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Quark-Colorization of Cabibbo-Kobayashi-Maskawa Matrix CKM

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Abstract- This paper proposes an interesting representation $V^{\text{CKM}}(q_{\text{RGB}}, \Phi, \xi)$ of Cabibbo-Kobayashi-Maskawa Matrix CKM, which based on scalar products of quark color quantum numbers $q_{\text{R}}, q_{\text{G}}, q_{\text{B}}$, or q_{RGB} (00.1). This representation is called colorization of CKM in weak interaction. The colors of down-type quarks q_w , ($w=d, s, b$) in the quarkcolor scalar products of CKM are "Color-Broken, $\xi \neq 0$, which results in isospin $I_3(q_w)$ to be violated in weak interaction, further charges $Q_{\text{dsb}}^{\text{CKM}}(\xi_{rw})$, to be a slight deviated from $-\frac{1}{3}e$ of SM theoretical value. A short discussion of possible existence of higher-charges of quark q is given in Epilogue.

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ShaoXu Ren

Abstraction-

This paper proposes an interesting representation $V^{\text{CKM}}(q_{\text{RGB}}, \Phi, \xi)$ of Cabibbo-Kobayashi-Maskawa Matrix CKM, which based on scalar products of quark color quantum numbers $q_{\text{R}}, q_{\text{G}}, q_{\text{B}}$, or q_{RGB} (00.1). This representation is called colorization of CKM in weak interaction. The colors of down-type quarks q_w , ($w=d, s, b$) in the quarkcolor scalar products of CKM are "Color-Broken, $\xi \neq 0$ ", which results in isospin $I_3(q_w)$ to be violated in weak interaction, further charges $Q_{\text{dsb}}^{\text{CKM}}(\xi_{rw})$, to be a slight deviated from $\frac{-1}{3}e$ of SM theoretical value. A short discussion of possible existence of higher-charges of quark q is given in Epilogue.

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0. INTRODUCTION

The three-colors R,G,B of quarks is really a curious and excellent concept in modern particle physics. In Standard Model SM, R,G,B are used to treat strong interaction quark classification and weak interaction flavor-transitions among particles in different generations.

In previous papers [1] when discussing SM, Colour Spectrum Diagram of Flavour CSDF is introduced by Spin Topological Space STS math frame [2], in which the concretization of color values q_R, q_G, q_B of each quark can be selected from the third components $\pi_3(q)$ of one-sixth spin $\vec{\pi}(q)$ below

$$\pi_3(q) = \dots, \frac{-29}{6}, \frac{-23}{6}, \frac{-17}{6}, \frac{-11}{6}, \frac{-5}{6}, \frac{+1}{6}, \frac{+7}{6}, \frac{+13}{6}, \frac{+19}{6}, \frac{+25}{6}, \dots \subseteq q_{RGB} \equiv q_R, q_G, q_B \quad (00.1)$$

$$\vec{\pi}(q) \times \vec{\pi}(q) = i\vec{\pi}(q) \quad (00.2)$$

To discuss hadronic constituents in strong interaction [3], *colored quark*, $q(\chi, \alpha) = q(\chi) + q_\alpha$ is introduced (where quark spin $q(\chi)$, $\chi = \uparrow, \downarrow$ and quark color $q_\alpha = q_{RGB}$ (00.1), $\alpha = R, G, B$). We will again make use of quark color q_α , turn to discuss weak interaction in this paper.

• **1)** One of the most distinguishing between weak interaction and strong interaction is the behavior of isospin of particle: The third isospin component I_3 and total isospin I are conserved in strong interaction. but both I_3 and I are not invariant in weak interaction, that means flavor are not " pure ", there are flavor-transitions among particles in different generations. Now I_3 be violated, be broken. How to devise a beautiful math platform to demonstrate such kind process of physical values of I_3 ?

Because these colors q_α or q_R, q_G, q_B can offer an unified isospin $I_3(q)$ representation [1] for all six quarks below. so we decide to use $I_3(q)$ (0.0) to research weak interaction following

$$I_3(q) = \frac{1}{3} (q_R + q_G + q_B) \equiv I_3(q_{RGB}) \quad (0.0)$$

Or

$$\vec{u} = (u_R, u_G, u_B) = \left(\frac{-5}{6}, \frac{+1}{6}, \frac{+13}{6} \right), \quad I_3(u) = \frac{1}{3} \left(\frac{-5}{6} + \frac{+1}{6} + \frac{+13}{6} \right) = \frac{+1}{2} \quad (0.1)$$

$$\vec{d} = (d_R, d_G, d_B) = \left(\frac{-11}{6}, \frac{-5}{6}, \frac{+7}{6} \right), \quad I_3(d) = \frac{1}{3} \left(\frac{-11}{6} + \frac{-5}{6} + \frac{+7}{6} \right) = \frac{-1}{2} \quad (0.2)$$

$$\vec{c} = (c_R, c_G, c_B) = \left(\frac{+1}{6}, \frac{+7}{6}, \frac{+19}{6} \right), \quad I_3(c) = \frac{1}{3} \left(\frac{+1}{6} + \frac{+7}{6} + \frac{+19}{6} \right) = \frac{+3}{2} \quad (0.3)$$

$$\vec{s} = (s_R, s_G, s_B) = \left(\frac{-17}{6}, \frac{-11}{6}, \frac{+1}{6} \right), \quad I_3(s) = \frac{1}{3} \left(\frac{-17}{6} + \frac{-11}{6} + \frac{+1}{6} \right) = \frac{-3}{2} \quad (0.4)$$

$$\vec{t} = (t_R, t_G, t_B) = \left(\frac{+7}{6}, \frac{+13}{6}, \frac{+25}{6} \right), \quad I_3(t) = \frac{1}{3} \left(\frac{+7}{6} + \frac{+13}{6} + \frac{+25}{6} \right) = \frac{+5}{2} \quad (0.5)$$

$$\vec{b} = (b_R, b_G, b_B) = \left(\frac{-23}{6}, \frac{-17}{6}, \frac{-5}{6} \right), \quad I_3(b) = \frac{1}{3} \left(\frac{-23}{6} + \frac{-17}{6} + \frac{-5}{6} \right) = \frac{-5}{2} \quad (0.6)$$

• **2)** Many weak interaction phenomena can be explained by V_{CKM} , CKM matrix (0) [4] that based on experimental observation.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.975 & 0.224 & 0.004 \\ 0.224 & 0.974 & 0.042 \\ 0.009 & 0.041 & 0.999 \end{pmatrix} \quad (0)$$

After that, CKM matrix is *parameterized* [5] to be written as a product of three rotation matrices, that called a smart Wolfenstein parametrization, one of its advantage is CP violation can be involved.

In this paper, CKM matrix is *colored* by means of quark color q_R, q_G, q_B . And scalar products of colors q_R, q_G, q_B and isospin broken $I_3(\xi_{rw})_{CKM}$ are concerned about into Cabibbo-Kobayashi-Maskawa Matrix CKM.

Outline Flowchart for Isospin Violated In CKM Matrix

I. QUARKCOLOR SCALAR PRODUCTS IN CKM MATRIX

$$V_{CKM} = V^{CKM}(q_{RGB}) \Rightarrow V(\vec{r} \cdot \vec{w}) \quad (1)\star \Rightarrow V(\vec{r}^j \cdot \vec{w}_r^j) \quad (2)\star \Rightarrow V(\vec{r}^j \cdot \vec{w}_r) \quad (3)\star$$

Here

【1】
$$\vec{r} \cdot \vec{w} = r_R w_R + r_G w_G + r_B w_B \quad (1.0)$$

【2】
$$\vec{r}^j \cdot \vec{w}_r^j = r'_R w'_{rR} + r'_G w'_{rG} + r'_B w'_{rB} \quad (2.0)$$

【3】
$$\vec{r}^j \cdot \vec{w}_r = r'_R w_{rR} + r'_G w_{rG} + r'_B w_{rB} \quad (3.0)$$

Symbols (1)★, (2)★, (3)★ respectively stand for the quarkcolor scalar products (1.0),(2.0),(3.0) of CKM Matrix in different interaction regions shown below

【1】 Strong interaction color representation of flavor, when $I_3(q)$ is conserved

$$\vec{r} = \vec{r}(q) = (r_R, r_G, r_B) \quad (1.1)$$

$$\vec{w} = \vec{w}(q) = (w_R, w_G, w_B) \quad (1.2)$$



【2】 Weak interaction color representation of flavor, when isospin $I_3(q)$ is conserved ($\xi = 0$).

$$\vec{r}^j = \vec{r}(q) + \frac{1}{6} \vec{\Phi}_r \tag{2.1}$$

$$\vec{w}_r = \vec{w}(q) + \frac{1}{6} \vec{\Phi}_{rw} = (w_R, w_G, w_B) + \frac{1}{6} ((\Phi_{rw})_R, (\Phi_{rw})_G, (\Phi_{rw})_B) \tag{2.2}$$

【3】 Weak interaction color representation of flavor, when isospin $I_3(q)$ is broken ($\xi \neq 0$)

$$\vec{r}^j = \vec{r}(q) + \frac{1}{6} \vec{\Phi}_r \tag{3.1}$$

$$\vec{w}_r \equiv \vec{w}_r(\xi) = \vec{w}(q) + \frac{1}{6} \vec{\Phi}_{rw}(\xi) \tag{3.2}$$

Where $r = u, c, t$ are *up-type quarks*, quark charge $Q_r = \frac{+2}{3} e$ and $w = d, s, b$ are *down-type quarks*, quark charge $Q_w = \frac{-1}{3} e$. It will be shown that in case **【3】**, the charge Q_w of *down-type quark* will be a slight deviated from $\frac{-1}{3}$ due to isospin broken $I_3(q)$. Superscript " ' ", that written on the top right of r and w , stands for quark being in weak interaction region.

Detail Processes for Isospin Violated In CKM Matrix

II. ISOSPIN I_3 BE CONSERVED IN CKM MATRIX

For clear logical route to quark-colorization of Cabibbo-Kobayashi-Maskawa Matrix CKM, in following an example (labelled by mark " \blacklozenge ") of color scalar product $\vec{u} \cdot \vec{d}$ is given, which (includes **2.1.** $\vec{u} \cdot \vec{d}_{\blacklozenge}$ and **2.2.** $\vec{u} \cdot \vec{d}'_{\blacklozenge}$) arranged in the top left element V_{11} of CKM matrix.

2.1. In **【1】** Strong interaction color representation of flavor, (4) is color scalar product of *u quark* and *d quark*

$$\vec{u} \cdot \vec{d}_{\blacklozenge} = u_R d_R + u_G d_G + u_B d_B = \left(\frac{-5}{6}\right)\left(\frac{-11}{6}\right) + \left(\frac{+1}{6}\right)\left(\frac{-5}{6}\right) + \left(\frac{+13}{6}\right)\left(\frac{+7}{6}\right) = \frac{1}{36} \{ +55 - 5 + 91 \} = \frac{+141}{36} \blacklozenge \tag{4}$$

obtain isospin $I_3(q_{RGB})$

$$I_3(u) = \frac{1}{3} \left(\frac{-5}{6} + \frac{+1}{6} + \frac{+13}{6} \right) = \frac{+1}{2} \tag{0.1}$$

$$I_3(d) = \frac{1}{3} \left(\frac{-11}{6} + \frac{-5}{6} + \frac{+7}{6} \right) = \frac{-1}{2} \tag{0.2}$$

In this way, for CKM Matrix we get following

$$\text{Matrix (1) } \star \quad V^{\text{CKM}}(q_{RGB}) = \begin{pmatrix} \vec{u} \cdot \vec{d}_{\blacklozenge} & \vec{u} \cdot \vec{s} & \vec{u} \cdot \vec{b} \\ \vec{c} \cdot \vec{d} & \vec{c} \cdot \vec{s} & \vec{c} \cdot \vec{b} \\ \vec{t} \cdot \vec{d} & \vec{t} \cdot \vec{s} & \vec{t} \cdot \vec{b} \end{pmatrix} = \frac{1}{36} \begin{pmatrix} +141\blacklozenge & +87 & +33 \\ +87 & -75 & -237 \\ +33 & -237 & -507 \end{pmatrix} \tag{5}$$

Matrix (5) always appears in strong interaction.

2.2. In **【2】** Weak interaction color representation of flavor, (10) is color scalar product of u' quark and d' quark

To research for the properties of quark color scalar product and quark isospin in Weak Interaction, so-called " *weak interaction pairing* $\vec{\Phi}$ " of CSDF is introduced and $\vec{\Phi}$ be attached to each of the six flavors $\vec{t}, \vec{c}, \vec{u}, \vec{d}, \vec{s}, \vec{b}$ of strong interaction.

(6),(7) are the concrete expressions of *weak interaction pairing*, for *up-type quark u* and *down-type quark d*, by which, \vec{u}' and \vec{d}' of express (2.1) and (2.2) are obtained.

$$\vec{\Phi} = \left(\frac{+1}{2}, \frac{+16}{2}, \frac{-17}{2} \right) \tag{6}$$

$$\vec{\Phi}_{ud} = \left(\frac{+5}{2}, \frac{+10}{2}, \frac{-15}{2} \right) \tag{7}$$

$$\vec{u}' = \vec{u} + \frac{1}{6} \vec{\Phi} = \left(\frac{-5}{6}, \frac{+1}{6}, \frac{+13}{6} \right) + \frac{1}{6} \left(\frac{+1}{2}, \frac{+16}{2}, \frac{-17}{2} \right) = \left(\frac{-9}{12}, \frac{+18}{12}, \frac{+9}{12} \right) \tag{8}$$

$$\vec{d}' = \vec{d} + \frac{1}{6} \vec{\Phi}_{ud} = \left(\frac{-11}{6}, \frac{-5}{6}, \frac{+7}{6} \right) + \frac{1}{6} \left(\frac{+5}{2}, \frac{+10}{2}, \frac{-15}{2} \right) = \left(\frac{-17}{12}, \frac{0}{12}, \frac{-1}{12} \right) \tag{9}$$

$$\vec{u}' \cdot \vec{d}' \blacklozenge = \left(\vec{u} + \frac{1}{6} \vec{\Phi} \right) \cdot \left(\vec{d} + \frac{1}{6} \vec{\Phi}_{ud} \right) = \left(\frac{-9}{12}, \frac{+18}{12}, \frac{+9}{12} \right) \cdot \left(\frac{-17}{12}, \frac{0}{12}, \frac{-1}{12} \right) = 1.000 \blacklozenge \tag{10}$$

Formulas (11) (12) show in case **【2】**, isospin I_3 is conserved too, as that I_3 (0.1) and (0.2) in strong interaction. (11) (12) are I_3 *crucial piont states* of weak interaction.

$$I_3(u') = \frac{1}{3} \left(\frac{-9}{12} + \frac{+18}{12} + \frac{+9}{12} \right) = \frac{+1}{2} \tag{11}$$

$$I_3(d') = \frac{1}{3} \left(\frac{-17}{12} + \frac{0}{12} + \frac{-1}{12} \right) = \frac{-1}{2} \tag{12}$$

There are nine *weak interaction pairings* $\vec{\Phi}$ in Matrix (2)★ below. Similar to *pairing* $\vec{\Phi}$ (6)-(7), after deliberate calculations, at last the eight other weak interaction pairings $\vec{\Phi}$ are found out, and using them, further previous Matrix (1)★ and (5) of CKM Matrix of strong interaction could be reconstructed into Matrix (2)★ and (13) of weak interaction. we get following

$$\text{Matrix (2)★ } V^{\text{CKM}}(q_{\text{RGB}}, \Phi) = \begin{matrix} \Downarrow & \Downarrow & \Downarrow & & & & \Downarrow & \Downarrow & \Downarrow \\ \left(\begin{array}{ccc} \vec{u} \cdot \vec{d}' \blacklozenge & \vec{u} \cdot \vec{s}' & \vec{u} \cdot \vec{b}' \\ \vec{c} \cdot \vec{d}'_c & \vec{c} \cdot \vec{s}'_c & \vec{c} \cdot \vec{b}'_c \\ \vec{t} \cdot \vec{d}'_t & \vec{t} \cdot \vec{s}'_t & \vec{t} \cdot \vec{b}'_t \end{array} \right) & = & \left(\begin{array}{ccc} 1.000 \blacklozenge & 0.000 & 0.000 \\ 0.000 & 1.000 & 0.000 \\ 0.000 & 0.000 & 1.000 \end{array} \right) \end{matrix} \quad (13)$$

(13) is the representation of crucial piont state of CKM in weak interaction and (13) is unitary obviously.

III. ISOSPIN I_3 BE BROKEN IN CKM MATRIX

Taking broken parameter $\xi_{\text{ud}} = \xi = 0.2$ into (7) of *pairing*- $\vec{\Phi}$ (6)-(7) obtain (14) and (15)

$$\vec{\Phi}_{\text{ud}}(\xi) = \left(\frac{+5}{2}, \frac{+10-\xi}{2}, \frac{-15}{2} \right) = \left(\frac{+5}{2}, \frac{+10-0.2}{2}, \frac{-15}{2} \right) = \left(\frac{+5}{2}, \frac{+9.8}{2}, \frac{-15}{2} \right) \quad (14)$$

Abbreviation

$$\vec{d} \equiv \vec{d}'(\xi) = \vec{d} + \frac{1}{6} \vec{\Phi}_{\text{ud}}(\xi) = \left(\frac{-11}{6}, \frac{-5}{6}, \frac{+7}{6} \right) + \frac{1}{6} \left(\frac{+5}{2}, \frac{+9.8}{2}, \frac{-15}{2} \right) = \left(\frac{-17}{12}, \frac{-0.2}{12} \blacklozenge, \frac{-1}{12} \right) \quad (15)$$

When broken parameter ξ appears in $\vec{\Phi}$, we call $\vec{\Phi}$ be "*Color-Broken*" and call the colors of down-type quark \vec{d} (15), or quarkcolor scalar products (16) of CKM be "*Color-Broken*".

From \vec{d} , simultaneously & respectively obtain two physical quantities $\vec{u}' \cdot \vec{d}$ (16) and $I_3(\vec{d})$ (18) below:

$$\vec{u}' \cdot \vec{d} \spadesuit = \left(\frac{-9}{12}, \frac{+18}{12}, \frac{+9}{12} \right) \left(\frac{-17}{12}, \frac{-0.2}{12} \spadesuit, \frac{-1}{12} \right) = \frac{1}{144} \{ +144 \quad - 3.6 \} = \frac{1}{144} \{ +140.4 \} = 0.975 \spadesuit \quad (16)$$

$$I_3(u') = \frac{1}{3} \left(\frac{-9}{12} + \frac{+18}{12} + \frac{+9}{12} \right) = \frac{+1}{2} \quad (17)$$

$$I_3(\vec{d}) \spadesuit = \frac{1}{3} \left(\frac{-18}{12} + \frac{-0.2}{12} \right) = \frac{-1}{3} \left(\frac{3}{2} + \frac{0.1}{6} \right) = \frac{-1}{2} \left(1 + \frac{0.1}{9} \right) = \frac{-1}{2} \left(1 + \frac{1}{90} \right) = \frac{-1}{2} (1.011) \spadesuit \quad (18)$$

• Formula (18) shows in case **【3】** $\xi \neq 0$, isospin I_3 is not conserved in weak interaction, there is a deviation 0.011 from $I_3(d) = \frac{-1}{2}$ (0.2)

There are nine independent real parameters ξ_{rw} (19) in CKM matrix, in which the third components I_3 are broken (20)

$$\xi_{rw} = \begin{pmatrix} \xi_{ud} & \xi_{us} & \xi_{ub} \\ \xi_{cd} & \xi_{cs} & \xi_{cb} \\ \xi_{td} & \xi_{ts} & \xi_{tb} \end{pmatrix} = \begin{pmatrix} +0.200 \spadesuit & - 1.792 & -0.032 \\ -1.0753 & +0.1248 & -0.2016 \\ -0.03086 & -0.140571 & +0.003429 \end{pmatrix} \quad (19)$$

$$I_3(\xi_{rw})_{CKM} = \begin{pmatrix} I_3(\vec{d}) \spadesuit & I_3(\vec{s}) & I_3(\vec{b}) \\ I_3(\vec{d}_c) & I_3(\vec{s}_c) & I_3(\vec{b}_c) \\ I_3(\vec{d}_t) & I_3(\vec{s}_t) & I_3(\vec{b}_t) \end{pmatrix} = \begin{pmatrix} \frac{-1}{2} \cdot (d) & \frac{-3}{2} \cdot (s) & \frac{-5}{2} \cdot (b) \\ \frac{-1}{2} \cdot (1.011) \spadesuit & \frac{-3}{2} \cdot (0.9668) & \frac{-5}{2} \cdot (0.9996) \\ \frac{-1}{2} \cdot (0.9403) & \frac{-3}{2} \cdot (1.0023) & \frac{-5}{2} \cdot (0.9978) \\ \frac{-1}{2} \cdot (0.9983) & \frac{-3}{2} \cdot (0.9974) & \frac{-5}{2} \cdot (1.00038) \end{pmatrix} \quad (20)$$

(20) is an elegant expression of isospin I_3 broken in CKM Matrix math frame. Respectively compare the first, the second and the third column of $I_3(\xi_{rw})_{CKM}$ (20) with $I_3(d)(0.2)$, $I_3(s)(0.4)$ and $I_3(b)(0.6)$ of $I_3(q_{RGB}) (0.0)$.

- Mindful of the deviated values of the third isospin components above: for diagonal terms $I_3(\vec{d}), I_3(\vec{s}_c), I_3(\vec{b}_t) > 1$ and for off-diagonal terms $I_3(\xi_{rw}) < 1$
- Formula (16) be filled in (21). After the fullness of the eight other elements in CKM Matrix, ultimately we complete the processes of CKM Matrix colorization below

$$\text{Matrix (3)★ } V^{CKM}(q_{RGB}, \Phi, \xi) = \begin{pmatrix} \vec{u} \cdot \vec{d} & \vec{u} \cdot \vec{s} & \vec{u} \cdot \vec{b} \\ \vec{c} \cdot \vec{d}_c & \vec{c} \cdot \vec{s}_c & \vec{c} \cdot \vec{b}_c \\ \vec{t} \cdot \vec{d}_t & \vec{t} \cdot \vec{s}_t & \vec{t} \cdot \vec{b}_t \end{pmatrix} = \begin{pmatrix} 0.975 & 0.224 & 0.004 \\ 0.224 & 0.974 & 0.042 \\ 0.009 & 0.041 & 0.999 \end{pmatrix} \quad (21)$$

IV. CONCLUSIONS

In interaction **【3】** region, isospin $I_3 = I_3(\xi_{rw})_{CKM}$, is not conserved, which lead to charge-deviated of quarks $Q_{dsb}^{CKM}(\xi_{rw})$ (Ref. (E))

$$Q_{d_u} = I_3(d) + \frac{+1}{6} = \frac{-1}{2} (1.011) + \frac{+1}{6} = \frac{-1}{3} (1.011)e$$

$$Q_{d_c} = I_3(d_c) + \frac{+1}{6} = \frac{-1}{2} (0.9403) + \frac{+1}{6} = \frac{-1}{3} (0.91045)e$$

$$Q_{d_t} = I_3(d_t) + \frac{+1}{6} = \frac{-1}{2} (0.9983) + \frac{+1}{6} = \frac{-1}{3} (0.99745)e$$

$$Q_{s_u} = I_3(\mathbf{s}_u) + \frac{+7}{6} = \frac{-3}{2} (0.9668) + \frac{+7}{6} = \frac{-1}{3} (0.8506)e$$

$$Q_{s_c} = I_3(\mathbf{s}_c) + \frac{+7}{6} = \frac{-3}{2} (1.0023) + \frac{+7}{6} = \frac{-1}{3} (1.01035)e$$

$$Q_{s_t} = I_3(\mathbf{s}_t) + \frac{+7}{6} = \frac{-3}{2} (0.9974) + \frac{+7}{6} = \frac{-1}{3} (0.9883)e$$

$$Q_b = I_3(\mathbf{b}_u) + \frac{+13}{6} = \frac{-5}{2} (0.9996) + \frac{+13}{6} = \frac{-1}{3} (0.997)e$$

$$Q_{b_c} = I_3(\mathbf{b}_c) + \frac{+13}{6} = \frac{-5}{2} (0.9978) + \frac{+13}{6} = \frac{-1}{3} (0.9835)e$$

$$Q_{b_t} = I_3(\mathbf{b}_t) + \frac{+13}{6} = \frac{-5}{2} (1.00038) + \frac{+13}{6} = \frac{-1}{3} (1.00285)e$$

OR

$$Q_{\text{dsb}}^{\text{CKM}}(\xi_{rw}) = \begin{pmatrix} Q_{d_u} & Q_{s_u} & Q_b \\ Q_{d_c} & Q_{s_c} & Q_{b_c} \\ Q_{d_t} & Q_{s_t} & Q_{b_t} \end{pmatrix} = \begin{pmatrix} \frac{-1}{3} \cdot (1.011)e & \frac{-1}{3} \cdot (0.8506)e & \frac{-1}{3} \cdot (0.997)e \\ \frac{-1}{3} \cdot (0.91045)e & \frac{-1}{3} \cdot (1.01035)e & \frac{-1}{3} \cdot (0.9835)e \\ \frac{-1}{3} \cdot (0.99745)e & \frac{-1}{3} \cdot (0.9883)e & \frac{-1}{3} \cdot (1.00285)e \end{pmatrix} \quad (22)$$

• Mindful of the deviated values in (22): diagonal terms $Q_{d_u}, Q_{s_c}, Q_{b_t} > 1$; off-diagonal terms $Q_{d_c}, Q_{d_t}, Q_{s_u}, Q_{s_t}, Q_b, Q_{b_c} < 1$ (22). In weak interaction, charges $Q_{\text{dsb}}^{\text{CKM}}(\xi_{rw})$ of *down-type quark* will be a slight deviated from $\frac{-1}{3}e$ (SM theoretical value), due to isospin broken $I_3(\xi_{rw})_{\text{CKM}}$ of CKM Matrix *colorized*.

EPILOGUE

• The charge Q_q of all known six quarks can be expressed by the sum (E) of isospin I_3 (0.0) and quark color q_{RGB} (00.1) below

$$Q_q = I_3(q) + q_{\text{RGB}} \tag{E}$$

$$q_{\text{RGB}} = \left(\frac{1}{6} + n\right), \quad n = 0, \pm 1, \pm 2 \dots \tag{E1}$$

For *up-type quark*, $n = 0, -1, -2 \dots$ (E2)

$$Q_t = I_3(t) + \frac{-11}{6} = \frac{+5}{2} + \frac{-11}{6} = \frac{+15}{6} + \frac{-11}{6} = \frac{+4}{6} = \frac{+2}{3} e \tag{E.5}$$

$$Q_c = I_3(c) + \frac{-5}{6} = \frac{+3}{2} + \frac{-5}{6} = \frac{+9}{6} + \frac{-5}{6} = \frac{+4}{6} = \frac{+2}{3} e \tag{E.3}$$

$$Q_u = I_3(u) + \frac{+1}{6} = \frac{+1}{2} + \frac{+1}{6} = \frac{+3}{6} + \frac{+1}{6} = \frac{+4}{6} = \frac{+2}{3} e \tag{E.1}$$

For *down-type quark*, $n = 0, +1, +2 \dots$ (E3)

$$Q_d = I_3(d) + \frac{+1}{6} = \frac{-1}{2} + \frac{+1}{6} = \frac{-3}{6} + \frac{+1}{6} = \frac{-2}{6} = \frac{-1}{3} e \tag{E.2}$$

$$Q_s = I_3(s) + \frac{+7}{6} = \frac{-3}{2} + \frac{+7}{6} = \frac{-9}{6} + \frac{+7}{6} = \frac{-2}{6} = \frac{-1}{3} e \tag{E.4}$$

$$Q_b = I_3(b) + \frac{+13}{6} = \frac{-5}{2} + \frac{+13}{6} = \frac{-15}{6} + \frac{+13}{6} = \frac{-2}{6} = \frac{-1}{3} e \tag{E.6}$$





Comparing (E) with Gell-Mann-Nishijima relation (E4) [12],[13], then obtain (E5) below

$$Q = I_3 + Y/2 \tag{E4}$$

$$Y = 2q_{\text{RGB}} \tag{E5}$$

Where, hypercharge $Y= B + S$. B baryon number and S strange number of quark q . We see Gell-Mann-Nishijima relation (E4) is a special situation of (E), The latter (E), *math-mysterious*, is the extension of the former (E4), *empirical*.

• The algebra symmetry of q_{RGB} of color representation of flavor *able1* [3] could permute many possible arrangements. Further a series of magic figures, those are multiples of $1/3$, are constructed, that may illuminate the hypothesis about possible existence of higher-charges of quark q .

Two examples of quark charge formula (E) with $q_{\text{RGB}}= \frac{+1}{6}$, and with the fourth general quark are given below

For $I_3 = \frac{+1}{2}, \frac{-1}{2}$

$$I_3 + \frac{+1}{6} = \frac{+1}{2} + \frac{+1}{6} = \frac{+3}{6} + \frac{+1}{6} = \frac{+4}{6} = \frac{+2}{3} e$$

$$I_3 + \frac{+1}{6} = \frac{-1}{2} + \frac{+1}{6} = \frac{-3}{6} + \frac{+1}{6} = \frac{-2}{6} = \frac{-1}{3} e$$

For $I_3 = \frac{+3}{2}, \frac{-3}{2}$

$$I_3 + \frac{+1}{6} = \frac{+3}{2} + \frac{+1}{6} = \frac{+9}{6} + \frac{+1}{6} = \frac{+10}{6} = \frac{+5}{3} e$$

$$I_3 + \frac{+1}{6} = \frac{-3}{2} + \frac{+1}{6} = \frac{-9}{6} + \frac{+1}{6} = \frac{-8}{6} = \frac{-4}{3} e$$

For $I_3 = \frac{+5}{2}, \frac{-5}{2}$

$$I_3 + \frac{+1}{6} = \frac{+5}{2} + \frac{+1}{6} = \frac{+15}{6} + \frac{+1}{6} = \frac{+16}{6} = \frac{+8}{3} e$$

$$I_3 + \frac{+1}{6} = \frac{-5}{2} + \frac{+1}{6} = \frac{-15}{6} + \frac{+1}{6} = \frac{-14}{6} = \frac{-7}{3} e$$

For $I_3 = \frac{+7}{2}, \frac{-7}{2}$

$$I_3 + \frac{+1}{6} = \frac{+7}{2} + \frac{+1}{6} = \frac{+21}{6} + \frac{+1}{6} = \frac{+22}{6} = \frac{+11}{3} e$$

$$I_3 + \frac{+1}{6} = \frac{-7}{2} + \frac{+1}{6} = \frac{-21}{6} + \frac{+1}{6} = \frac{-20}{6} = \frac{-10}{3} e$$

Charges of the fourth general quark $I_3 = \frac{+7}{2}, \frac{-7}{2}$

I_3		$\frac{+7}{2}$	$\frac{-7}{2}$	$\frac{+7}{2}$	$\frac{-7}{2}$	$\frac{+7}{2}$	$\frac{-7}{2}$	$\frac{+7}{2}$	$\frac{-7}{2}$	$\frac{+7}{2}$	$\frac{-7}{2}$	$\frac{+7}{2}$	$\frac{-7}{2}$	$\frac{+7}{2}$	$\frac{-7}{2}$
q_{RGB}		$\frac{+19}{6}$	$\frac{+19}{6}$	$\frac{+13}{6}$	$\frac{+13}{6}$	$\frac{+7}{6}$	$\frac{+7}{6}$	$\frac{+1}{6}$	$\frac{+1}{6}$	$\frac{-5}{6}$	$\frac{-5}{6}$	$\frac{-11}{6}$	$\frac{-11}{6}$	$\frac{-17}{6}$	$\frac{-17}{6}$
Q		$\frac{+20}{3} e$	$\frac{-1}{3} e$	$\frac{+17}{3} e$	$\frac{-4}{3} e$	$\frac{+14}{3} e$	$\frac{-7}{3} e$	$\frac{+11}{3} e$	$\frac{-10}{3} e$	$\frac{+8}{3} e$	$\frac{-13}{3} e$	$\frac{+5}{3} e$	$\frac{-16}{3} e$	$\frac{+2}{3} e$	$\frac{-19}{3} e$

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