Global Journal of Science Frontier Research: F
Mathematics and Decision Sciences
Volume 23 Issue 4 Version 1.0 Year 2023
Type: Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

By an Extended Iteration Method to Adequate Solutions of Jerk Oscillator Containing Displacement Times Velocity Time's Acceleration and Velocity

By Md. Ishaque Ali & B. M. Ikramul Haque
Khulna University of Engineering & Technology

Abstract- Mathematics has applications in every aspect of real life. So many such type of real-life problems are modeled by differential equations. Therefore, differential equations are used as tools to solve many complex situations. With the help of differential equations, we can find the formula to solve many significant issues in many areas of the Anatomy and Physiology of the human body like physical, mental physical, and medical principles. Differential equations can be linear, nonlinear, autonomous, or non-autonomous. Practically, most of the differential equations involving physical phenomena are nonlinear. Hence nonlinear differential equations play a vital role in case of science and engineering. Nonlinear systems are differently classified, and the 'nonlinear jerk oscillator' is one of the most essential parts of a nonlinear system. Different types of nonlinear jerk oscillators will be analyzed using Extended Iteration Method, and the outcome may leave an impact to be better than the current results.

Keywords: jerk equation; truncated fourier series; newton's method; angular frequency; extended iteration technique.

GJSFR-F Classification: LCC: QA371.5

BYANEXTENDEDITERATIONMETHODTOADEQUATESOLUTIONSOFJERKOSCILLATORCONTAININGDISPLACEMENTTIMESVELOCITYTIMESACCELERATIONANDVELOCITY

Notes

By an Extended Iteration Method to Adequate Solutions of Jerk Oscillator Containing Displacement Times Velocity Time's Acceleration and Velocity

Md. Ishaque Ali [α] & B. M. Ikramul Haque [σ]

Abstract- Mathematics has applications in every aspect of real life. So many such type of real-life problems are modeled by differential equations. Therefore, differential equations are used as tools to solve many complex situations. With the help of differential equations, we can find the formula to solve many significant issues in many areas of the Anatomy and Physiology of the human body like physical, mental physical, and medical principles. Differential equations can be linear, nonlinear, autonomous, or non-autonomous. Practically, most of the differential equations involving physical phenomena are nonlinear. Hence nonlinear differential equations play a vital role in case of science and engineering. Nonlinear systems are differently classified, and the 'nonlinear jerk oscillator' is one of the most essential parts of a nonlinear system. Different types of nonlinear jerk oscillators will be analyzed using Extended Iteration Method, and the outcome may leave an impact to be better than the current results. The principal advantage of this method is that it paves a suitable and smooth way and is more accurate, practical, easy, and straightforward.

Keywords: jerk equation; truncated fourier series; newton's method; angular frequency; extended iteration technique.

I. Introduction

The use of quadratic nonlinear terms, cubic nonlinear terms, and asymmetric behavior illustrate the researchers' concentration to a great extent. Similarly, in the case of studying elastic force, structural dynamics and elliptic curve cryptography oscillators are found in use. The demands of such oscillators with strong nonlinearities are very prevalent among researchers because of their importance. Because ofsolid nonlinearities, it isn't easy to get solutions to such oscillators. In these circumstances, researchers have developed different procedures to solve and describe the physical properties of different types of nonlinear equations. Such as the Perturbation Method (PM) (Nayfeh, 1973); Homotopy Perturbation Method (HPM) (Bélendez et al., 2007; Anjumet al., 2021);Differential Transform Method (DTM) (Alquran & Al-Khaled, 2012);Harmonic Balance Method (HBM) (Mickens, 1984; Hu & Tang, 2006; Hosen et al., 2016); Enhanced Cubication Method (ECM) (Elias-Zuniga et al., 2012); Energy Balance Method (EBM) (Ozis &Yildirim, 2007); Variational Iteration Method (VIM)

Author α: Ph. D student, Department of Mathematics, Khulna University of Engineering & Technology, Khulna-9203.
e-mail: ishaqueali72@yahoo.com
Author σ: Professor and Head, Department of Mathematics, Khulna University of Engineering & Technology, Khulna-9203.
e-mail: ikramul@math.kuet.ac.bd

(Alquran & Doğan 2010); Direct Iteration Method (DIM) (Mickens, 1987; Hu & Tang, 2007; Mickens, 2010); Extended Iteration Method (EIM) (Mickens, 2010; Haque, B.I, 2014;Haque &Flora, 2020; Haque, B.I & Iqbal, 2021) etc.
The general form of the jerk equation is

$$\dddot{x} = J(x, \dot{x}, \ddot{x}) \tag{1}$$

where $J(x, \dot{x}, \ddot{x})$ denotes the jerk functions.

In this article, to determine the approximate solution of the nonlinear jerk equation, we have used Mickens' extended iteration method where the jerk function is the velocity times the acceleration-squared.

II. The Methodology

The vital base of the extended iteration method is to reconvey the prestige's solution to obtain the present stage's solution and its sequence of extensions. The most essential apprehension is

(i) How to reprocess the preliminary shape of the given nonlinear oscillator so that it will help us to handle step-by-step iterations with simplification of a term including the large number of harmonics.

(ii) How to reprocess the obtained solutions for the case of harmonic terms so that there is no algebraic complexity. The outline of the presented procedure is as follows:

Step 1: considering the general form of the oscillator in the following way:

$$\ddot{x} + f(x) = 0 \tag{2}$$

Step 2: setting initial conditions as

$$x(0) = A, \tag{3}$$

$$\dot{x}(0) = 0$$

Step 3: making standard form by adding$\Omega^2 x$on both sides, we have

$$\ddot{x} + \Omega^2 x + \Omega^2 x - f(x) \equiv G(x) \tag{4}$$

Step 4: formulizing the suitable iteration scheme as

$$\ddot{x}_{k+1} + \Omega_k^2 x_{k+1} = G(x_k) + G_x(x_0, \Omega_k)(x_k - x_0); \quad k = 1, 2, 3, \ldots.. \tag{5}$$

Step 5: setting initial guess as

$$x_0(t) = A\, cos\, (\Omega_0 t) \tag{6}$$

Step 6: setting iterated initial conditions as

$$x_{k+1}(0) = A, \tag{7}$$

$$\dot{x}_{k+1}(0) = 0$$

Notes

Step 7: executing the successive iteration from

$$k = 1, 2, 3, \ldots$$

III. Solution Procedure

The nonlinear jerk oscillator, which we consider

$$.\dddot{x} + \dot{x} = x\,\dot{x}\ddot{x} \quad (8)$$

Let $\dot{x} = y$ where $y(t)$be the space variable. Then equation (8) becomes

$$\ddot{y} + y = y\,\dot{y}y' \quad (9)$$

Let $\dot{y} = z$ and $y' = w$Then equation (9) becomes

$$.\,\ddot{y} + \mathrm{y} = y\,z\,w \quad . \quad (10)$$

For getting standard form, adding $\Omega^2 y$on both sides of equation (10), we have

$$\ddot{y} + \Omega^2 y = \Omega^2 y - y + y\,\mathrm{z}\,\mathrm{w} = \mathrm{G}(y, z, w) \quad (11)$$

where $G(y, \mathrm{z},\ \mathrm{w}) = \Omega^2 y - y + y\,z\,w$

Then $G_y = \frac{\partial G}{\partial y} = \Omega^2 - 1 + z\,w, G_{\mathrm{z}} = \frac{\partial G}{\partial \mathrm{z}} = y\,w$ and $G_{\mathrm{w}} = \frac{\partial G}{\partial \mathrm{w}} = y\,z$

Thus, the extended iteration scheme of equation (11) is

$$\ddot{y}_{k+1} + \Omega_k^2 y_{k+1} = (\Omega_k^2 y_0 - y_0 + y_0 z_0 w_0) + (\Omega_k^2 - 1 + z_0 w_0)(y_k - y_0) + y_0 w_0 (z_k - z_0) + y_0 z_0 (w_k - w_0); k = 1,2,3, \ldots . \quad (12)$$

wherethe initial guess is

$$y_0 = y_0(t) = A\cos\theta\,;\ \theta = \Omega t \quad . \quad (13)$$

$$z_0 = -A\,\Omega_0 \sin\theta \quad (14)$$

$$w_0 = \frac{1}{\Omega_0} A\ \sin\theta \quad (15)$$

First Iteration Step: Putting $k = 0$ in equation (12) and using initial guess (13), (14) and (15), we get

$$\ddot{y}_1 + \Omega_0^2 y_1 = (\Omega_0^2\, y_0 - y_0 + y_0 z_0 w_0) \quad (16)$$

Now, expanding the right-hand side of equation (16), then it reduces to

$$\ddot{y}_1 + \Omega_0^2 y_1 = \mathrm{a11}\cos\theta + \mathrm{a13}\cos 3\theta \quad (17)$$

where,

$$\text{a11} = \frac{1}{4}(-4A - A^3 + 4A\Omega_0^2) \tag{18}$$

$$\text{a13} = \frac{A^3}{4} \tag{19}$$

Now, solving equation (17), we have,

$$y_1 = \left(A + \text{a13}\frac{1}{8\Omega_0^2}\right)\cos\theta - \text{a13}\frac{1}{8\Omega_0^2}\cos 3\theta \tag{20}$$

$$z_1 = -\Omega_1\left(\left(A + \text{a13}\frac{1}{8\Omega_0^2}\right)\sin\theta - \text{a13}\frac{3}{8\Omega_0^2}\sin 3\theta\right) \tag{21}$$

$$w_1 = \frac{1}{\Omega_1}\left(\left(A + \text{a13}\frac{1}{8\Omega_0^2}\right)\sin\theta - \text{a13}\frac{1}{24\Omega_0^2}\sin 3\theta\right) \tag{22}$$

To avoid secular term from equation (17), we obtain

$$\Omega_0 = \frac{2}{\sqrt{4-A^2}} \tag{23}$$

Equation (20) is known as the first iterated approximate analytic solution, and equation (23) is known as the first approximate frequency of the oscillator (8).

Second Iteration Step: Putting $k = 1$ in equation (12), we get,

$$\ddot{y}_2 + \Omega_1^2 y_2 = (\Omega_1^2 y_0 - y_0 + y_0 z_0 w_0) + (\Omega_1^2 - 1 + z_0 w_0)(y_1 - y_0) + y_0 w_0 (z_1 - z_0) + y_0 z_0 (w_1 - w_0) \tag{24}$$

Now, substituting the value of y_0, z_0, w_0, y_1, z_1, and w_1 from equations (13), (14), (15), (20), (21), and (22) into (24) and expanding the right-hand side of equation (24), then it reduces to

$$\ddot{y}_2 + \Omega_1^2 y_2 = \text{a21}\cos\theta + \text{a23}\cos 3\theta + \text{a25}\cos 5\theta \tag{25}$$

where

$$\text{a21} = -\frac{4A}{4+A^2} - \frac{A^3}{8(4+A^2)} + \frac{3A^5}{16(4+A^2)} - \frac{2A^3}{(4+A^2)^{3/2}\Omega_1} - \frac{25A^5}{24(4+A^2)^{3/2}\Omega_1} - \frac{13A^7}{96(4+A^2)^{3/2}\Omega_1} - \frac{2A^3\Omega_1}{(4+A^2)^{3/2}} - \frac{3A^5\Omega_1}{8(4+A^2)^{3/2}} + \frac{4A\Omega_1^2}{4+A^2} + \frac{9A^3\Omega_1^2}{8(4+A^2)} \tag{26}$$

$$\text{a23} = -\frac{7A^3}{8(4+A^2)} - \frac{5A^5}{32(4+A^2)} + \frac{2A^3}{(4+A^2)^{3/2}\Omega_1} + \frac{17A^5}{16(4+A^2)^{3/2}\Omega_1} + \frac{9A^7}{64(4+A^2)^{3/2}\Omega_1} + \frac{2A^3\Omega_1}{(4+A^2)^{3/2}} + \frac{9A^5\Omega_1}{16(4+A^2)^{3/2}} - \frac{A^3\Omega_1^2}{8(4+A^2)} \tag{27}$$

$$\text{a25} = -\frac{A^5}{32(4+A^2)} - \frac{A^5}{48(4+A^2)^{3/2}\Omega_1} - \frac{A^7}{192(4+A^2)^{3/2}\Omega_1} - \frac{3A^5\Omega_1}{16(4+A^2)^{3/2}} \tag{28}$$

Notes

Now, solving equation (25), we have,

$$y_2 = \left(A + a23\frac{1}{8\Omega_1^2} + a25\frac{1}{24\Omega_1^2}\right)\cos\theta + a23\frac{1}{-8\Omega_1^2}\cos3\theta + a25\frac{1}{-24\Omega_1^2}\cos5\theta \qquad (29)$$

Notes

$$z_2 = -\Omega_2\left(\left(A + a23\frac{1}{8\Omega_1^2} + a25\frac{1}{24\Omega_1^2}\right)\sin\theta + a23\frac{3}{-8\Omega_1^2}\sin3\theta + a25\frac{5}{-24\Omega_1^2}\sin5\theta\right) \qquad (30)$$

$$w_2 = \frac{1}{\Omega_2}\left(\left(A + a23\frac{1}{8\Omega_1^2} + a25\frac{1}{24\Omega_1^2}\right)\sin\theta - a23\frac{1}{24\Omega_1^2}\sin3\theta - a25\frac{1}{120\Omega_1^2}\sin5\theta\right) \qquad (31)$$

To avoid secular term from equation (29), we obtain

$$\Omega_1 = \frac{16A^2+3A^4}{3(32\sqrt{4+A^2}+9A^2\sqrt{4+A^2})} - (-24576 - 13824A^2 - 1496A^4 + 366A^6 + 63A^8)/ (3(32\sqrt{4+A^2} + 9A^2\sqrt{4+A^2})(9437184A^2 + 11206656A^4 + 5405696A^6 + 1321920A^8 + 163836A^{10} + 8235A^{12}(9437184A^2 + 11206656A^4 + 5405696A^6 + 1321920A^8 + 163836A^{10} + 8235A^{12} + 3\sqrt{(-13194139533312 - 22265110462464A^2 - 5037996638208A^4 + 19032610701312A^6 + 25146509230080A^8 + 16524372606976A^{10} + 6955567370240A^{12} + 2014431843328A^{14} + 409363828992A^{16} + 57734182080A^{18} + 5408255088A^{20} + 303693624A^{22} + 7757289A^{24}})^{1/3}) + \frac{1}{6(32\sqrt{4+A^2}+9A^2\sqrt{4+A^2})}(9437184A^2 + 11206656A^4 + 5405696A^6 + 1321920A^8 + 163836A^{10} + 8235A^{12} + 3\sqrt{(-13194139533312 - 22265110462464A^2 - 5037996638208A^4 + 19032610701312A^6 + 25146509230080A^8 + 16524372606976A^{10} + 6955567370240A^{12} + 2014431843328A^{14} + 409363828992A^{16} + 57734182080A^{18} + 5408255088A^{20} + 303693624A^{22} + 7757289A^{24}})^{1/3} \qquad (32)$$

Equation (29) is known as the second iterated approximate analytic solution, and equation (32) is known as the second approximate frequency of the oscillator (8).

Third Iteration Step: Putting $k = 2$ in equation (12), we get,

$$\ddot{y}_3 + \Omega_2^2 y_3 = (\Omega_2^2 y_0 - y_0 + y_0 z_0 w_0) + (\Omega_2^2 - 1 + z_0 w_0)(y_2 - y_0) + y_0 w_0 (z_2 - z_0) + y_0 z_0 (w_2 - w_0) \qquad (33)$$

Now, substituting the value of y_0, z_0, w_0, y_2, z_2, and w_2 from equations (13), (14), (15), (29), (30), and (31) into (33) and expanding the right-hand side of equation (33), then it reduces to

$$\ddot{y}_3 + \Omega_2^2 y_3 = \text{a31}\cos\theta + \text{a33}\cos3\theta + \text{a35}\cos5\theta + \text{a37}\cos7\theta \qquad (34)$$

where,

$$a31 = (4720203418042368A^5)/((4 + A^2)^2 (-49152 - 27648A^2 - 2992A^4 + 732A^6 + 126A^8 - 32A^2(9437184A^2 + 11206656A^4 + 5405696A^6 + 1321920A^8 + 163836A^{10} + 8235A^{12} + 3\sqrt{}(-13194139533312 - 22265110462464A^2 - 5037996638208A^4 + 19032610701312A^6 + 25146509230080A^8 + 16524372606976A^{10} + 6955567370240A^{12} + 2014431843328A^{14} + 409363828992A^{16} + 57734182080A^{18} + 5408255088A^{20} + 303693624A^{22} + 7757289A^{24}))^{1/3} - 6A^4(9437184A^2 + 11206656A^4 + 5405696A^6 + 1321920A^8 + 163836A^{10} + 8235A^{12} \ldots . -(9437184A^2 + 11206656A^4 + 5405696A^6 + 1321920A^8 + 163836A^{10} + 8235A^{12} + 3\sqrt{}(-13194139533312 - 22265110462464A^2 - 5037996638208A^4 + 19032610701312A^6 + 25146509230080A^8 + 16524372606976A^{10} + 6955567370240A^{12} + 2014431843328A^{14} + 409363828992A^{16} + 57734182080A^{18} + 5408255088A^{20} + 303693624A^{22} + 7757289A^{24}))^{2/3})^3) \quad (35)$$

$$a33 = -(445302209249280A^5)/((4 + A^2)^2(-49152 - 27648A^2 - 2992A^4 + 732A^6 + 126A^8 - 32A^2(9437184A^2 + 11206656A^4 + 5405696A^6 + 1321920A^8 + 163836A^{10} + 8235A^{12} + 3\sqrt{}(-13194139533312 - 22265110462464A^2 - 5037996638208A^4 + 19032610701312A^6 + 25146509230080A^8 + 16524372606976A^{10} + 6955567370240A^{12} + 2014431843328A^{14} + 409363828992A^{16} + 57734182080A^{18} + 5408255088A^{20} + \cdots + 3\sqrt{}(-13194139533312 - 22265110462464A^2 - 5037996638208A^4 + 19032610701312A^6 + 25146509230080A^8 + 16524372606976A^{10} + 6955567370240A^{12} + 2014431843328A^{14} + 409363828992A^{16} + 57734182080A^{18} + 5408255088A^{20} + 303693624A^{22} + 7757289A^{24}))^{2/3})^3) \quad (36)$$

$$a35 = (117819542863872A^7)/((4 + A^2)^2(-49152 - 27648A^2 - 2992A^4 + 732A^6 + 126A^8 - 32A^2(9437184A^2 + 11206656A^4 + 5405696A^6 + 1321920A^8 + 163836A^{10} + 8235A^{12} + \cdots + 19032610701312A^6 + 25146509230080A^8 + 16524372606976A^{10} + 6955567370240A^{12} + 2014431843328A^{14} + 409363828992A^{16} + 57734182080A^{18} + 5408255088A^{20} + 303693624A^{22} + 7757289A^{24}))^{2/3})^3 \quad (37)$$

$$a37 = -(1623497637888A^9)/((4 + A^2)^2 (-49152 - 27648A^2 - 2992A^4 + 732A^6 + 126A^8 - 32A^2(9437184A^2 + 11206656A^4 + 5405696A^6 + 1321920A^8 + 163836A^{10} + 8235A^{12} + \cdots + 16524372606976A^{10} + 6955567370240A^{12} + 2014431843328A^{14} + 409363828992A^{16} + 57734182080A^{18} + 5408255088A^{20} + 303693624A^{22} + 7757289A^{24}))^{2/3})^3 \quad (38)$$

After solving equation (34), we have,

Notes

$$y_3 = \left(A + a33\frac{1}{8\Omega_2^2} + a35\frac{1}{24\Omega_2^2} + a37\frac{1}{48\Omega_2^2}\right)\cos\theta + a33\frac{1}{-8\Omega_2^2}\cos 3\theta + a35\frac{1}{-24\Omega_2^2}\cos 5\theta + a37\frac{1}{-48\Omega_2^2}\cos 7\theta \tag{39}$$

$$z_3 = -\Omega_3\left(\left(A + a33\frac{1}{8\Omega_2^2} + a35\frac{1}{24\Omega_2^2} + a37\frac{1}{48\Omega_2^2}\right)\sin\theta + a33\frac{3}{-8\Omega_2^2}\sin 3\theta + a35\frac{5}{-24\Omega_2^2}\sin 5\theta + a37\frac{7}{-48\Omega_2^2}\sin 7\theta\right) \tag{40}$$

$$w_3 = \frac{1}{\Omega_3}\left(\left(A + \mathrm{a}33\frac{1}{8\Omega_2^2} + \mathrm{a}35\frac{1}{24\Omega_2^2} + \mathrm{a}37\frac{1}{48\Omega_2^2}\right)\sin\theta - \mathrm{a}33\frac{1}{24\Omega_2^2}\sin 3\theta - \mathrm{a}35\frac{1}{120\Omega_2^2}\sin 5\theta - \mathrm{a}37\frac{1}{336\Omega_2^2}\sin 7\theta\right) \tag{41}$$

To avoid secular term from equation (39), we obtain

$$\begin{aligned}\Omega_2 = {} & (18049582881570816A^6 + 44465899494703104A^8 + 50000686410104832A^{10} + 33868321094893568A^{12} + 15353161074081792A^{14} + 4891022626127872A^{16} + \\ & 1117554239127552A^{18} + 183226816945152A^{20} + 21138775350336A^{22} + 1635969435120A^{24} + 76541101308A^{26} + 1642907205A^{28} + \\ & 5737807872A^4\sqrt{((128 + 68A^2 + 9A^4)^2(-805306368 - 503316480A^2 + 567803904A^4 + 831356928A^6 + 452990208A^8 + 135794368A^{10} + 23789664A^{12} + 2302128A^{14} + 95769A^{16}))} + \\ & 7321681920A^6\sqrt{((128 + 68A^2 + 9A^4)^2(-805306368 - 503316480A^2 + 567803904A^4 + 831356928A^6 + 452990208A^8 + 135794368A^{10} + 23789664A^{12} + 2302128A^{14} + 95769A^{16}))} + \\ & 3913629696A^8\sqrt{((128 + 68A^2 + 9A^4)^2(-805306368 - 503316480A^2 + 567803904A^4 + 831356928A^6 + 452990208A^8 + 135794368A^{10} + 23789664A^{12} + 2302128A^{14} + 95769A^{16}))} + \\ & 1121367552A^{10}\sqrt{((128 + 68A^2 + 9A^4)^2(-805306368 - 503316480A^2 + 567803904A^4 + 831356928A^6 + 452990208A^8 + 135794368A^{10} + 23789664A^{12} + 2302128A^{14} + 95769A^{16}))} + \cdots + \\ & 675A^6\sqrt{4 + A^2}\sqrt{((128 + 68A^2 + 9A^4)^2(-805306368 - 503316480A^2 + 567803904A^4 + 831356928A^6 + 452990208A^8 + 135794368A^{10} + 23789664A^{12} + 2302128A^{14} + 95769A^{16}))}(9437184A^2 + \\ & 11206656A^4 + 5405696A^6 + 1321920A^8 + 163836A^{10} + 8235A^{12} + 3\sqrt{((128 + 68A^2 + 9A^4)^2(-805306368 - 503316480A^2 + 567803904A^4 + 831356928A^6 + 452990208A^8 + 135794368A^{10} + 23789664A^{12} + 2302128A^{14} + 95769A^{16}))})^{2/3}))\end{aligned} \tag{42}$$

Equation (39) is known as the third iterated approximate analytic solution, and equation (42) is known as the third approximate frequency of the oscillator (8).

IV. Results and Discussions

To verify the exactness, we considered the percentage error (%) according to the definition:

$$\text{Error} = \left|\frac{T_e - T_k}{T_e}\right| \times 100\%$$

Where the various approximate periods obtained by $T_0; k = 0, 1, 2, \dots\dots$ are illustrated by the modified method and T_e denotes the exact period of the oscillator.

Haque's extended iterative method (Haque, 2014); has been presented based on Mickens extended iteration method (Mickens, 1987); to obtain the approximate analytic solution of the jerk oscillator containing "velocity times acceleration-squared, "The frequency, in addition to amplitude, has been earned by a modified approach and compared with those made by another existing process. We have calculated the first, second, and third approximate frequencies and corresponding periods of the oscillator denoted by $\Omega_0, \Omega_1, \Omega_2$ and T_0, T_1, T_2, respectively. It is noteworthy that the analytical solutions of algebraic equations produced by the executed method are straightforward to calculate. So, the iteration steps can be preceded to a finite necessary level. In this modification, we have found the solution up to the third iteration step.

All the obtained results are presented in Table 1. Table 2 it has been shown the comparison of results. A graph is provided in figure 1 and figure 2, where the comparative diagram of the modified development and the exact result is presented. We have compared the solution with the numerical solution obtained by Ranjikutta's 4th order method graphically; our solution shows good agreement with the numerical solution. A graph is provided in figure 1 and figure 2, where the comparative diagram of the modified result and the exact result is presented. We have compared the solution with the numerical solution obtained by Ranjikutta's 4th order method graphically; our solution shows good agreement with the numerical solution.

Table 1: Analyzing the differences between the approximate and exact periods T_e of $\ddot{x} + \dot{x} = x\dot{x}\ddot{x}$:

A	T_{exact}	Modified T_0 Er(%)	Modified T_1 Er(%)	Modified T_2 Er(%)
0.1	6.275347	6.275346 1.56 e^{-5}	6.275347 1.54 e^{-5}	6.275347 2.60 e^{-6}
0.2	6.252016	6.252003 2.07 e^{-4}	6.252016 3.10 e^{-6}	6.252016 9.53 e^{-8}
0.5	6.096061	6.095585 1.17 e^{-1}	6.096021 6.54 e^{-4}	6.096062 1.18 e^{-5}
1	5.626007	5.619852 1.09 e^{-1}	5.624396 2.86 e^{-2}	5.625880 2.26 e^{-3}
2	4.491214	4.442883 1.08	4.463964 6.07 e^{-1}	4.614498 2.74

Initial, second, and third modified approximate periods are denoted respectively by T_0, T_1 and T_2 and percentage error indicates by Er(%).

Notes

Notes

Table 2: Approximate periods obtained by our method compared with exact periods T_e and other existing periods of $\dddot{x} + \dot{x} = x\dot{x}\ddot{x}$:

A	T_{exact}	Modified $T_{2(\text{Second})}$ Er(%)	Gottlieb T_G Er(%) (2004) [6]	Ma et al. T_M Er(%) (2008) [15]	Ramos T_R (2010) [21] Er(%)	Karahan K_R (2017) [14] Er(%)	Gamal T_I (2021) [13] Er(%)
0.1	6.275347	6.275347 2.60 e-6	6.275346 1.3 e-5	6.275347 2.5 e-6	6.275329 7.2 e-5	6.275347 3.19 e-6	6.275334 2.11 e-4
0.2	6.252016	6.252016 9.53 e-8	6.252003 2.11 e-4	6.252016 1.6 e-7	6.251740 1.1 e-3	6.252016 3.20 e-6	6.252016 1.59 e-7
0.5	6.096061	6.096062 1.18 e-5	6.095585 7.08 e-3	6.096059 3.21 e-5	6.085649 4.6 e-2	6.275334 2.94	6.275334 3.28 e-5
1	5.626007	5.625880 2.26 e-3	5.619852 1.09 e-1	5.625795 3.8 e-3	5.477174 9.0 e-1	5.624549 2.60 e-2	6.275334 2.31 e-4
2	4.491214	4.614498 2.74	4.442883 1.08	4.482081 2.03	4.466205 5.6 e-1	4.466455 5.51 e-1	4.47661 3.25 e-1

Approximate period obtained by Gottlieb, Ma et al. Ramos, Karahan, and Ismail, respectively denoted by T_G, T_M, T_R, K_R, T_I and the modified second approximate periodreceived by us is represented by T_2. Percentage error is indicated by Er(%).

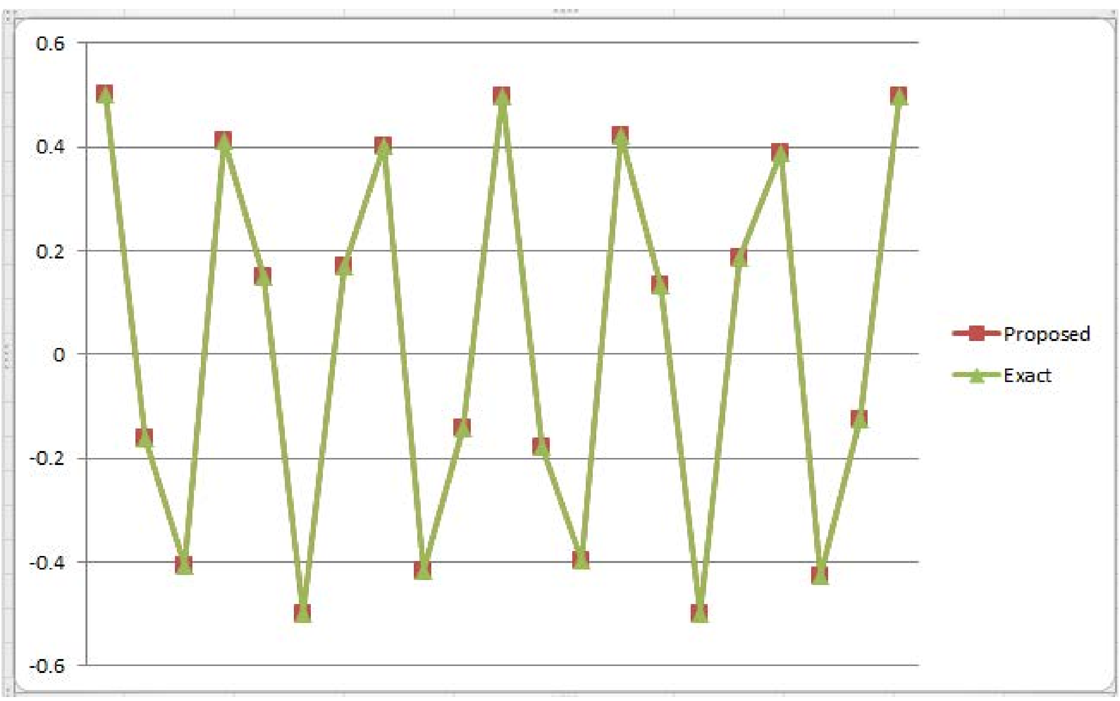

Figure 1: Approximate solutions of $\dddot{x} + \dot{x} = x\,\dot{x}\ddot{x}$ for $A = 0.5$ Compare with the corresponding numerical solution.

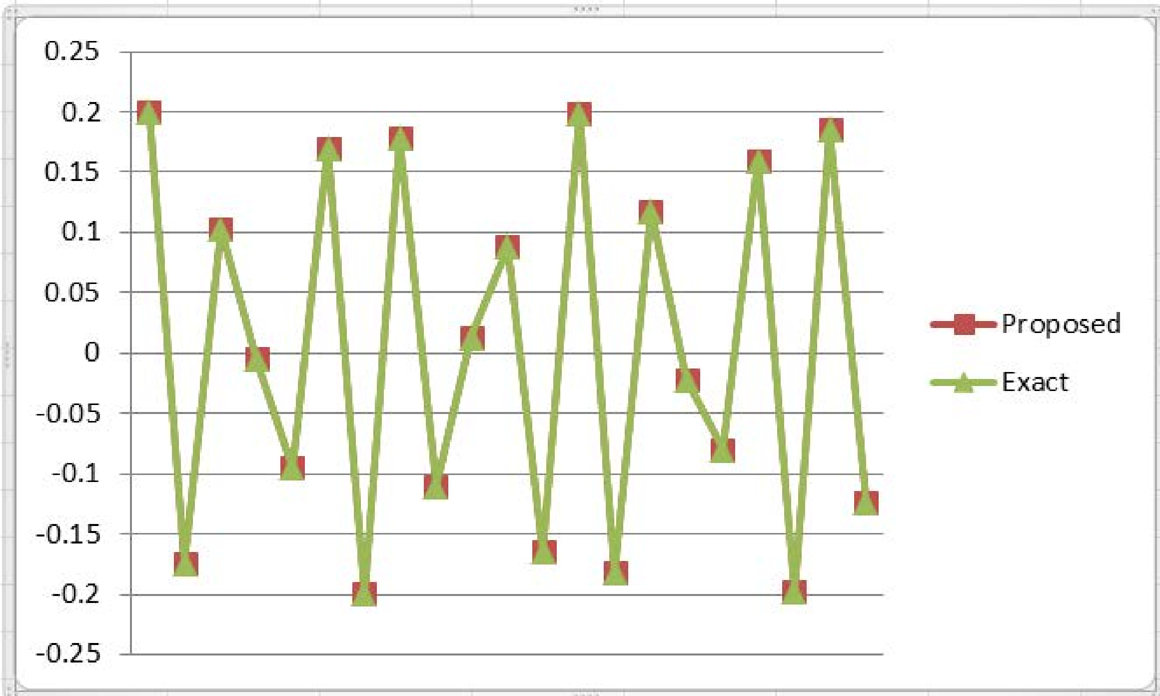

Figure 2: Approximate solutions of $\dddot{x} + \dot{x} = x\,\dot{x}\ddot{x}$ for $A = 0.1$Compare with the corresponding numerical solution.

V. Convergence and Consistency Analysis

Test of convergence: The iteration method will be convergent if the set of solutions x_k (or frequencies $\Omega_k$or amplitudes T_k) in ascending order satisfy the following property.

$x_E = \mathrm{Lim}(x_k)$ or, $\Omega_E = \mathrm{Lim}(\Omega_k)$ or, $T_E = \mathrm{Lim}(T_k)$, k $\rightarrow\infty$ Here x_E is considered as the exact solution, Ω_k denotes the frequencies and T_k denotes the corresponding periods of the nonlinear oscillator.

In the obtained solutions, it has been indicated that there is less error to iterative steps in ascending order and finally it has been shown that $|T_2 - T_E| < \varepsilon$, where ε is a small positive number. Hence the presented extended iteration method is convergent.

Test of consistency: The iterative method will be consistent if the set of solutions x_k(or frequencies $\Omega_k$or amplitudes T_k) in ascending order satisfy the following property

$$\mathrm{Lim}|x_k - x_E| = 0\text{or, } \mathrm{Lim}|\Omega_k - \Omega_E| = 0\text{or, } \mathrm{Lim}|T_k - T_E| = 0,\ k \rightarrow \infty$$

In the obtained solutions, it has been indicated that there is less error to iterative steps in ascending order, and finally, it has been shown that,

$$\mathrm{Lim}|T_k - T_E| = 0,\ k \rightarrow \infty \text{ as} |T_2 - T_E| = 0.$$

Hence the presented extended iteration method is consistent.

VI. Conclusion

In this research, it has been seen that the most significant part of solutions has been enhanced drastically. The modified solutions show that this modification is more

Notes

precise than other existing solution methods and resolution is valid for the large amplitude of oscillation for the jerk system. We have seen that it is not always true that the extended iteration method yields better results than the direct iteration method. It can be accomplished that the adopted modification is steadfast, effectual, and conformable also, it, also present sufficient well-suited solutions to the nonlinear jerk equations arising in mathematical physics, applied mathematics, and different field of engineering, specially in Mechanical, Electrical, and Space engineering.

Notes

Availability of Data

This study was not supported by any data.

Conflict of Interest

The authors affirm that they are impartial.

References Références Referencias

1. Alquran, M. T., & Al-Khaled, K. (2012). Effective approximate methods for strongly nonlinear differential equations with oscillations. *Mathematical Sciences*, 6(1), 32-6.
2. Alquran, M. T., & Doğan, N. (2010). Variational Iteration method for solving two-parameter singularly perturbed two point boundary value problem. *Applications and Applied Mathematics: An International Journal (AAM)*, *5*(1), 7.
3. Anjum, Naveed, et al. "Li-He's modified homotopy perturbation method for doubly-clamped electrically actuated microbeams-based microelectromechanical system." *Facta Universitatis, Series: Mechanical Engineering* 19.4 (2021): 601-612.
4. Beléndez, A., Hernandez, A., Beléndez, T., Fernández, E., Alvarez, M. L., & Neipp, C. (2007). Application of He's homotopy perturbation method to the Duffing-harmonic oscillator. *International Journal of Nonlinear Sciences and Numerical Simulation*, *8*(1), 79-88.
5. Elías-Zúñiga, A., Martínez-Romero, O., & Córdoba-Díaz, R. K. (2012). Approximate solution for the Duffing-harmonic oscillator by the enhanced cubication method. *Mathematical problems in Engineering*, *2012*.
6. Gottlieb, H. P. W. "Harmonic balance approach to periodic solutions of non-linear jerk equations." *Journal of Sound and Vibration* 271.3-5 (2004): 671-683.
7. Haque, B. I. (2014). A New Approach of Mickens' Extended Iteration Method for Solving Some Nonlinear Jerk Equations. *British journal of Mathematics & Computer Science*, *4*(22), 3146.
8. Haque, B. I., & Flora, S. A. (2020). On the analytical approximation of the quadratic non-linear oscillator by modified extended iteration method. *Applied Mathematics and Nonlinear Sciences*, *6*(1), 527-536.
9. Haque, BM Ikramul, and M. Iqbal Hossain. "An analytical approach for solving the nonlinear jerk oscillator containing velocity times acceleration-squared by an extended iteration method." *journal of Mechanics of Continua and Mathematical Sciences* 16.2 (2021): 35-47.
10. Hosen, M. A., Chowdhury, M. S. H., Ali, M. Y., & Ismail, A. F. (2016).A comparison study on the harmonic balance method and rational harmonic balance method for the Duffing-harmonic oscillator. *Global Journal of Pure and Applied Mathematics*, *12*(3), 2721-2732.
11. Hu, H., & Tang, J. H. (2006).Solution of a Duffing-harmonic oscillator by the method of harmonic balance. *Journal of sound and vibration*, *294*(3), 637-639.

12. Hu, H., & Tang, J. H. (2007).A classical iteration procedure valid for certain strongly nonlinear oscillators. *Journal of sound and vibration*, *299*(1-2), 397-402.
13. Ismail, Gamal M., and Hanaa Abu-Zinadah. "Analytic approximations to non-linear third order Jerk equations via modified global error minimization method." *Journal of King Saud University-Science* 33.1 (2021): 101219.
14. Karahan, MM Fatih. "Approximate solutions for the nonlinear third-order ordinary differential equations." *Zeitschrift für Naturforschung A* 72.6 (2017): 547-557.
15. Ma, X., Wei, L. and Guo, Z.: He's homotopy perturbation method to periodic solutions of nonlinear jerk equations, J. Sound Vib., vol. 314, pp. 217-227, 2008.
16. Mickens, R. (1984). Comments on the method of harmonic balance. *Journal of sound and vibration*, *94*(3), 456-460.
17. Mickens, R. E. (1987). Iteration procedure for determining approximate solutions to non-linear oscillator equations. *Journal of Sound Vibration*, *116*(1), 185-187.
18. Mickens, R. E. (2010). *Truly nonlinear oscillations: harmonic balance, parameter expansions, iteration, and averaging methods*. World Scientific.
19. Nayfeh, A. H. Perturbation Methods. John Wiley and Sons, New York, 1973.
20. Ozis, T., & Yildirim, A. (2007). Determination of the frequency-amplitude relation for a Duffing-harmonic oscillator by the energy balance method. *Computers and Mathematics with Applications*, *54*(7), 1184-1187.
21. Ramos, J.I.: Analytical and approximate solutions to autonomous, nonlinear, thirdorder ordinary differential equations, Nonlinear Anal. Real., vol. 11, pp. 1613-1626, 2010.

Notes