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To Foundations of General Theory Relativity

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Abstract- It is shown that the foundations of the generally accepted theory contain a number of contradictory and unfounded statements. The geometric properties of the three-dimensional Riemannian space are successively described. It is shown that this space is locally Euclidean. The metric tensor can be algebraically diagonalized. In this case, all diagonal elements are equal to each other and differ from unity by a function of coordinates, which is called the gravitational potential. It is shown that this function is a solution of the differential equation of potential theory. If this solution is such that the potential can be represented by equipotential surfaces, then the trajectory of free motion of a material particle lies on this surface. The trajectory on the surface is a geodesic line determined by the initial conditions. The numerical value of the potential on the surface is included in the definition of the maximum possible speed on this surface, that is, a constant equal to the speed of light in vacuum is multiplied by a value less than one, determined by the value of the potential.

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To Foundations of General Theory Relativity

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I. Introduction

The General Theory of Relativity is based on the description of the geometry and properties of space-time [1]. The author is not an expert in this field. He worked for fifty-four years at the Institute for Metal Physics National Academy of Science Ukraine. He currently lives in the State of Israel and does not work in any research institution. Reading the monograph [1], he found that the authors point out many contradictions inherent in modern theory. Some thoughts arose on how to resolve these contradictions, and not being able to discuss them, the author outlined them in the article "Kinematics and Dynamics of a Particle in Gravitation Field" and sent it to the Global Journal with a request for a review. The reviewers did not raise any objections, recognized the article as interesting and recommended to publish it. (I do not provide links to this article, because I present all its results in this article and consistently interpret in accordance with the new theory. In particular, what was the main assumption has become a mathematically substantiated fact). Then the author decided to study the question more deeply and used for this purpose the monographs [2], in which the Riemannian geometry is presented in depth and comprehensively for the general case *n*-dimensional manifold.

The authors of works on the general theory of relativity use the results of mathematicians for the four-dimensional space-time. Sometimes attempts are made to introduce additional dimensions of space to introduce fields other than the gravitational one. But it turned out that the consistent formulation of the theory proposed by the author requires considering the Riemannian geometry of three-dimensional space and introducing the time axis additionally, just as it is done in the Special Theory of Relativity.

In the second section of this work, a brief critical review of the theories of space-time adopted in the Special Theory of Relativity and the General Theory of Relativity is given.

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In the third section, we show that in three-dimensional space the metric tensor, when diagonalized, becomes the unit diagonal matrix multiplied by a scalar function of coordinates. In the case of a Riemannian space, this function differs from unity by an amount that we have called the gravitational potential. The diagonalized Ricci tensor turns out to be the Laplacian of the gravitational potential. In a flat space, the Ricci tensor is equal to zero, then the gravitational potential does not depend on the coordinates. If a mass is distributed in some region of space, creating a gravitational field, then for the gravitational potential we obtain a field equation similar to the Poisson equation in electrostatics.

In the fourth section, the concepts of time and motion are introduced and a rigorous construction of a four-dimensional pseudo-Riemannian space is proposed based on the construction of a four-dimensional pseudo-Euclidean Minkowski space.

The fifth section considers a region of three-dimensional space in which each point is located on an equipotential surface, determined by one parameter, the value of the gravitational potential. Then the initial conditions define the trajectory as a geodesic line on this surface. In this case, the differential of the world line arc is proportional to dt and is determined by a formula similar to the formula of special relativity with the world constant c_0 replaced by the product c_0Q, where Q is less than 1 and is determined by the value of the gravitational potential on the corresponding equipotential surface.

II. Space and Time in the Theory of Relativity. Critical Review

Physical space is the space of positions. In the absence of gravity, it is a three-dimensional affine space in which a three-dimensional orthogonal frame of the curvilinear coordinate system can be introduced at any point. Point coordinates determine its position and can be changed by introducing another system of curvilinear coordinates. If in an affine space a Cartesian system of orthogonal coordinates describing the entire space can also be introduced, then such a space is called Euclidean. The complete axiomatics of an affine space can be found in the monograph [2].

Changing the position of a material point in an unchanged system of spatial coordinates is called movement. A point can be in various states of motion, in particular, in a state of rest. The state is described in a three-dimensional state space, the three dimensions of which x^1, x^2, x^3 form the space of positions. A trajectory $l(\boldsymbol{r}(t))$ is a line that a point describes in position space when its coordinates are continuous functions of time. The fourth dimension of the space of states - time, has completely different physical properties, but in the modern generally accepted theory of relativity, this is described only by the fact that the time is measured by an imaginary number $\mathrm{i}c_0t$, where c_0 is the world constant, the speed of a plane electromagnetic wave in vacuum. The coordinate axis of time is perpendicular to all three Cartesian axes of space. Such a space, called the pseudo-Euclidean rank one, was introduced by Minkowski and well described all the phenomena of Einstein's Special Theory of Relativity (STR). A detailed description of this geometry can be found in [1,2] and any other monograph where STR is described.

Trajectory arc differential is

$$dl = dt\sqrt{v_1^2 + v_2^2 + v_3^2} = |\mathbf{v}|\,dt\,. \tag{1}$$

In Minkowski space, the history of the existence of a material particle in space and time is described by a world line, whose arc differential is

$$dS = dt\sqrt{-c_0^2 + (v^1)^2 + (v^2)^2 + (v^3)^2} = \mathrm{i}c\,dt\sqrt{1 - \mathbf{v}^2/c_0^2}\,. \tag{2}$$

(Here and below, we will denote by capital letters the quantities related to the four-dimensional space-time). Einstein, based on the similarity of gravitational forces with inertial forces and the equality of inertial mass and mass in the law of gravitation, suggested that gravitation is a change in the geometry of four-dimensional space-time, the replacement of pseudo-Euclidean geometry with pseudo-Riemannian one. But the sequential development of the geometry of the four-dimensional pseudo-Riemannian space was not carried out, it was assumed that the imaginary coordinate can be treated in the same way as with the real one. This led to incorrect results.

The set of elements of the four-dimensional Riemannian space, called points, is one-to-one mapped onto a connected region of change of four real variables x^i up to an arbitrary transformation of these variables into new variables according scheme $x^i = f^i(x'^0, x'^1, x'^2, x'^3)$ (Latin indices take the values 0, 1, 2, 3). This transformation must be reversible and differentiable. The special properties of the time coordinate are not considered in any way. They cannot be preserved at a certain coordinate with arbitrary transformations of all coordinates. If we determine that in some coordinate system one of the coordinates is an imaginary number, then when transforming coordinates, all coordinates become complex numbers. In a Riemannian space, an orthogonal frame leading to the differential form (2) can only be introduced locally, in a Euclidean space that is tangent at a certain point in the Riemannian space. Therefore, the four-dimensional space of general relativity is not a direct generalization of the four-dimensional space of STR, the Minkowski space. This leads to many contradictions, noted, for example, in the monograph [1].

The main contradiction, which is usually ignored, is that when the curvature tensor tends to zero, the transition of the four-dimensional space of general relativity into the space-time of Minkowski does not occur. In the monograph [1], the differential quadratic form (2) (interval) of a pseudo-Euclidean space, which is invariant under coordinate transformations, is transferred without proof to a four-dimensional pseudo-Riemannian space, the mathematical description of which is not given. In the monograph [2], a pseudo-Riemannian space is defined by the fact that the tangent spaces at each point of the Riemannian space are pseudo-Euclidean. But the tangent spaces of a Riemannian space can only be Euclidean spaces, and vice versa, pseudo-Euclidean spaces can only be tangent to a pseudo-Riemannian space, which is not defined. Therefore, in the chapter of the monograph [2] devoted to the mathematical foundations of general relativity, the change in geometry is described by introducing an $\gamma_{ij} dx_i dx_j$ where $\gamma_{ij} \ll 1$ into the differential form (2). Similar assumptions about small gravitational potential, not related to geometry, are made by other authors, in particular, in the monograph [1].

III. Gravitational Potential in Three-Dimensional Riemannian space

The positional space is a three-dimensional Riemannian space R_3, 3-manifold in which the metric tensor field $\left\| \boldsymbol{g}\,(x^1, x^2, x^3) \right\|$ is given. Let's consider some special properties of this matrix. Quadratic form of three-dimensional Riemannian space $dl^2 = g_{\alpha\beta} dx^\alpha dx^\beta$ can be diagonalized algebraically. The characteristic equation of the matrix of the third rank is the equation of the third degree $y^3 + ay^2 + by + c = 0$. Its coefficients are certain functions of the six components of the metric tensor at the considered point in space:

$$a = g_{11} + g_{22} + g_{33}$$

$$b = g_{12}{}^2 + g_{13}{}^2 + g_{23}{}^2 - g_{11}g_{22} - g_{33}g_{22} - g_{33}g_{11} \quad . \tag{3}$$

$$c = g_{11}g_{22}g_{33} - \left[g_{11}g_{23}{}^2 + g_{22}g_{13}{}^2 + g_{33}g_{12}{}^2 \right] + 2g_{12}g_{23}g_{31}$$

The values of the coefficients change depending on the coordinates of the point, but from (3) it is obvious that they are always real. The roots of a cubic equation are certain functions of its three coefficients. They can be real or complex, but it is known that one of the roots of a cubic equation is real regardless of the values of the coefficients. We will further denote the only always real root of the characteristic equation $G(a,b,c,)$. Hence the matrix

$$\left\| \boldsymbol{g}\,(x^1, x^2, x^3) \right\| = \left[1 + G(a,b,c) \right] \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix}, \; g_{\mu\nu} = \delta_{\mu\nu} \left[1 + G\,(x^1, x^2, x^3) \right] \quad . \tag{4}$$

The dependence of the metric tensor on coordinates is included in the definition of $G(a,b,c,)$ through the definition of three coefficients of the characteristic equation (formula (3)). Since the components of the metric tensor in the Riemannian space are function of the coordinates, $G(a,b,c,)$ is also definite function of the spatial coordinates $G(\boldsymbol{r})$. In Euclidean space $G(\boldsymbol{r})=0$. We will further call the function $G(\boldsymbol{r})$ the gravitational potential.

The result expressed by formula (4) is a general theorem: in three-dimensional space, a bivalent symmetric tensor reduces to a scalar function of coordinates multiplied by the identity matrix. The quadratic form (arc length element) in the Riemannian space under consideration has the form:

$$dl^2 = \left[1 + G(\boldsymbol{r})\right]\left[\left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2\right]. \tag{5}$$

Such a space is called locally Euclidean.

The metric tensor is six arbitrary and independent functions of coordinates that satisfy the differentiability condition. Therefore, the gravitational potential is still an arbitrary function. To find an equation whose solution can be the gravitational potential, let's move on to considering the Riemann tensor for three-dimensional space. Riemannian geometry asserts that a necessary and sufficient condition for a space to be flat is the equality of the Riemann tensor to zero. It is easy to show (see to [2]) that in three-dimensional space the Riemann tensor reduces to the bivalent symmetric Ricci tensor. According to the theorem proved above, such a tensor reduces to an identity matrix multiplied by a scalar function. This function is expressed in terms of the second derivatives with respect to the coordinates of the diagonal components of the metric tensor (4). As a result, we get:

$$\left\|R_{\mu\nu}\right\| = \delta_{\mu\nu}\sum_{\alpha=1}^{3}\frac{\partial^2 G}{\left(\partial x^\alpha\right)^2}\begin{Vmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{Vmatrix} = \Delta G\left(x^1, x^2, x^3\right)\begin{Vmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{Vmatrix}. \tag{6}$$

Therefore, three-dimensional space is flat if the Laplacian of the gravitational potential is identically equal to zero. This means that there is no body that creates a gravitational field.

If the mass density is not equal to zero in some region of space, then the gravitational potential is a solution of the Poisson equation.

This statement can be taken as the basis of the theory of the gravitational field.

IV. Time and Movement

Time is a physical parameter on which all physical quantities depend. Time increases uniformly from an arbitrarily chosen zero, $dt > 0$. Change in physical quantity $a(t)$ with time is determined by the speed da/dt. The physical quantity can also be stationary over a certain period of time, that is $da/dt = 0$. This article will consider only the position of a material point in space and its change in time. An example of another physical quantity, the change in which with time is studied by another section of physics, is entropy.

As shown in Section 3, the geometry of space is determined by gravity. The mass of the particle is so small compared to the mass of the body that creates the gravitational field that its influence on the field can be neglected. Motion is the change in the position of a particle over time. These changes can be divided into the change in time of the geometry of space, i.e., its

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metric tensor, and the motion of a particle in stationary space. Obviously, a change in time of the mass-energy density distribution in a body that creates a gravitational field should also cause a change in time of the gravitational potential. For example, vibrations in the body should generate gravitational waves. An important special case is the rotation of a body, which creates a gravitational field. Its influence on the field is manifested in the fact that the three-dimensional space becomes non-isotropic: the axis of rotation retains its direction.

In a four-dimensional pseudo-Euclidean space of rank 1, the time axis is perpendicular to the three spatial axes and $dx^0 = \mathrm{i}cdt$. As shown in Section 3, the three-dimensional Riemannian space is conformal to the Euclidean space only locally, in a small neighborhood of the chosen point. For the time axis to remain perpendicular to the spatial axes, it must be perpendicular to the three-dimensional Euclidean tangent space at the point in question. Therefore, the four-dimensional event space is not just a four-dimensional Riemannian space. It must be locally conformal to a four-dimensional pseudo-Euclidean space of rank 1.

A four-dimensional pseudo-Riemannian space is an essentially new concept in mathematics. Therefore, for clarity, we describe a simple three-dimensional pseudo-Riemannian space. A two-dimensional Riemannian space is also locally Euclidean [2]. An example of such a space is the surface of a sphere. At each point of this surface, we introduce the time axis on the continuation of the radius, measured by the imaginary coordinate. The result is a three-dimensional pseudo-Riemannian space.

Next, consider the motion of a particle in space and time in a stationary gravitational field. The laws of motion of particles in the absence of gravity, confirmed by experiments, were formulated by Einstein. Their convenient formulation in the form of the geometry of a four-dimensional pseudo-Euclidean space was proposed by Minkovski. An attempt to formulate such a theory for a four-dimensional Riemannian space, the geometry of which changes under the action of gravity, should be recognized as unsuccessful, as shown in Section 2.

An event is the presence of a particle at a certain point at a certain moment in time. Let a material point pass for some time a path in a three-dimensional Riemannian space (a trajectory) $l\left[x^1(t), x^2(t), x^3(t)\right]$. In the four-dimensional pseudo-Riemannian space, each point of the trajectory corresponds to the time axis, perpendicular to the tangent three-dimensional Euclidean space at this point. On this axis, the time interval elapsed from the beginning of the movement to the moment of reaching this point is plotted. The space of events must be four-dimensional, consisting of a three-dimensional space of places and coordinate axes of time, orthogonal at each point to the three-dimensional tangent space.

V. Equations of Motion of a Particle in a Gravitational Field

The main task of mechanics in the macroworld is to determine the world lines of particles in various conditions. In pseudo-Euclidean space, in the absence of forces, this is always a straight line determined by the initial conditions: the starting point and the velocity vector. This is formulated in Newton's first law - the law of inertia: "A particle maintains a state of rest or uniform, rectilinear motion, if no an external force acts on it." In the pseudo-Riemannian event space in this law, only the words "uniform, rectilinear motion" should be replaced by "movement with constant speed."

The transfer of the laws of motion to the pseudo-Riemannian space requires a consistent generalization of concepts and definitions. As shown above, no consistent introduction of the time axis has been made. Therefore, it was believed that the element of the arc ds is proportional dt and built a theory of geodesic lines, along which a point in the absence of acting forces moves at a constant speed. This is unfounded and, as will be shown, incorrect. Next, we correct this theory by using the notion of a pseudo-Riemannian space.

The differential equation of a geodesic curve in a Riemannian space was mathematically rigorously derived [2]. In this case, the parameter that determines the position of the point on the curve is the length of the arc of the curve, counted from the starting point. A geodesic line is a line on which the modulus of the tangent vector with components dx^{α}/ds remains constant along the line. The arc length is called the canonical parameter [2] and it is proved that another parameter can be canonical only if it is associated with s by a linear transformation with constant coefficients.

To consider physical problems, it is necessary to use time as a parameter. In pseudo-Euclidean space (in the special theory of relativity) in curvilinear coordinates, taking into account the fact that the metric tensor is diagonal and does not depend on the coordinates, the world line length differential

$$dS = \mathrm{i}dt\sqrt{c_0{}^2 - g_{kj}dv^k dv^j} = \mathrm{i}c_0 dt\sqrt{1-|\mathbf{v}|^2/c_0{}^2}\ . \tag{7}$$

In this case, dt differs from dS by a constant factor if $|\mathbf{v}|$ is preserved during movement. If we substitute the metric tensor in the gravitational field (4) into formula (7), then we obtain

$$dS = \mathrm{i}dt\sqrt{c_0{}^2 - [1+G\ \boldsymbol{r}\ ]|\mathbf{v}|^2}\ . \tag{8}$$

This formula makes sense if we assume that the movement occurs along an equipotential surface.

Let us consider a region of three-dimensional space in which each point is located on an equipotential surface, determined by one parameter P, the value of the gravitational potential in the point of the initial conditions. Then the initial conditions define the trajectory as a geodesic line on this surface. In this case, the differential of the world line arc dS is proportional to dt and is determined by a formula similar to the formula of special relativity

$$G(\boldsymbol{r}) = P = \text{const}, \ (1+P)^{-1/2} = Q < 1. \quad dS = \frac{\mathrm{i}dt}{Q}\sqrt{(c_0 Q)^2 - |\boldsymbol{v}|^2} \tag{9}$$

with the world constant c_0 replaced by the product $c_0 Q$, where $(1+P)^{-1/2} = Q < 1$.

Then the assignment of this surface, the initial point and the initial velocity vector completely determine the world line.

Recall that the gravitational potential is a solution to the Laplace equation, which is determined by the shape of body that creates the gravitational field, and the mass-energy distribution in it. The origin of coordinates is at the center of mass of the body, the point $\boldsymbol{r}$ is outside the body and $|G(\boldsymbol{r})|$ decreases with increasing $|\boldsymbol{r}|$. Such a correction seems to be quite reasonable: the speed of light increases with distance from the center of gravity, tending to the limit c_0.

$$\frac{dS}{dt} = \sqrt{c_0 Q^{\ 2} + |\boldsymbol{v}|^2} \ . \tag{10}$$

“For $|\boldsymbol{v}| < c_0 Q$ the arc element of the world line is an imaginary number. This means that the world line is time like. But it is possible that $|\boldsymbol{v}| > c_0 Q$. Then the world line is space like and the particle cannot move away from the center that creates gravity. This means that particle is in a black hole”.

The transition from kinematics to dynamics, that is, the introduction of the mass of the particle and the replacement of velocities by impulses, is carried out in the same way as in STR.

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