin{pmatrix} -9 \\ 12 \\ 18 \\ 12 \\ 9 \\ 12 \end{pmatrix} \cdot (\begin{pmatrix} -17 \\ 12 \\ 0 \\ 12 \\ -1 \\ 12 \end{pmatrix}) = 1.000$$

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Formulas (11) (12) show in case [2], isospin $I_3$ is conserved too, as that $I_3$ (0.1) and (0.2) in strong interaction. (11) (12) are $I_3$ crucial piont states of weak interaction.

\[
I_3(u') = \frac{1}{3} \left( -\frac{9}{12} + \frac{18}{12} + \frac{9}{12} \right) = \frac{1}{2}
\]  

(11)

\[
I_3(d_u) = \frac{1}{3} \left( -\frac{17}{12} + \frac{0}{12} + \frac{1}{12} \right) = \frac{-1}{2}
\]  

(12)

There are nine weak interaction pairings $\vec{\Phi}$ in Matrix (2)★ below. Similar to pairing $\vec{\Phi}$ (6)-(7), after deliberate calculations, at last the eight other weak interaction pairings $\vec{\Phi}$ are found out, and using them, further previous Matrix (1)★ and (5) of CKM Matrix of strong interaction could be reconstructed into Matrix (2)★ and (13) of weak interaction.

\[
\begin{pmatrix}
\quad \downarrow \quad \downarrow \quad \downarrow \quad \\
\quad u' \cdot d_u & u' \cdot s_u & u' \cdot b_u \\
\quad c' \cdot d_c & c' \cdot s_c & c' \cdot b_c \\
\quad t' \cdot d_t & t' \cdot s_t & t' \cdot b_t
\end{pmatrix}

\begin{pmatrix}
1.000 \star & 0.000 & 0.000 \\
0.000 & 1.000 & 0.000 \\
0.000 & 0.000 & 1.000
\end{pmatrix}
\]

(13)

(13) is the representation of crucial piont state of CKM in weak interaction and (13) is unitary obviously.
III. ISOSPIN $I_3$ BE BROKEN IN CKM MATRIX

Taking broken parameter $\xi_{ud} = \xi = 0.2$ into (7) of pairing-\(\Phi\) (6)-(7) obtain (14) and (15)

$$\Phi_{ud}(\xi) = \left( \frac{-5}{2}, \frac{-10-\xi}{2}, \frac{-15}{2} \right) = \left( \frac{-5}{2}, \frac{-10-0.2}{2}, \frac{-15}{2} \right) = \left( \frac{-5}{2}, \frac{-9.8}{2}, \frac{-15}{2} \right)$$

(14)

Abbreviation

$$\Phi_u \equiv \Phi_u(\xi) = \frac{1}{6} \Phi_{ud}(\xi) = \left( \frac{-11}{6}, \frac{-5}{6}, \frac{+7}{6} \right) + \frac{1}{6} \left( \frac{-5}{2}, \frac{-9.8}{2}, \frac{-15}{2} \right) = \left( \frac{-17}{12}, \frac{-0.2}{12}, \frac{-1}{12} \right)$$

(15)

When broken parameter $\xi$ appears in $\Phi$, we call $\Phi$ be "Color-Broken" and call the colors of down-type quark $\phi$ (15), or quarkcolor scalar products (16) of CKM be "Color-Broken".

From $\Phi_u$, simultaneously & respectively obtain two physical quantities $\Phi' \cdot \Phi_u$ (16) and $I_3(\Phi_u)$ (18) below:

$$\Phi' \cdot \Phi_u = \left( \frac{-9}{12}, \frac{-18}{12}, \frac{+9}{12} \right) \left( \frac{-17}{12}, \frac{-0.2}{12}, \frac{-1}{12} \right) \equiv \frac{1}{144} \left\{ +144 - 3.6 \right\} = \frac{1}{144} \left\{ +140.4 \right\} = 0.975$$

(16)

$$I_3(u') = \frac{1}{3} \left( \frac{-9}{12} + \frac{-18}{12} + \frac{+9}{12} \right) = -\frac{1}{2}$$

(17)

$$I_3(\Phi_u) = \frac{1}{3} \left( \frac{-18}{12} + \frac{-0.2}{12} \right) = \frac{1}{3} \left( \frac{3}{2} + \frac{0.1}{6} \right) = \frac{1}{2} \left( 1 + \frac{0.1}{90} \right) = \frac{1}{2} \left( 1.011 \right)$$

(18)

- Formula (18) shows in case $[3] \xi \neq 0$, isospin $I_3$ is not conserved in weak interaction, there is a deviation 0.011 from $I_3(d) = \frac{-1}{2} (0.2)$
There are nine independent real parameters $\xi_{rw}$ (19) in CKM matrix, in which the third components $I_3$ are broken (20)

$$\xi_{rw} = \begin{pmatrix}
\xi_{ud} & \xi_{us} & \xi_{ub} \\
\xi_{cd} & \xi_{cs} & \xi_{cb} \\
\xi_{td} & \xi_{ts} & \xi_{tb}
\end{pmatrix} = \begin{pmatrix}
+0.200 & -1.792 & -0.032 \\
-1.075 & +0.1248 & -0.2016 \\
-0.03086 & -0.140571 & +0.003429
\end{pmatrix}$$ (19)

$$I_3(\xi_{rw})_{\text{CKM}} = \begin{pmatrix}
I_3(\vec{d}_u) & I_3(\vec{s}_u) & I_3(\vec{b}_u) \\
I_3(\vec{d}_c) & I_3(\vec{s}_c) & I_3(\vec{b}_c) \\
I_3(\vec{d}_t) & I_3(\vec{s}_t) & I_3(\vec{b}_t)
\end{pmatrix} = \begin{pmatrix}
\frac{-1}{2} \cdot (d) & \frac{-3}{2} \cdot (s) & \frac{-5}{2} \cdot (b) \\
\frac{-1}{2} \cdot (1.011) & \frac{-3}{2} \cdot (0.9668) & \frac{-5}{2} \cdot (0.9996) \\
\frac{-1}{2} \cdot (0.9403) & \frac{-3}{2} \cdot (1.0023) & \frac{-5}{2} \cdot (0.9978) \\
\frac{-1}{2} \cdot (0.9983) & \frac{-3}{2} \cdot (0.9974) & \frac{-5}{2} \cdot (1.00038)
\end{pmatrix}$$ (20)

(20) is an elegant expression of isospin $I_3$ broken in CKM Matrix math frame. Respectively compare the first, the second and the third column of $I_3(\xi_{rw})_{\text{CKM}}$ (20) with $I_3(d)(0.2)$, $I_3(s)(0.4)$ and $I_3(b)(0.6)$ of $I_3(q_{RGB})(0.0)$.

- Mindful of the deviated values of the third isospin components above: for diagonal terms $I_3(\vec{d}_u), I_3(\vec{s}_c), I_3(\vec{b}_t) > 1$ and for off-diagonal terms $I_3(\xi_{rw}) < 1$

- Formula (16) be filled in (21). After the fullness of the eight other elements in CKM Matrix, ultimately we complete the processes of CKM Matrix colorization below

$$\text{Matrix (3)★} \quad V_{\text{CKM}}(q_{RGB}, \Phi, \xi) = \begin{pmatrix}
\vec{u} \cdot \vec{d}_u & \vec{u} \cdot \vec{s}_u & \vec{u} \cdot \vec{b}_u \\
\vec{c} \cdot \vec{d}_c & \vec{c} \cdot \vec{s}_c & \vec{c} \cdot \vec{b}_c \\
\vec{t} \cdot \vec{d}_t & \vec{t} \cdot \vec{s}_t & \vec{t} \cdot \vec{b}_t
\end{pmatrix} = \begin{pmatrix}
0.975 & 0.224 & 0.004 \\
0.224 & 0.974 & 0.042 \\
0.009 & 0.041 & 0.999
\end{pmatrix}$$ (21)
IV. Conclusions

In interaction [3] region, isospin $I_3 = I_3(\xi_{rw})_{\text{CKM}}$, is not conserved, which lead to charge-deviated of quarks $Q_{\text{dU}}^{\text{CKM}}(\xi_{rw})$ (Ref. (E)).

\[
Q_{\text{dU}} = I_3(\text{d}_u) + \frac{4}{6} = \frac{-1}{3} (1.011) + \frac{4}{6} = \frac{-1}{3} (1.011) e
\]

\[
Q_{\text{dc}} = I_3(\text{d}_c) + \frac{4}{6} = \frac{-1}{3} (0.9403) + \frac{4}{6} = \frac{-1}{3} (0.91045) e
\]

\[
Q_{\text{dt}} = I_3(\text{d}_t) + \frac{4}{6} = \frac{-1}{3} (0.9983) + \frac{4}{6} = \frac{-1}{3} (0.99745) e
\]

\[
Q_{\text{su}} = I_3(\text{s}_u) + \frac{4}{6} = \frac{-1}{2} (0.9668) + \frac{4}{6} = \frac{-1}{3} (0.8506) e
\]

\[
Q_{\text{sc}} = I_3(\text{s}_c) + \frac{4}{6} = \frac{-3}{2} (1.0023) + \frac{4}{6} = \frac{-1}{3} (1.01035) e
\]

\[
Q_{\text{st}} = I_3(\text{s}_t) + \frac{4}{6} = \frac{-3}{2} (0.9974) + \frac{4}{6} = \frac{-1}{3} (0.9883) e
\]

\[
Q_{\text{bu}} = I_3(\text{b}_u) + \frac{5}{6} = \frac{-5}{2} (0.9996) + \frac{5}{6} = \frac{-1}{3} (0.997) e
\]

\[
Q_{\text{bc}} = I_3(\text{b}_c) + \frac{5}{6} = \frac{-5}{2} (0.9978) + \frac{5}{6} = \frac{-1}{3} (0.9835) e
\]

\[
Q_{\text{bt}} = I_3(\text{b}_t) + \frac{5}{6} = \frac{-5}{2} (1.00038) + \frac{5}{6} = \frac{-1}{3} (1.00285) e
\]
OR

\[
Q^{\text{CKM}}_{\text{dsb}}(\xi_{r_{\text{w}}}) = \begin{pmatrix}
Q_{dU} & Q_{sU} & Q_{bU} \\
Q_{dC} & Q_{sC} & Q_{bC} \\
Q_{dI} & Q_{sI} & Q_{bI}
\end{pmatrix} = \begin{pmatrix}
\frac{1}{3} \cdot (1.011)e & \frac{1}{3} \cdot (0.8506)e & \frac{1}{3} \cdot (0.997)e \\
\frac{1}{3} \cdot (0.91045)e & \frac{1}{3} \cdot (1.01035)e & \frac{1}{3} \cdot (0.9835)e \\
\frac{1}{3} \cdot (0.99745)e & \frac{1}{3} \cdot (0.9883)e & \frac{1}{3} \cdot (1.00285)e
\end{pmatrix}
\] (22)

- Mindful of the deviated values in (22): diagonal terms \(Q_{dU}, Q_{sC}, Q_{bI} > 1\); off-diagonal terms \(Q_{dC}, Q_{dI}, Q_{sU}, Q_{sI}, Q_{bu}, Q_{bc} < 1\) (22). In weak interaction, charges \(Q^{\text{CKM}}_{\text{dsb}}(\xi_{r_{\text{w}}})\) of down-type quark will be a slight deviated from \(\frac{1}{3} e\) (SM theoretical value), due to isospin broken \(I_3(\xi_{r_{\text{w}}})_{\text{CKM}}\) of CKM Matrix colorized.
The charge $Q_q$ of all known six quarks can be expressed by the sum (E) of isospin $I_3$ (0.0) and quark color $q_{RGB}$ (00.1) below

$$Q_q = I_3(q) + q_{RGB}$$

(E)

$$q_{RGB} = \left( \frac{1}{6} + n \right), \quad n = 0, \pm 1, \pm 2\ldots$$

(E1)

For up-type quark, $n = 0, -1, -2\ldots$ (E2)

$$Q_t = I_3(t) + \frac{-11}{6} = \frac{15}{2} + \frac{-11}{6} = \frac{-15}{6} + \frac{11}{6} = \frac{4}{6} = \frac{2}{3} e$$

(E5)

$$Q_c = I_3(c) + \frac{-5}{6} = \frac{3}{2} + \frac{-5}{6} = \frac{-9}{6} + \frac{-5}{6} = \frac{-14}{6} = \frac{-7}{3} e$$

(E3)

$$Q_u = I_3(u) + \frac{1}{6} = \frac{1}{2} + \frac{1}{6} = \frac{-3}{6} + \frac{1}{6} = \frac{-2}{6} = \frac{-1}{3} e$$

(E1)

For down-type quark, $n = 0, +1, +2\ldots$ (E3)

$$Q_d = I_3(d) + \frac{1}{6} = \frac{-1}{2} + \frac{1}{6} = \frac{-3}{6} + \frac{1}{6} = \frac{-2}{6} = \frac{-1}{3} e$$

(E2)

$$Q_s = I_3(s) + \frac{7}{6} = \frac{3}{2} + \frac{7}{6} = \frac{-9}{6} + \frac{-7}{6} = \frac{-2}{6} = \frac{-1}{3} e$$

(E4)

$$Q_b = I_3(b) + \frac{-13}{6} = \frac{-5}{2} + \frac{-13}{6} = \frac{-15}{6} + \frac{13}{6} = \frac{-2}{6} = \frac{-1}{3} e$$

(E6)
Comparing (E) with Gell-Mann-Nishijima relation (E4) [13],[14], then obtain (E5) below

\[ Q = I_3 + Y/2 \]  \hspace{1cm} (E4)

\[ Y = 2q_{RGB} \]  \hspace{1cm} (E5)

Where, hypercharge \( Y = B + S \). \( B \) baryon number and \( S \) strange number of quark \( q \). We see Gell-Mann-Nishijima relation (E4) is a special situation of (E), The latter (E), math-mysterious, is the extension of the former (E4), empirical.

• The algebra symmetry of \( q_{RGB} \) of color representation of flavor Table 1 [3] could permute many possible arrangements. Further a series of magic figures, those are multiples of 1/3, are constructed, that may illuminate the hypothesis about possible existence of higher-charges of quark \( q \).

Two examples of quark charge formula (E) with \( q_{RGB} = \frac{-1}{6} \), and with the fourth general quark are given below

For \( I_3 = \frac{-1}{2}, \frac{1}{2} \)

\[ I_3 + \frac{-1}{6} = \frac{-1}{2} + \frac{-1}{6} = \frac{-3}{6} + \frac{-1}{6} = \frac{-4}{6} = \frac{-2}{3} e \]

\[ I_3 + \frac{1}{6} = \frac{-1}{2} + \frac{1}{6} = \frac{-3}{6} + \frac{1}{6} = \frac{-2}{6} = \frac{-1}{3} e \]

For \( I_3 = \frac{3}{2}, \frac{3}{2} \)

\[ I_3 + \frac{-1}{6} = \frac{3}{2} + \frac{-1}{6} = \frac{9}{6} + \frac{-1}{6} = \frac{8}{6} = \frac{4}{3} e \]

\[ I_3 + \frac{1}{6} = \frac{-3}{2} + \frac{1}{6} = \frac{-9}{6} + \frac{1}{6} = \frac{-8}{6} = \frac{-4}{3} e \]
For $I_3 = \frac{+5}{2}, \frac{-5}{2}$

$$I_3 + \frac{+1}{6} = \frac{+5}{2} + \frac{+1}{6} = \frac{+15}{6} + \frac{+1}{6} = \frac{+16}{6} = \frac{8}{3} \, e$$

$$I_3 + \frac{-1}{6} = \frac{-5}{2} + \frac{+1}{6} = \frac{-15}{6} + \frac{-1}{6} = \frac{-14}{6} = \frac{-7}{3} \, e$$

For $I_3 = \frac{+7}{2}, \frac{-7}{2}$

$$I_3 + \frac{+1}{6} = \frac{+7}{2} + \frac{+1}{6} = \frac{+21}{6} + \frac{+1}{6} = \frac{+22}{6} = \frac{-11}{3} \, e$$

$$I_3 + \frac{-1}{6} = \frac{-7}{2} + \frac{+1}{6} = \frac{-21}{6} + \frac{-1}{6} = \frac{-20}{6} = \frac{-10}{3} \, e$$

Charges of the fourth general quark $I_3 = \frac{+7}{2}, \frac{-7}{2}$

<table>
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<th>$I_3$</th>
<th>$\frac{+7}{2}$</th>
<th>$\frac{-7}{2}$</th>
<th>$\frac{+7}{2}$</th>
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<tbody>
<tr>
<td>$q_{RGB}$</td>
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<td>$\frac{-19}{6}$</td>
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<td>$\frac{-5}{6}$</td>
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</tr>
<tr>
<td>$Q$</td>
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<td>$\frac{-1}{3} , e$</td>
<td>$\frac{-17}{3} , e$</td>
<td>$\frac{-4}{3} , e$</td>
<td>$\frac{+14}{3} , e$</td>
<td>$\frac{-7}{3} , e$</td>
<td>$\frac{+11}{3} , e$</td>
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<td>$\frac{+8}{3} , e$</td>
<td>$\frac{-13}{3} , e$</td>
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REFERENCES Références Referencias


