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Highlights

- Intuitionistic Fuzzy Structures
- Systems using the Kalman Algorithm
- Bessel Differential Equation
- Convergence Theorems of Multi-Valued

Discovering Thoughts, Inventing Future

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Obtaining the Sinusoid for Working with Membrane Vibration from the Bessel Differential Equation

By Jose Mujica EE & Ramon A. Mata-Toledo

Abstract- To study membranes' acoustic behavior there are several equations concerning their vibration and resonance which help us to understand better their physical properties. In the present paper we want to show the results of the Bessel sinusoid expressions when we change the values of their variables. Because the procedures to solve the Bessel equation are usually not shown in their entirety in books or the Internet this paper explains in detail each step to go from its differential expression to the sinusoid one.

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Notes

Obtaining the Sinusoid for Working with Membrane Vibration from the Bessel Differential Equation

Jose Mujica EE [α] & Ramon A. Mata-Toledo [σ]

Abstract- To study membranes' acoustic behavior there are several equations concerning their vibration and resonance which help us to understand better their physical properties. In the present paper we want to show the results of the Bessel sinusoid expressions when we change the values of their variables. Because the procedures to solve the Bessel equation are usually not shown in their entirety in books or the Internet this paper explains in detail each step to go from its differential expression to the sinusoid one.

I. Introduction

It is well known that the expression J_0 of Bessel equations is the graphic most widely published in Differential Equation textbooks. Every time we talk about the vibration of a circular membrane the graphics of Figure 1is always referred to. Mathematical software packages such as Maple™ always includes this curve in their libraries. If you hit once a circular membrane, it is easy to assume that the first undulation will have a higher amplitude than the following undulations which will decrease gradually as shown also in Figure 1.

$plot([\text{BesselJ}(0, x), \text{BesselJ}(1, x)], x = 0..30, y = -2..2, color = [red, blue]);$

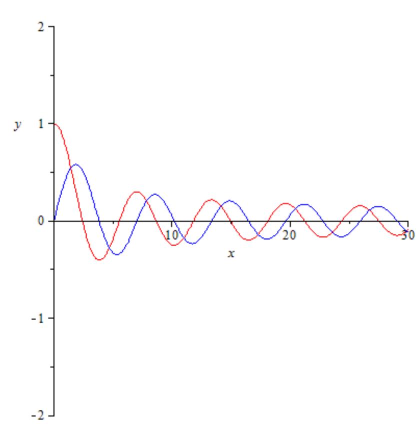

Figure 1: Plot of J0 and J1 of Bessel

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Author σ: Ph.D, Rollins College, Winter Park, U.S.A. e-mail: matatora@jmu.edu

In this work we proceed to find the Jv(x) functions of the first kind and order v to satisfy the second order differential equation of Bessel.

$$x^2\frac{d^2y(x)}{dx^2}+x\frac{dy(x)}{dx}+(x^2-v^2)y(x)=0$$

To study acoustical radiation and vibration analysis one usually solves problems in cylindrical coordinates that are associated with Besselfunctions of integer order.We can solve also spherical problems with half-integer order Bessel Function.

II. Bessel Functions

a) Differential Equation of Oder v

Wefocus this work on Besselfunctions of the first kind of positive order and real arguments[1]for

$$x^2y"+xy'+(x^2-v^2)y=0$$

Assuming

$$y=\sum_{0}^{\infty}c_n\,x^{n+r}$$

which leads to

Differentiating and substituting

$$y'=c_{n(n+r)X^{n+r-1}}$$

$$y"=c_{n(n+r)(n+r-1)X^{n+r-2}}$$

$$x^2y"+xy'+(x^2-v^2)y=0$$

$$x^2c_n(n+r)(n+r-1)x^{n+r}x^{-2}$$

$$c_n(n+r)(n+r-1)x^{n+r}$$

$$x^2y"+xy'+(x^2-v^2)y=0$$

$$xc_n(n+r)x^{n+r}x^{-1}$$

$$x^2y"+xy'+(x^2-v^2)y=0$$

$$c_n(x^2-v^2)x^{n+r}$$

Writing out the differential equation give us

$$\sum_{n=0}^{\infty}c_n(n+r)(n+r-1)\,x^{n+r}+\sum_{n=0}^{\infty}c_n(n+r)x^{n+r}+\sum_{n=0}^{\infty}c_nx^{n+r+2}-v^2\sum_{n=0}^{\infty}c_nx^{n+r}$$

Ref

1. Dennis Zill, A First course in Differential Equations with Applications, 2nd. Edition, PWS Publishers, Wadsworth, Inc. Belmont, California, 1982.

For n=0

$$c_0(0+r)(0+r-1)x^{0+r}+c_0(0+r)x^{0+r}+c_0x^{0+r+2}-c_0v^2x^{0+r}$$

$$c_0(r)(r-1)x^r+c_0(r^2-r)x^r+c_0rx^r+c_0x^{r+2}-c_0v^2x^r$$

$$c_{0(x^rr^2-x^rr+rx^r+x^rx^2-v^2x^r)}$$

$$c_0(r^2-r+r+x^2-v^2)x^r$$

Substituting this back to the differential equation gives

$$c_0(r^2-r+r-v^2)x^r+x^r\sum_{n=1}^{\infty}c_n[(n+r)(n+r-1)+(n+r)-v^2]x^n+x^r\sum_{n=0}^{\infty}c_nx^{n+2}$$

$$(n+r)(n+r-1)+(n+r)$$

$$(n+r)(n+r+1)=n^2+2nr+r^2-n-r$$

$$n^2+2nr+r^2-n-r+(n+r)$$

$$n^2+2nr+r^2$$

$$(n+r)^2$$

Leading to

$$c_0(r^2-v^2)x^r+x^2\sum_{n=1}^{\infty}c_n[(n+r)^2-v^2]x^n+x^r\sum_{n=1}^{\infty}c_nx^{n+2}=0$$

If $r_1=v$

$$[(n+v)^2-v^2]=n^2+2nv+v^2-v^2$$

$$n(n+2v)$$

giving

$$x^v\sum_{n=1}^{\infty}c_nn(n+2v)x^n+x^v\sum_{n=0}^{\infty}c_nx^{n+2}$$

or

$$x^v\left[c_1(1+2v)+\sum_{n=2}^{\infty}c_nn(n+2v)x^n+\sum_{n=0}^{\infty}c_nx^{n+2}\right]$$

Notes

Using k =n-2 in

$$\sum_{n=2}^{\infty} c_n n(n+2v)x^n$$

And k =n in

$$\sum_{n=0}^{\infty} c_n x^{n+2}$$

leads to

$$\sum_{k=0}^{\infty}[(k+2)(k+2+2v)c_{k+2}+c_k]x^{k+2}=o$$

Writing this out

$$x^v\left[c_1(1+2v)+\sum_{k=0}^{\infty}[(k+2)(k+2+2v)c_{k+2}+c_k]x^{k+2}\right]=0$$

So that

$$(1+2\text{v})=0$$

$$(k+2)(k+2+2v)c_{k+2}+c_k=0$$

Or

$$c_{k+2}=\frac{-c_k}{(k+2)(k+2+2v)} \qquad \text{k=0,1,2..} \qquad (1)$$

$c_1=0$ in (1) leads to$c_3=c_5=c_7=\ldots=0$ so, for k=0,2,4. after makingk+2=2n, n=1, 2, 3..we will have that

$$c_{2n}=\frac{-c_{2n-2}}{2n(2n+2v)}$$

$$\frac{-c_{2n-2}}{2^2 n(n+v)}$$

The coefficients with even index are determined by the following formula:

$$c_2=-\frac{c_0}{2^2*1*(1+v)}$$

$$c_4=-\frac{c_2}{2^2*2*(2+v)}=\frac{c_0}{2^4*1*2(1+v)(2+v)}$$

$$c_6=-\frac{c_4}{2^2*3*(3+v)}=\frac{c_0}{2^6*1*2*3(1+v)(2+v)(3+v)}$$

.
.
.

$$c_{2n}=\frac{(-1^n)c_0}{2^n n!\,(1+v)(2+v)..(n+v)}$$

Notes

i. *The Gamma Function*

Since we can select a_0, we take it to be [2]

$$c_0 = \frac{1}{2^v \Gamma(1+\mathrm{v})}$$

The Gamma function Γ is defined as

$$\Gamma(n) = \int_0^\infty e^{-t} t^{n-t} dt$$

then

$$\Gamma(n+1) = n!$$

Where

$$\Gamma(1+\mathrm{v}+1) = (1+\mathrm{v})\,\Gamma(1+v)$$

$$\Gamma(1+\mathrm{v}+2) = (2+\mathrm{v})\,\Gamma(2+v)$$

$$= (2+\mathrm{v})(1+\mathrm{v})\,\Gamma(1+v)$$

Then we can write

$$c_{2n} = \frac{(-1^n)c_0}{2^{n+v} n!\,(1+v)(2+v)..(n+v)\Gamma(1+v)}$$

$$c_{2n} = \frac{(-1^n)c_0}{2^{n+v} n!\,\Gamma(1+v+n)}$$

The solution for this c_n is

$$y = \sum_{n=0}^{\infty} c_{2n} x^{2n+v}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(1+v+n)} \left(\frac{x}{2}\right)^{2n+v}$$

$$J_v(X) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(1+v+n)} \left(\frac{x}{2}\right)^{2n+v}$$

b) *Half Integer Order [3]*

$$j_{\frac{1}{2}}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(1+\frac{1}{2}+n)} \left(\frac{x}{2}\right)^{2n+\frac{1}{2}}$$

Ref

2. Dennis Zill, Advance Engineering Mathematics, 7th Edition, Jones & Barlett Learning, Burlington, MA, 2022.

$$j_{\frac{1}{2}}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(\frac{3}{2}+n)} \left(\frac{x}{2}\right)^{2n} \left(\frac{x}{2}\right)^{\frac{1}{2}}$$

$$j_{\frac{1}{2}}(x) = \sqrt{\frac{1}{2}} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!\,\Gamma(\frac{3}{2}+n)} \left(\frac{x}{2}\right)^{2n}$$

$$j_{\frac{1}{2}}(X) = \sqrt{\frac{1}{2}} \left[\frac{(1)}{\Gamma\left(\frac{3}{2}\right)} - \left(\frac{x}{2}\right)^2 \frac{1}{\Gamma\left(\frac{5}{2}\right)1!} + \left(\frac{x}{2}\right)^4 \frac{1}{\Gamma\left(\frac{7}{2}\right)2!} - \cdots \right]$$

$$\Gamma_{\frac{1}{2}} = \sqrt{\pi}$$

$$\Gamma_{\frac{3}{2}} = \frac{3\sqrt{\pi}}{4}$$

$$\Gamma_{\frac{5}{2}} = \frac{15\sqrt{\pi}}{8}$$

$$j_{\frac{1}{2}}(x) = \sqrt{\frac{1}{2}} \left[\frac{2}{\sqrt{\pi}} - \frac{x^2}{4}\frac{4}{3\sqrt{\pi}} + \frac{x^4}{16}\frac{8}{15\sqrt{\pi*2}} - \cdots \right]$$

$$= \sqrt{\frac{x}{2\pi}} \left[2 - \frac{x^2}{3} + \frac{x^4}{60} - \cdots \right]$$

$$= \sqrt{\frac{x}{2\pi}} * \frac{2}{x} \left[x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots \right]$$

$$= \sqrt{\frac{2}{\pi x}} \left[x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \right]$$

$$j_{1/2(x)} = \sqrt{\frac{2}{\pi x}} * \sin(x) \tag{2}$$

c) Spherical Bessel [4]

If we solve the Helmholtz's equation we obtain the following differential

$$\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + \left[1 + \frac{l(l+1)}{x^2}\right] y = 0 \tag{3}$$

One of the solutions of (3) is

$$Jj_l(x) = \sqrt{\frac{\pi}{2x}} J_{l+\frac{1}{2}}(x)$$

The Bessel function of half-integral order is used to define the important function:

Ref

4. Teboho A. Moloi, Spherical Bessel Functions, Nelson Mandela University, Port Elizabeth, South Africa, 2022.

$$J_n(x) = \sqrt{\frac{\pi}{2x}} J_{n+\frac{1}{2}}(x)$$

J_n is called the spherical Bessel function of the first kind. For n=0 we can see that (1) becomes [5]

$$J_0(x) = \sqrt{\frac{\pi}{2x}} J_{\frac{1}{2}}(x) = \sqrt{\frac{\pi}{2x}} \sqrt{\frac{2}{\pi x}} \sin x = \frac{\sin x}{x}$$

III. Plotting with Maple

a) Plotting sin(x)/x

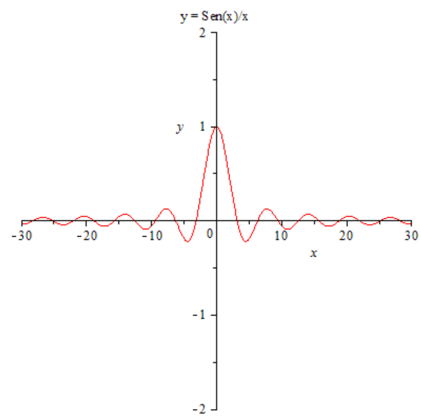

Figure 2: Plot of sin(x)/x

b) Plotting of J(-1/2)

$$plot\left(\sqrt{\frac{2}{\pi \cdot x}} \cdot \cos\left(x - \frac{1}{4}(1) \cdot \pi\right) + \sqrt{\frac{2}{\pi \cdot x}} \cdot \cos\left(x - \frac{1}{4}(2) \cdot \pi\right) + \sqrt{\frac{2}{\pi \cdot x}} \cdot \cos\left(x - \frac{1}{4}(3) \cdot \pi\right), x = 1..20, y = -1..5, discont = true, color = red\right)$$

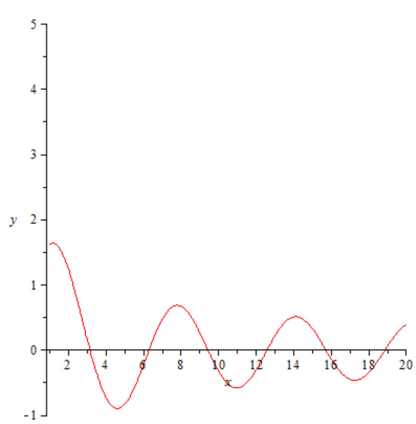

Figure 2: Plot of Sin (2/πx)^2*Sin(x)

Ref

5. Annie Cuyt1, Wen-shin Lee1; 2, and Min Wu3. High accuracy trigonometric approximations of the real Bessel functions of the First kind, University of Stirling, Scotland, UK,2019.

c) Plotting J(1/2) with variations

$$plot\left(\left[\sqrt{\frac{2}{\pi \cdot x}} \cdot \sin\left(x - \frac{1}{4}(1) \cdot \pi\right) + \sqrt{\frac{2}{\pi \cdot x}} \cdot \sin\left(x - \frac{1}{4}(2) \cdot \pi\right) + \sqrt{\frac{2}{\pi \cdot x}} \cdot \sin\left(x - \frac{1}{4}(3) \cdot \pi\right), 2 \cdot \left(\sqrt{\frac{2}{\pi \cdot x}} \cdot \sin\left(x - \frac{1}{4}(1) \cdot \pi\right) + \sqrt{\frac{2}{\pi \cdot x}} \cdot \sin\left(x - \frac{1}{4}(2) \cdot \pi\right) + \sqrt{\frac{2}{\pi \cdot x}} \cdot \sin\left(x - \frac{1}{4}(3) \cdot \pi\right)\right)\right], x = 1..20, y = -5..5, discont = true, color = [red, blue]\right)$$

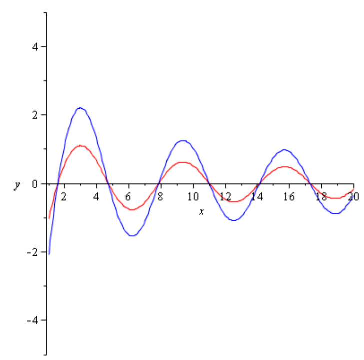

Figure 4: Plot of Two variations of J(1/2)

$$plot\left(2 \cdot \left(\frac{\sin(x)}{8x}\right), x = -30..30, y = -2..2, discont = true, title = \text{"y = Sen(x)"}\right);$$

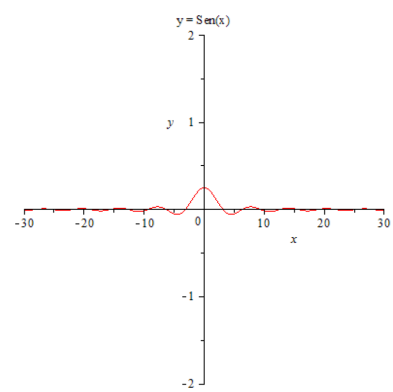

Figure 5: In this plot you can assume a change in the frequency

$$plot\left(2 \cdot \left(\frac{\sin(x)}{x}\right), x = -30..30, y = -2..2, discont = true, title = \text{"y = Sen(x)"}\right);$$

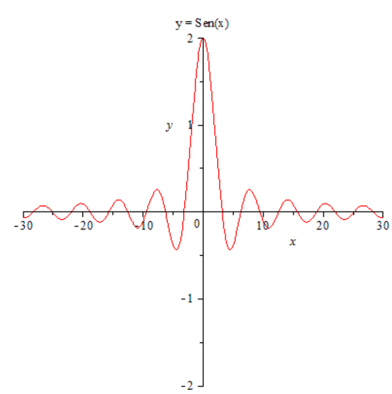

Figure 6: In this plot you can assume a change in Amplitude

Notes

Notes

IV. Comparing Bessel with a Linear Model Underdamped

Given that

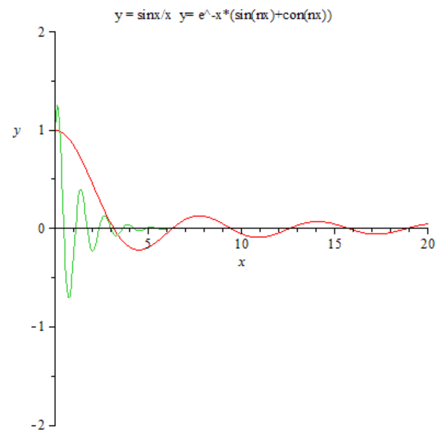

Figure 7: Bessel J(1/2) vs Over damped

V. Animation of Bessel on Maple Software

$$animate(wave, [uC(0,3)], t = 0..P(0,3));$$

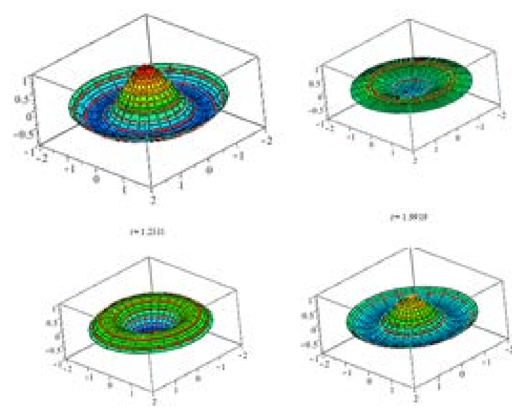

Figure 8: Four snapshots of the Bessel animation

VI. Conclusion

A study of Bessel equations can lead us to new applications in acoustics. By changing their parameters, we can determine the ratio between a membrane dimension and the acoustic power it can radiate or the way the membrane will vibrate. For example, we can find analogies between the elasticity of a trampoline jumper and a loudspeaker membrane. We can also study suspension, material thickness, and surfaces and compare them to computer animations which allow us to explore these analogies. Thus, educational institutions with low budgets can make physics experiments on vibration and mechanics at more affordable prices using computer animations. Knowing Bessel equation and its relationship with Helmholtz's formula we can extend the application to the resonance phenomena too.

Acknowledgment

The authors thank Dr. James S. Sochacki for his invaluable assistance and his thorough review of this paper. Dr. Sochacki is a James Madison University Emeritus

Professor in the department of Mathematics and Statistics. Dr. Sochacki is also co-author of the Parker-Sochacki's algorithm widely for solving systems of ordinary Differential Equations (https://en.wikipedia.org/wiki/Parker%E2%80%93Sochacki_method).

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5. Annie Cuyt1, Wen-shin Lee1; 2, and Min Wu3. High accuracy trigonometric approximations of the real Bessel functions of the First kind, University of Stirling, Scotland, UK,2019.
6. Maple Software plot. https://www.maplesoft.com/

Notes

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Intuitionistic Fuzzy Structures

By Khalid Ebadullah
Mai Nefhi College of Science

Abstract- In this article we introduce to certain class of intuitionistic fuzzy structures.

Keywords: *ideal, filter, i-convergence, intuitionistic fuzzy normed spaces.*

GJSFR-F Classification: *AMS 2010: 40A05, 40A35, 40C05*

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35. L.A. Zadeh. Fuzzy Sets. Inform Control (8) (1965), 338-353.

Intuitionistic Fuzzy Structures

Khalid Ebadullah

Abstract- In this article we introduce to certain class of intuitionistic fuzzy structures.

Keywords: ideal, filter, i-convergence, intuitionistic fuzzy normed spaces.

I. Introduction

The fuzzy theory has emerged as the most active area of research in many branches of science and engineering. Among various developments of the theory of fuzzy sets[35] a progressive development has been made to find the fuzzy analogues of the classical set theory. In fact the fuzzy theory has become an area of active research for the last 50 years. It has a wide range of applications in the field of science and engineering, e.g. application of fuzzy topology in quantum particle physics that arises in string and $e^{(\infty)}$-theory of El-Naschie[7-11], electronic engineering, chaos control, computer programming, electrical engineering, nonlinear dynamical system, population dynamics and biological engineering etc.

In [35], Zadeh introduced the theory of fuzzy sets. Atanassov[1-2] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Later on the theory of intuitionistic fuzzy sets caught the interest of various mathematicians round the globe and a huge literature in this direction has been produced in forms of books and research papers published in famous journals round the globe. By reviewing the literature, one can reach them easily, (e. g., see, [1-6], [12], [13], [15],[26], [30] and [31] and the references there in.)

Sequences play an important role in various fields of Real Analysis, Complex Analysis, Functional Analysis and Topology. These are very useful tools in demonstrating abstract concepts through constructing examples and counter examples. The topic "Sequence spaces" is very broad in its own sense as one can study from various point of views. In [22] Kostyrko, Salat and Wilczynski introduced and studied the concept of I-Convergence and later on there has been much progress and development in the study in this direction. (e. g., see, [12], [14-19], [32] and [34] and the references there in.)

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In many branches of science and engineering we often come across with different type of sequences and certainly there are situations of inexactness where the idea of ordinary convergence does not work. So to deal with such situations we have to introduce new measures and tools which are suitable to the said situation. Here we give the preliminaries.

II. Preliminaries

Notes

Now we quote the following definitions which will be needed in the sequel

Definition 2.1. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is said to be continuous t-norm if it satisfies the following conditions;

(a) * is associative and commutative.

(b) * is continuous.

(c) a*1=a for all a∈[0,1]

(d) a*c≤b*d whenever a≤b and c≤d for each a,b,c,d∈[0,1].

For example, $a * b = a \cdot b$ is a continuous t-norm.

Definition 2.2. A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is said to be continuous t-conorm if it satisfies the following conditions;

(a) $\diamond$ is associative and commutative.

(b) $\diamond$ is continuous.

(c) a$\diamond$0=a for all a∈[0,1]

(d) a$\diamond$c≤b$\diamond$d whenever a≤b and c≤d for each a,b,c,d∈[0,1].

For example, a$\diamond$b=min$\{a + b, 1\}$ is a continuous t-conorm.

Definition 2.3. Let * be a continuous t-norm and $\diamond$ be a continuous t-conorm and X be a linear space over the field(R or C). If μ and ν are fuzzy sets on X×(0,∞) satisfying the following conditions, the five- tuple (X,μ,ν,*,$\diamond$) is said to be an intuitionistic fuzzy normed space(IFNS) and (μ,ν) is called an intuitionistic fuzzy norm. For every x, y ∈ X and s, t>0,

(a)μ(x,t)+ν(x,t)≤ 1,

(b)μ(x,t)>0,

(c)μ(x,t)=1 iff x=0,

(d)μ(ax,t)=μ(x, $\frac{t}{|a|}$) for each a ≠0,

(e)μ(x,t)*μ(y,s)≤μ(x+y,t+s)

(f)μ(x,·) : (0,∞)→[0,1] is continuous,

(g)$\lim_{t\to\infty} \mu(x, t) = 1$ and $\lim_{t\to 0} \mu(x, t) = 0$,

(h)ν(x,t)< 1,

(i)ν(x,t)=0 iff x=0,

(j)ν(ax,t)=ν(x, $\frac{t}{|a|}$) for each a $\neq$0,

(k)ν(x,t)$\diamond\nu$(y,s)$\geq\nu$(x+y,t+s)

(l)ν(x,$\cdot$) : (0,∞)$\rightarrow$[0,1] is continuous,

(m)$\lim_{t\to\infty} \nu(x,t) = 1$ and $\lim_{t\to 0} \nu(x,t) = 0$,

(n)a*a=a, a$\diamond$a=a for all a$\in$[0,1].

Definition 2.4. Let (X, μ,ν,*,$\diamond$) be IFNS and (x_n) be a sequence in X. Sequence (x_n) is said to be convergent to L$\in$ X with respect to the intuitionistic fuzzy norm (μ,ν) if for every $\epsilon > 0$ and $t > 0$, there exists a positive integer n_0 such that $\mu(x_n$-L,t)$> 1-\epsilon$ and $\nu(x_n$-L,t)$< \epsilon$ whenever $n > n_0$. In this case we write (μ,ν)- limx_n=L as $n \to \infty$.

Definition 2.5. If X be a non- empty set, then a family of set $I \subset P(X)(P(X)$ denoting the power set of X) is called an ideal in X if and only if

(a) $\phi \in I$;

(b) For each $A, B \in I$, we have $A \cup B \in I$;

(c) For each $A \in I$ and $B \subset A$ we have $B \in I$.

Deffinition 2.6. If X be a non- empty set. A non- empty family of sets $F \subset P(X)(P(X)$ denoting the power set of X) is called a filter on X if and only if

(a) $\phi \notin F$;

(b) For each $A, B \in F$, we have $A \cap B \in F$;

(c) For each $A \in F$ and $A \subset B$ we have $B \in F$.

Definition 2.7. Let $I \subset P(N)$ be a non trivial ideal and (X, μ,ν,*,$\diamond$) be an IFNS. A sequence $x = (x_n)$ of elements in X is said to be I- convergent to L$\in$ X with respect to the intuitionistic fuzzy norm (μ,ν) if for every $\epsilon > 0$ and $t > 0$, The set

$$\{n \in N : \mu(x_n - L, t) \geq 1 - \epsilon \text{ or } \nu(x_n - L, t) \leq \epsilon\} \in I$$

In this case L is called the I-limit of the sequence (x_n) with respect to the intuitionistic fuzzy norm (μ,ν) and we write $I_{(\mu,\nu)}$ - limx_n=L.

Definition 2.8. A convergence field of I-convergence is a set

$$F(I) = \{x = (x_n) \in \ell_\infty : \text{there exists } I - \lim x \in R\}.$$

The convergence field $F(I)$ is a closed linear subspace of ℓ_∞ with respect to the supremum norm, $F(I) = \ell_\infty \cap c^I$.

As an insight, while working in the direction of fuzzy theory and Ideal Convergence the author has the following observations

Let N, R and C be the sets of all natural, real and complex numbers respectively. We write

$$\omega = \{x = (x_k) : x_k \in R \; or \; C\},$$

the space of all real or complex sequences.
Listed below are few complementary structures which are intuitionistic fuzzy in nature.

$c_0 = \{x \in \omega : \lim_k |x_k| = 0\}$, the space of null sequences.

$c = \{x \in \omega : \lim_k x_k = l, \quad \text{for some} \quad l \in C\}$, the space of convergent sequences.

$\ell_\infty = \{x \in \omega : \sup_k |x_k| < \infty\}$, the space of bounded sequences.

Recently Kostyrko, Šalát and Wilczyński[22], Šalát Tripathy and Ziman[32] introduced and studied the following sequence spaces

$c_0^I = \{(x_k) \in \omega : \{k \in N : |x_k| \geq \epsilon\} \in I\}$,

$c^I = \{(x_k) \in \omega : \{k \in N : |x_k - L| \geq \epsilon\} \in I$, for some L$\in C$ },

$\ell_\infty^I = \{(x_k) \in \omega : \{k \in N : |x_k| \geq M\} \in I,$ for each fixed M>0}.

Ruckle [27-29] used the idea of a modulus function f to construct the sequence space

$$X(f) = \{x = (x_k) : \sum_{k=1}^{\infty} f(|x_k|) < \infty\}$$

Khan and Ebadullah[16] introduced the following sequence spaces

$c_0^I(f) = \{(x_k) \in \omega : I - \lim f(|x_k|) = 0\}$;

$c^I(f) = \{(x_k) \in \omega : I - \lim f(|x_k|) = L$ for some L$\in C$ };

$\ell_\infty^I(f) = \{(x_k) \in \omega : \sup_k f(|x_k|) < \infty\}$.

Lindenstrauss and Tzafriri[23] used the idea of Orlicz functions to construct the sequence space

$$\ell_M = \{x \in \omega : \sum_{k=1}^{\infty} M(\tfrac{|x_k|}{\rho}) < \infty, \text{for some } \rho > 0\}$$

Tripathy and Hazarika[34] introduced the following sequence spaces

$c_0^I(M) = \{x = (x_k) \in \omega : I - \lim M(\frac{|x_k|}{\rho}) = 0$ for some $\rho > 0\}$;

$c^I(M) = \{x = (x_k) \in \omega : I - \lim M(\frac{|x_k - L|}{\rho}) = 0$ for some L$\in C$ and $\rho > 0\}$;

$\ell_\infty^I(M) = \{x = (x_k) \in \omega : \sup_k M(\frac{|x_k|}{\rho}) < \infty$ for some $\rho > 0\}$;

Ref

32. T.Šalát, B.C.Tripathy, M.Ziman, On some properties of I-convergence. *Tatra Mt. Math. Publ.*,(28) (2004), 279-286.

Kolk[20-21] gave an extension of $X(f)$ by considering a sequence of modulii $F=(f_k)$ and defined the sequence space

$$X(F)=\{x=(x_k):(f_k(|x_k|))\in X\}.$$

Khan, Suantai and Ebadullah[17] introduced the following sequence spaces

$c_0^I(F)=\{(x_k)\in\omega: I-\lim f_k(|x_k|)=0\};$

$c^I(F)=\{(x_k)\in\omega: I-\lim f_k(|x_k|)=L \text{ for some } L\in C\};$

$\ell_\infty^I(F)=\{(x_k)\in\omega: \sup_k f_k(|x_k|)<\infty\}.$

The σ-convergent sequences

$V_\sigma=\{x=(x_k): \sum_{m=1}^{\infty} t_{m,k}(x)=L \text{ uniformly in } k, \text{L}=\sigma-\lim x\},$
where $m\geq 0, k>0$

Khan and Ebadullah[19] introduced the following sequence spaces

$V_{0\sigma}^I(m,\epsilon)=\{(x_k)\in\ell_\infty:(\forall m)(\exists\epsilon>0)\{k\in N:|t_{m,k}(x)|\geq\epsilon\}\in I\}$

$V_\sigma^I(m,\epsilon)=\{(x_k)\in\ell_\infty:(\forall m)(\exists\epsilon>0)\{k\in N:|t_{m,k}(x)-L|\geq\epsilon\}\in I, \text{for some } L\in C\}$

Mursaleen [24-25] defined the sequence space BV_σ, the space of all sequences of σ-bounded variation

$$BV_\sigma=\{x\in\ell_\infty:\sum_m|\phi_{m,k}(x)|<\infty, \text{uniformly in k}\}.$$

Khan, Ebadullah and Suantai [18] introduced the following sequence space

$$BV_\sigma^I=\{(x_k)\in\ell_\infty:\{k\in N:|\phi_{m,k}(x)-L|\geq\epsilon\}\in I, \text{for some } L\in C\}$$

Şengönül[33] introduced the Zweier sequence spaces$\mathcal{Z}_0$ and $\mathcal{Z}$ as follows

$$\mathcal{Z}_0=\{x=(x_k)\in\omega: Z^p x\in c_0\}.$$

$$\mathcal{Z}=\{x=(x_k)\in\omega: Z^p x\in c\}$$

Khan, Ebadullah and Yasmeen [14] introduced the following classes of sequence spaces

$\mathcal{Z}_0^I=\{x=(x_k)\in\omega: I-\lim Z^p x=0\};$

$\mathcal{Z}^I=\{x=(x_k)\in\omega: I-\lim Z^p x=L \text{ for some } L\in C\};$

$\mathcal{Z}_\infty^I=\{x=(x_k)\in\omega: \sup_k |Z^p x|<\infty\}.$

Ref

17. V. A.Khan, S. Suantai, and K. Ebadullah. On Some I-Convergent Sequence Spaces Defined By a Sequence of Moduli, Journal of Nonlinear Analysis and Optimization,3(2) (2012), 145-152.

III. Main Results

The approach is to construct and study new intuitionistic fuzzy I-convergent sequenence spaces

$$c^I_{(\mu,\nu)} = \{\{k \in N : \mu(x_k - L, t) \leq 1 - \epsilon \text{ or } \nu(x_k - L, t) \geq \epsilon\} \in I\},$$

$$c^I_{0(\mu,\nu)} = \{\{k \in N : \mu(x_k, t) \leq 1 - \epsilon \text{ or } \nu(x_k, t) \geq \epsilon\} \in I\},$$

$$V^I_{\sigma(\mu,\nu)} = \{\{k \in N : \mu(t_{m,k}(x) - L, t) \leq 1 - \epsilon \text{ or } \nu(t_{m,k}(x) - L, t) \geq \epsilon\} \in I\},$$

$$V^I_{0\sigma(\mu,\nu)} = \{\{k \in N : \mu(t_{m,k}(x), t) \leq 1 - \epsilon \text{ or } \nu(t_{m,k}(x), t) \geq \epsilon\} \in I\},$$

$$BV^I_{\sigma(\mu,\nu)} = \{\{k \in N : \mu(\phi_{m,k}(x) - L, t) \leq 1 - \epsilon \text{ or } \nu(\phi_{m,k}(x) - L, t) \geq \epsilon\} \in I\},$$

$$\mathcal{Z}^I_{(\mu,\nu)} = \{\{k \in N : \mu(x^/_k - L, t) \leq 1 - \epsilon \text{ or } \nu(x^/_k - L, t) \geq \epsilon\} \in I\},$$

$$\mathcal{Z}^I_{0\,(\mu,\nu)} = \{\{k \in N : \mu(x^/_k, t) \leq 1 - \epsilon \text{ or } \nu(x^/_k, t) \geq \epsilon\} \in I\}.$$

using the (μ,ν) intuitionistic fuzzy norm.

IV. Conclusion

We can study the algebraic, topological and elementary properties of intuitionistic fuzzy I-convergent sequenence spaces $c^I_{(\mu,\nu)}$, $c^I_{0(\mu,\nu)}$, $V^I_{\sigma(\mu,\nu)}$, $V^I_{0\sigma(\mu,\nu)}$, $BV^I_{\sigma(\mu,\nu)}$, $Z^I_{(\mu,\nu)}$,$Z^I_{0(\mu,\nu)}$.

Further, as a future research directions, the sequence spaces $c^I_{(\mu,\nu)}$, $c^I_{0(\mu,\nu)}$, $V^I_{\sigma(\mu,\nu)}$, $V^I_{0\sigma(\mu,\nu)}$, $BV^I_{\sigma(\mu,\nu)}$, $Z^I_{(\mu,\nu)}$,$Z^I_{0(\mu,\nu)}$. can be studied using the Lacunary, Modulus function, Orlicz function, Sequence of moduli, Musielak-Orlicz function and Fibonacci sequences.

The proposed new intuitionistic fuzzy I-convergent sequence spaces are quite pathological from algebraic and topological point of view and one can study it in the direction of double sequences also.

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Adaptive and Minimax Methods of Prediction Dynamic Systems using the Kalman Algorithm

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Abstract- In the article we consider the problem of linear extrapolation of zero-mean wide-sense-stationary random process both discrete-time and continuous-time cases under conditions of the absence of a priori information about the statistical characteristics of disturbance in the absence of measurement errors under scalar observation only the restricted disturbance is assumed. We investigate a minimax approach, which guarantees the prediction of high quality at the least favorable disturbance spectrum. The simple implementation of an optimal adaptive minimax predictor and prediction based on Kalman – Bucy filter and their comparative characteristics has been obtained. Examples are given.

Keywords: minimax, filtering, linear, extrapolation, stationary, saddle-point, disturbance, dispersion.

GJSFR-F Classification: MSC 2010: 00A69

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Abstract- In the article we consider the problem of linear extrapolation of zero-mean wide-sense-stationary random process both discrete-time and continuous-time cases under conditions of the absence of a priori information about the statistical characteristics of disturbance in the absence of measurement errors under scalar observation only the restricted disturbance is assumed. We investigate a minimax approach, which guarantees the prediction of high quality at the least favorable disturbance spectrum. The simple implementation of an optimal adaptive minimax predictor and prediction based on Kalman – Bucy filter and their comparative characteristics has been obtained. Examples are given.

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I. Introduction

The solution to a similar problem for the continuous-time processes was obtained for the first time by U. Grenander [1], where the problem of predicting a stationary continuous in time occasional process observed without noises with only its dispersion to be known, was solved. This paper should be marked as the first one where the minimax approach to extrapolation problem for stationary processes was proposed. In the papers by M. Moklyachuk [2] the minimax approach is applied toextrapolation, interpolation and filtering problems for functionals which depend on the unknown values of stationary processes and sequences. Many investigators have been interested in minimax extrapolation problems for the stationarystochastic sequences. J. Franke [4], J. Franke and H. V. Poor [5] investigated the minimax (robust) extrapolation and filtering problems for the stationary sequences with the help of convex optimization methods. This approach makes it possible to find equations that determine the least favourable spectral densities for various classes of densities.

Unlike the Kalman method [11,13] or more general case of Bayes evaluation, the essence of the minimax method is that the disturbances are not considered probability –described for the full, i.e., as for instance in the Kalman model, the disturbances are defined by a no correlated in time process. Such a problem, as well as other more general problems may be solved using the minimax approach which guaranties the prediction of high quality at the least favorable spectrum.

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In engineering motivation, the article differs from previous publications in that it contains an analytical study of the problem of predicting a scalar stationary process in a broad sense with the worst disturbance spectrum, about which only the variances in continuous and discrete time are known a priori, both in the medium and long term.

Unlike previous works [3,4] devoted to methods and problems of forecasting using parametric time series models, this work uses a minimax approach to obtain more correct guaranteed results. In this case, the prediction algorithm, in contrast to the Kalman method or the more general case of Bayesian estimation, consists in the fact that disturbances are not considered probabilistically described in full, that is, for example, as in the Kalman model, disturbances are set by a process uncorrelated in time and represents a correction of the forecast by the model and therefore does not coincide with simple forecasting. At the same time, the prediction efficiency increases with an increase in the depth of the filter memory in N steps.

To compare the results, as well as to specifically explain how adaptation is carried out when finding an approximating model, the most general method based on the Kalman filter is given in the paper. At the same time, an algorithm is synthesized and the worst disturbance. spectrum is found.

The aim of this work is to present new methods of solution of the linear extrapolation of zero-mean wide-sense-stationary random process for both discrete-time and continuous-time cases under conditions of the absence of a priori information about the statistical characteristics of disturbance in the absence of measurement errors under scalar observation. The existence of the saddle point of the extrapolation game in terms of the extreme properties of permissible spectral densities of linear filter and nature is also discussed.

The above setting of the problem is characterized, in particular, for the tasks of determining the movements of the space object, the aircraft on radar or optical measurements. Here, fluctuational (noise) measurement errors are in nature "white noise." The remaining components of errors and indignations are indefinite in nature - their dispersion is known or, in a more general case, dispersion matrix, i.e. an ellipsoid containing these errors at a fixed level of confidential information $\alpha \approx 0.1$.

II. Statement of a Problem of Minimax Extrapolation in Case of the Absence of Measurement Errors in Continuous Time

Let us assume that the real component of the measured signal $y(t)$ was formed from a certain disturbance $u(t)$ by means of the dynamic system:

$$\dot{\vec{x}}(t) = A\vec{x}(t) + \vec{b}u(t) \quad (1)$$

Here A is the constant matrix of $n \times n$ dimension; $t \in (-\infty, +\infty)$; $\vec{x}(t) \in R^n$ is the state vector of the system; $\vec{b} \in R^n$ is the constant vector; $u(t)$ is the unknown disturbance.

Representing a scalar stationary occasional process with zero mean value with the only information concerning its correlation function about the constraint on its dispersion, which satisfies inequality.

Notes

$$Mu^2(t) \le a,$$

Where M denotes the mathematical expectation of $u^2(t)$; $a < \infty$ is a fixed disturbance power and, perhaps, the constraint on the concentration area of its spectral density- $H_u(\lambda)$ at $\lambda \in \Lambda$, Λ is the given subset of the frequency axis. The measured signal upon the the results of the observations on the time interval $\tau \in (-\infty, t)$ are given by

$$y(\tau) = \vec{C}^T \vec{x}(\tau), \tag{2}$$

where $\vec{C} \in R^n$ - is the constant column vector.

The linear equations input-state-output (1) and (2) will be called in the future the meter-object system(1), (2).

Let us make the following assumptions concerning A, $\vec{b}$ and $\vec{C}$ matrixes:

1. The meter-object system (1),(2) in the absence of measurement errors and disturbances is the system, i.e.:

$$rank(\vec{C}, A^T\vec{C}, ..., (A^{n-1})^T \vec{C}) = n$$

Throughout this paper, let «$rank$» be the matrix operator of the taking the rank over correspondent compound matrix stated below here as:

$$\vec{C}, A^T\vec{C}, ..., (A^{n-1})^T \vec{C}$$

It means thatif at least one of its the minors of order n is different from zero while Every minor of order (n+1) is zero.

2. System (1) is "masked" by the disturbance, i.e.:

$$\vec{x}(t) = \int_{-\infty}^{t} e^{A(t-\tau)} \vec{b} u(\tau) d\tau \tag{3}$$

The concept of "mask ability" is similar to the concept of controllability in systems with control signals, therefore, the necessary and sufficient condition for "maskability" is mathematically expressed for our case as [6]

$$rank(\vec{b}, A\vec{b}, ..., A^{n-1}\vec{b}) = n$$

Due to the observ ability of the useful signal, the set of functions satisfying the unbiased condition [10]is not empty.

From the vector record of the meter-object system (1), (2) let's move on to its equivalent scalar representation record.

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6. Livshits N.A., Vinogradov V.N., Golubev G.A. Korrelyatsionnaya teoriya optimalnogo upravleniya mnogomernymi protsessami [Correlation theory of the optimal control of multidimensional processes].Moscow, Sovetskoye Radio, 1974.

It is required to find out the process:

$$\hat{s}(t+T)=\int_{-\infty}^{t} g(t+T-s)y(s)ds$$

i.e., to find the transitional function $g(t)$ of a physically realized filter which evaluates linear functional atthe moment of time $t+T$ which can be calculated by the formula

$$s(t+T)=\int_{-\infty}^{t} \vec{q}^{\,T}(t+T-s)\vec{x}(s)ds$$

where T is the time of extrapolation of the useful vectorsignal $\vec{x}(s)$ $(s \le t)$ and $\vec{q}^{\,T}(t)$ is an $1\times n$ real-valued and mean-square-integrablerow vector that is the transpose of an $n\times 1$ column vector $\vec{q}(t)$ which is set by a row vector of frequency characteristics $Q(\lambda)$ associated by means of this transformation function.

The quality criterion is the dispersion of the extrapolation error with the time period of extrapolation T is given by

$$\min_{G^{ext}} \max_{u} M[\hat{s}(t+T)-s(t+T)]^2 = \min_{G^{ext}} \max_{h} D(G^{ext},h), \quad (4)$$

where G^{ext} and Q are the transfer functions associated with transformations g and $\vec{q}$ respectively, h is the spectrum of the unknown disturbance $u(t)$.

Then for fixed spectral density $h(\lambda)\in K$, the optimum linear extrapolator $G^{ext}(\lambda)$ is found by solving the problem

$$\min_{G^{ext}\in K^{ext}} D(G^{ext},h) \quad (5)$$

Let K^{ext} denote the class of complex-valued linear extrapolators, K denote a class $K\subseteq L_1(R)$ of disturbance spectrum where $L_1(R)$ denotes the class of absolutely integrated real-value functions on R.

Thus, the problem comes to finding the frequency characteristics $G^{ext}(\lambda)$ according to minimax criterion (4). It is of interest to consider situations in which the exact form of the power spectral density of the signal is known, but the form of the power spectral density of the disturbance is unknown.
In particular,we consider disturbance spectral class K defined by

Notes

$$K = \left\{ h(\lambda) \in L_1(\lambda): \quad \frac{1}{2\pi} \int_{-\infty}^{\infty} h(\lambda)\Psi(\lambda) d\lambda \le D_u \right\}$$

$$h(\lambda) = 0 \ \text{if} \ \ \lambda \notin \Lambda,$$

where $D_u < \infty$ is a fixed perturbation power and $\Psi(\lambda)$ is the non-negative even upon λ pre assigned function satisfying the Paley – Wiener condition

$$\int_{-\infty}^{\infty} \frac{|\ln \Psi(\omega)|}{1+\omega^2} d\omega < \infty, \qquad (6)$$

Λ is the given subset of the frequency axis of positive measure (the infinite measure is possible), We shall consider in further K^{ext} as the subspace of $L_2(\lambda)$ the Hilbert space of complex –valued functions on $[-\infty, \infty]$ that are square integrable with respect to the Lebesque measure with density $h(\lambda)$ be not equal to zero.

The paper by Ulf Grenander [1] was the first one where this approach to extrapolation problem for stationary processes was proposed. The analogous causal case has been considered for interval fuzziness of the linear dynamic system with the parametric uncertainty of the specification only in the state matrix in the frameworks of the form of H-stability with the restricted variance of the random disturbance in the useful component of the signal model in [11]. Some aspects of a related causal case have been considered in [2,3] using convex optimization and sub differential computation methods. For this problem we may now state the following theorem which gives a technique for searching for least-favorable spectra in certain cases.

Theorem 2.1: The given problem has the saddle point due to the fact that $D(G^{ext}, h)$ is linear over h and the set of the all $h(\lambda)$ with restricted integral dispersion

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} h(\lambda)\Psi(\lambda) \le D_u$$

is convex weak compact and $D(G^{ext}, h)$ is square over G^{ext}, i.e. the conditions of convexity-concavity which are required when the well-known theorems from the game theory are used [14] are met and the classical condition of the existence of the saddle point is satisfied, i.e. we have the corresponding relation determining the saddle point:

Ref

1. Grenander U. A prediction problem in game theory//Endless antagonistic games. M. Fizmatgiz, 1963, pp. 403-413.

$$\min_{G^{ext}\in K^{ext}} \max_{h\in K} D(G^{ext},h) = \max_{h\in K} \min_{G^{ext}\in K^{ext}} D(G^{ext},h) \tag{7}$$

Thus, the problem should be interpreted as an antagonistic interplay $\Gamma(D^{ext},K,K^{ext})$, where the gain functional is $D(G^{ext},h)$ with corresponding strategy spaces of two players : space of the first player named by nature K striving to maximize $D(G^{ext},h)$ and space of the second player named by investigator K^{ext} striving to minimize $D(G^{ext},h)$.

III. The System of the Relations, Determining the Saddle Point in the Problem of Minimax Extrapolation for the Situation of Uncertainty Information about the Statistical Characteristics of the Disturbance

Theorem 2.1 can be applied for finding solutions to extrapolation problem for the stationary stationary occasional process in the case of spectral uncertainty, when spectral density of disturbance is not known exactly. In this section without loss of generality we assume that the useful vector signal $\vec{x}(s)$ may be presented in the form of the scalar signal $x(s)$ and the spectral density of $x(s)$ signal may be presented as by:

$$X_u(\lambda) = T(\lambda) + h(\lambda),$$

where $T(\lambda)$ is known non-negative component which satisfies the Paley – Wiener condition (6) and $h(\lambda)\in K$ is unknown component in the spectrum of signal. The solution to the problem of (5) can be found for this case by applying the results of [7] concerning the relationship of the minimax filtering problem with the solution to the Markov moments problem. Let us confine ourselves to considering the case when $T(\lambda)=0$. A more general case treated similarly to this.

The system of the relations, determining the saddle point in this case is defined by the formulas (8) - (10)

$$\Psi(\lambda) = |\varphi(\lambda)|^2; \tag{8}$$

$$X_u^+(\lambda) = \frac{|[Q^*(\lambda)X_u^-(\lambda)]_+|^2}{\sqrt{\alpha}\;\varphi(\lambda)}; \tag{9}$$

Ref

7. Crane M.G.,Nudelman A.AThe problem of Markov moments and extreme problems. M.:Nauka, 1973.

$$\int_{-\infty}^{+\infty} \frac{|[Q(\lambda)X_u^+(\lambda)]_-|^2}{\alpha\Psi(\lambda)} d\lambda = a; \qquad (9.1)$$

$$G^{ext}(\lambda) = \frac{[Q(\lambda)X_u^+(\lambda)]_+}{X_u^+(\lambda)} = Q(\lambda) - \mu \frac{X_u^-(\lambda)}{X_u^+}\varphi^*(\lambda) \qquad (10)$$

Notes

where α is the Lagrange factor satisfying system of the relations, determining the saddle point of the minimax extrapolator [8, see paragraph 3.6.1]; $\varphi(\lambda)$ is the result of factorization $\Psi(\lambda) = \varphi^+(\lambda)\varphi^-(\lambda)$, $\varphi^*(\lambda) = \varphi^-(\lambda)$, μ is the maximum positive eigen value corresponding to the eigen function $X_u(\lambda)$ in the equation (9.1). The dispersion of the extrapolation error with the time period of extrapolation T can be represented in the form

$$D^{ext}(\lambda) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |G^{ext}(\lambda) - Q(\lambda)|^2 h(\lambda) d\lambda = a\mu^2 \qquad (11)$$

Here the equations(9-10) symbols for function $A_+(\lambda)$ and $A_-(\lambda)$ were introduced as correspond for the separation operation of $A(\lambda)$ function in the lower and upper analytic semi-plane correspondingly, where $A^+(\lambda)$ and $A^-(\lambda)$ are causal and noncausal parts of $A(\lambda)$ satisfying factorization $A(\lambda) = A^+(\lambda)A^-(\lambda)$, $h(\lambda)$ is the solution to the Markov moments problem concerning the spectral density perturbation is not known exactly related to minimax mean_square error criterion (4). In order to demonstrate the development the technique we propose the following example.

Example 3.1

Let consider the problem of optimal linear extrapolation when the spectral density of the perturbation is not known exactly about the object of the second order

$$\ddot{s} = u(t)$$

It is required, using the values of the measured signal $s(\tau)$ upon the results of the observations at $\tau \le t$, to evaluate the value of the signal in the moment of time at $t_{ext} = t + T$. In fact, since $\varphi(\lambda) = (i\lambda)^2$; $\Psi(\lambda) = \lambda^4$; $Q(\lambda) = e^{i\lambda T}$; $a(t) = \delta(t-\tau)$.

Differentiation two times reduces the integral equation of Grenander [1, p.156] determining the unknown eigenvalue function $X_u(\lambda)$ to the differential equation with the boundary conditions

$$\mu \ddot{s}(t) = s(T-t); s(0) = \dot{s}(0) = 0; 0 \le t \le T.$$

From this relation we have the differential equation

$$\mu^2 s^{(4)}(t) = s(t),$$

where μ is the maximum positive eigen value corresponding to the eigen function $X_u(\lambda)$ in the equation (9.1). The common solution of the last equation can be represented in the form

$$s(t) = A\cos\frac{t}{\sqrt{\mu}} + B\sin\frac{t}{\sqrt{\mu}} + Csh\frac{t}{\sqrt{\mu}} + Dch\frac{t}{\sqrt{\mu}}$$

From the initial and boundary conditions $s(0) = \dot{s}(0) = 0; \ddot{s}(T) = \dddot{s}(T) = 0$ we will have $B + C = 0; A + D = 0;$

$$\begin{cases} -A\cos\frac{t}{\sqrt{\mu}} - B\sin\frac{t}{\sqrt{\mu}} + Csh\frac{t}{\sqrt{\mu}} + Dch\frac{t}{\sqrt{\mu}} = 0; \\ A\cos\frac{t}{\sqrt{\mu}} - B\sin\frac{t}{\sqrt{\mu}} + Csh\frac{t}{\sqrt{\mu}} + Dch\frac{t}{\sqrt{\mu}} = 0. \end{cases}$$

Then a necessary and sufficient condition by virtue of which not the trivial solution of this system is possible is the condition

$$\cos\frac{T}{\sqrt{\mu}} ch\frac{T}{\sqrt{\mu}} + 1 = 0$$

The maximum positive eigenvalue μ corresponding to this equation is

$$\mu = \frac{T^2}{x^2} \approx 0.284T^2; \quad \cos x \cdot chx = -1.$$

Carrying out simple, but rather cumbersome calculations will get the expression for the extrapolator

$$G^{ext}(\lambda) = \frac{(1+\alpha\tilde{p})(1-\beta^2\tilde{p}^4)}{(1-\alpha\tilde{p})\tilde{p}^2\beta + (1+\alpha\tilde{p})e^{-\tilde{p}}},$$

where

Notes

$$\tilde{p} = i\lambda T;\ \gamma = \frac{\cos x + chx}{\sin x - shx} \approx -1.362;$$

$$\alpha = -\gamma / x \approx 0.726;\ \ \beta = \frac{1}{x^2} \approx 0.284.$$

Notes

Guaranteed accuracy is determined by the standard extrapolation error

$$\sigma_{ext} = \mu\sqrt{a}\ T^2 \approx 0.284\sqrt{a}\ T^2.$$

Comparison with the simplest extrapolates chosen for reasonable reasons shows that the obtained filter has a marked advantage. So linear prediction

$$s(t+T) = s(t) + \dot{s}(t)T$$

gives the mean square extrapolation error at the worst perturbation

$$h(\lambda) = a\delta(\lambda),\ \text{or}\ \ u(t) = A,\ MA = 0,\ MA^2 = a$$

$$\sigma_{ext} = 0.5\sqrt{a}\ T^2,$$

i.e. loses in 1.75 times. Square prediction

$$s(t+T) = s(t) + \dot{s}(t)T + \frac{\ddot{s}(t)}{2}T^2$$

gives the mean square extrapolation error at the worst disturbance

$$h(\lambda) = \frac{a}{2}[\delta(\lambda - \lambda_r) + \delta(\lambda + \lambda_r)],\ \lambda_r \approx 5.5/\mathrm{T}\ \ \text{or}\ \ u(t) = A\cos(\lambda_r t),$$

$$MA = 0,\ MA^2 = a; \sigma_{ext} 0.53\sqrt{a}\ T^2,$$

here λ_r is the resonance frequency of the minimax filter of the extrapolation, at this point the least favorable spectral measure of an unknown scalar disturbance $h(\lambda)$ is concentrated mostly and loses in 1.87 times. The best forecast is the forecast being the average of linear and quadratic forecasts. At frequency $\lambda_r \approx 4.25 / T$ the mean- square extrapolation error does not exceed

$$\sigma_{ext} \approx 0.33\sqrt{a}\ T^2,$$

i.e. loss of the optimal extrapolation does not exceed 17%.

IV. Case of Extrapolation for Discrete Time

Let us consider the dynamic model of a step-by-step vector process:

$$\vec{\lambda}_n = A\vec{\lambda}_{n-1} + \vec{\xi}_n \qquad (12)$$

$$y_n = \vec{C}^T \vec{\lambda}_n$$

where $\vec{\lambda}_n$ - is the column – vector of the model condition, y_n - is the measurement quantity; ξ_n - is the column – vector of the excitation in the model; A - is the transition matrix of the order of r; $\vec{C}$ - is the bonding column – vector ; n - is the time-moment.

Let us pass from vector record (12) to the scalar one by choosing the corresponding coefficients A, $\vec{C}$ and p_i in the case if (12) is the observable system [6]. Let us confine ourselves to the case when

$$\sum_{i=0}^{n} p_i x_{n-i} = u_n, \qquad (13)$$

where u_n -is the scalar disturbance with restricted dispersion. In z - symbolization the latter representation is given by $\sigma_{ext} = \gamma\sigma_u, \sigma_u = \sqrt{D_u}$, where $P(z) = \sum_{i=0}^{\infty} p_i z^i$.

Let us label the desired evaluation y_{n+N} by l_n. Then

$$l_n = \sum_{i=0}^{\infty} g_i y_{n-i} \qquad (14)$$

In z - representation expression (14) will read as:

$$l(z) = G^{ext}(z)y(z)$$

If the spectrum of $u(z)$ disturbance is presented

$$h_u(\omega) = \sum_{n=-\infty}^{+\infty} e^{-j\omega n} K_{u_n}$$

where $K_{u_n} = M(u_i u_{i+n})$ - correlated moments, then the expression of the prediction error dispersion will be as follows:

$$D_N(G^{ext}, h_u(\omega)) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{h_u(\omega)}{|P(e^{-j\omega})|^2} |G^{ext}(e^{-j\omega}) - e^{j\omega N}|^2 \, d\omega \qquad (15)$$

The given problem has a saddlepoint due to the fact that $D_N(G, h_u)$ is linear over h_u and the set of all $h_u(\omega)$ with restricted integral dispersion $D = \frac{1}{2\pi}\int_{-\pi}^{\pi} h_u(\omega)d\omega$ is convex weak compact and $D_N(G^{ext}, h_u)$ is

Notes

square over $G^{ext}(e^{-j\omega})$, i. e. the conditions of convexity – concavity, which required when the well known theorem from the game theory is used [14,15], are met and

$$\min_{G^{ext}} \max_{h_u} D_N(G^{ext}, h_u) = \max_{h_u} \min_{G^{ext}} D_N(G^{ext}, h_u) \quad (16)$$

Notes

Minimum (15) gives Winer-Kolmogorov filter [9]:

$$G^{ext}(z) = \frac{\left[\frac{1}{z^N} x \right]_+}{x} \quad (17)$$

where $x = x(z)$ - is analytical function (together with the inversed one $\frac{1}{x(z)}$) inside an isolated unit circle, obtained by factorizing the function $X(\omega)$

$$X(\omega) = \frac{h_u(\omega)}{|P(e^{-j\omega})|^2} = x(e^{-j\omega})x(e^{j\omega})$$

The maximum of expression (16) provides for an arbitrary spectral density $h_u(\omega)$ satisfying the constraint

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} h_u(\omega)d\omega = D_u < \infty$$

only in the case [9]

$$\gamma^2 X(\omega) = |e^{j\omega N}x^+ - [e^{j\omega N}x^+]_+|^2 \cdot \frac{1}{|P(e^{-j\omega})|^2}. \quad (18)$$

Lagrange factor γ determines mean square error of the extrapolation

$$l_n = Y_n(N) - \sum_{i=1}^{N-1} x_{N-i-1}(l_{n-i} - Y_n(N-i)).$$

Equation for γ and nonzero $x^+(z)$ follows from expression (18):

$$\gamma \sum_{i=0}^{n} p_i x_{n-i} = X_{N-n-1} 1(N-n-1), \quad (19)$$

where $1(i)$ is a step function:

$$1(i)=\begin{cases}0, & i<0,\\ 1, & i\geq 0.\end{cases}$$

Thus minimax prediction method concerning the algorithm does not depend on the level D_u, which makes its applicability domain wider.

Now let us consider a more compact representation of the minimax prediction process. Let us study the prediction in accordance with (13) in the absence of perturbations.

Equations (19) have a solution relative to an accuracy of an arbitrary multiplier. Therefore, $x_{N-1}=1$ may be added to (19). Then at N=1

$$l_n = Y_n(N),$$

there is no difference between the minimax prediction for one step and the prediction without taking into consideration the perturbation and at N>1:

$$l_n = Y_n(N)-\sum_{i=1}^{N-1} x_{N-i-1}(l_{n-i}-Y_n(N-i)), \tag{20}$$

Minimax prediction is calculated according to recurrent ratios with the filter memory depth of N-1 steps, as in the evaluation process only N-1 last values of the previous prediction estimations are memorized and the same number of values of prediction estimations according to the model, the perturbations being neglected. Thus, minimax predictions require corrections which should be more profound at long – term prediction.

However, it should be noted, that unlike in the Kalman filter, there should be a separate time-set run over recurrent ratios (20) for each N.

The initial outset of algorithm (20) is evident : it is assumed that the first N-1 steps residuals in (20) $l_{n-i}-Y_n(N-i)$ equal zero. Formula (20) is the main in the suggested algorithm of minimax prediction. As will be seen from analytical examples, minimax predictions may not prove to be effective at low N.

As N increases the efficiency of the prediction grows.

Let us consider the examples.

1. $y_n = y_{n-1}+u_n$. In this case equation(19)can be expressed as:

$$\gamma(x_n - x_{n-1}) = x_{N-n-1}.$$

It's solution at N>1 is as follows:

$$x_n = \sin\frac{\pi(n+1)}{2N+1}\cdot\frac{1}{\sin(\pi N/2N+1)}$$

Notes

$$\gamma = \frac{1}{2\sin(\pi/2(2N+1))}$$

The prediction equation can be written as

Notes

$$l_n = y_n - \sum_{i=0}^{N-1} x_{N-i-1}(l_{n-i} - y_{n-i})$$

At high N the maximal gain obtained by the minimax filter in comparison with the model prediction totals $\pi/2$. The criterion of the comparison of the two filters is mean square error of the prediction under the most favorable perturbation spectrum.

2. $y_n = 2y_{n-1} - y_{n-2} + u_n$. Equation(19) can be expressed as

$$\gamma(x_n - 2x_{n-1} + x_{n-2}) = x_{N-n-1}.$$

Their solutions at N > 2 may be presented by

$$x_n = \frac{1}{2}\left[\frac{\sin\alpha(n+1-N/2)}{\sin(\alpha N/2)} + \frac{\text{ch}\,\beta(n+1-N/2)}{\text{ch}(\beta N/2)}\right]$$

where $\cos\alpha = 1-(1/2\gamma); ch\beta = 1+(1/2\gamma)$ and γ may be calculated using the equation

$$\sqrt{\gamma - \frac{1}{4}}ctg(\alpha N/2) - \sqrt{\gamma + \frac{1}{4}}th(\beta N/2) = 1$$

At N=2, $\gamma = \sqrt{2}+1, x_0 = \sqrt{2}-1, x_1 = 1$. The maximum gain for our case is given by

$$k = 3/(\sqrt{2}+1) \approx 1.21.$$

The prediction equations are given by

$$l_n = (N+1)y_n - Ny_{n-1} - \sum_{i=0}^{N-1} x_{N-i-1}(l_{n-i} - (N+1-i)y_n + (N-i)y_{n-1})$$

At $N \to \infty$ a symptotically $\gamma \to \infty, \alpha \to \beta \to 0, ctg(\alpha N/2) = th(\beta N/2)$. Therefore from the equation $th(\alpha N/2) = tg(\alpha N/2) = 1$ we obtain $(\alpha N/2) \to \xi = 0.937552, \gamma \square 1/\alpha^2$, the maximal gain obtained by the minimax filter is $k = 1.87$.

V. Adaptive and Minimax Prediction

a) Adaption based on the Kalman filter

In the assumptions that the constant transition matrix in is Hurwitz and known and the measurements are correct, the Kalman filter [12,14], giving the evaluations of the vector in (12) has the recurrent form:

$$\vec{\lambda}_n = \vec{\lambda}_n^e + \frac{1}{\vec{c}^T K_n^e \vec{c}} K_n^e \vec{c}(y_n - \vec{c}^T \vec{\lambda}_n^e)$$

$$K_n = K_{n-1}^e - K_n^e \vec{c}\vec{c}^T K_n^e \frac{1}{\vec{c}^T K_n^e \vec{c}} \quad (21)$$

$$\vec{\lambda}_n^e = A\vec{\lambda}_{n-1}, K_n^e = AK_{n-1}A^T + K_{\xi_n}$$

Here the extrapolation of y into N steps , denoted by $y_n^e(N) = \vec{c}^T A^N \hat{\vec{\lambda}}_n$. The adaptation is based on the selection of the elements of the matrix K_{ξ_n} at every moment n, for example, the selection of the minimized sum of residuals squared

$$\varphi(K_{\xi_n}) = \sum_{k=0}^{n-1}[y_{n-k} - y_{n-r}^e(r-k)] \to \min_{K_{\xi_n}}, \quad (22)$$

where $r = \dim \vec{\lambda}_n$.

The condition of nonnegative definiteness should be imposed on K_{ξ_n}. Thus the time segment, measured back from the present moment, during which the measurements are used effectively, is regulated.

b) Adaption based on the approximating model coefficients

Let us present formula (13) in the following way

$$\vec{a}^T \vec{y}^n = u_n, \quad (23)$$

where u_n are perturbations with zero mean and restricted dispersion and the following columns are introduced $\vec{a} = [a_0, ..., a_N, ...]^T, \vec{y}^n = [y_n, ..., y_{n-N_y}, ...]^T$.

Here it is assumed that Y_i are precisely measured and coefficients are unknown.

In a number of cases the following representation may be known a priori

$$\vec{a} = A_{a\alpha}\vec{\alpha} \quad (24)$$

where $\vec{\alpha}$ is column vector smaller than $\vec{a}$; $A_{\alpha a}$ is known matrix.

The adaption of filter in ref. [12] which provides $\vec{\alpha}$ estimations, is given by

$$\hat{\vec{a}}_n = \hat{\vec{a}}_{n-1} + K_n y^n (0 - \hat{\vec{a}}_{n-1} A_{a\alpha}^T y^n) = (E - K_n y^n y^{nT} A_{a\alpha})\hat{\vec{a}}_{n-1},$$

Ref

12. Sidorov I.G. Linear minimax filtering of a stationary random process under the condition of the interval fuzziness in the state matrix of the system with the restricted variance//Radio technique and electronics. 2018.- Vol. 63. - № 8, - P. 831-836.

$$K_n = K_{n-1} - K_{n-1} y^n y^{nT} K_{n-1} \frac{1}{1 + y^{nT} K_{n-1} y^n} \quad , \tag{25}$$

Notes

where $\hat{\vec{\alpha}}_n$ is the estimation of $\vec{\alpha}_n$. E is a unit matrix. K_n is the covariance error matrix of estimations of the order r_α. The initialization the algorithm is conducted according to the first r_α measurements, where $r_\alpha = \dim \vec{\alpha}$, in the assumption of uncorrelated u_n. The minimax approach is used after the adoption of coefficients of the under discussion model. In this case we are dealing with a combined adaptive-minimax model.

VI. Conclusion

In this paper, various new techniques related to the adaptive and minimax methods of extrapolationof a stationary occasional sequence in the presence of disturbancewith restricted dispersionhave been presented along with some practical application examples. The results presented here are intended mainly to reduce the complexity involved in the prediction problem in game theory inthe case of the absence of a priori information about the statistical characteristics of disturbance in the absence of measurement errors under scalar observation. Another important problem involved with the realization robust, steady minimax predictors becomes quite demanding in the conditions of interval fuzziness in the model parameters in the presenceor absence of measurement errors under scalar observation.The problem of the existence of the interval saddle point in the extrapolation game in terms of the extreme properties of permissible interval spectral densities of the robust linear filters and nature is actualized in the context of prediction estimation of discrete and/or continuous-time economic processes in the presence of uncertainty.We also expect that the application results, which were discussed in this paper, will motivate further potential applications of Kalman and minimax filtering techniques in various fields of economic,engineering and econometric forecasting.

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Notes

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Convergence Theorems of Multi-Valued Generalized Nonexpansive Mappings in Banach Spaces

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Abstract- In this paper, we study multi-valued generalized nonexpansive mappings in uniformly convex Banach spaces. We introduce multi-valued version of the iterative scheme presented in [4] and prove some weak and strong convergence results in uniformly convex Banach space. Further, we present an illustrative numerical example of approximating fixed point of multi-valued generalized nonexpansive mappings satisfying condition (E) considering multivalued version of the iterative scheme presented in [4].

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Convergence Theorems of Multi-Valued Generalized Nonexpansive Mappings in Banach Spaces

Seyit Temir

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I. Introduction and Preliminaries

Fixed point theory for multi-valued mappings has many useful applications in various fields, control theory, convex optimization, game theory and mathematical economics.Therefore, it is natural to extend the known fixed point results for single-valued mappings to the setting of multi-valued mappings.The theory of multi-valued nonexpansive mappings is more difficult than the corresponding theory of single-valued nonexpansive mappings. The convergence of a sequence of fixed points of a convergent sequence of set valued contractions was investigated by [8] and [9]. In the last few decades, the numerous numbers of researchers attracted in these direction and developed the study of multi-valued version of iterative processes have been investigated to approximate fixed point for not only nonexpansive mapping, but also for some wider class of nonexpansive mappings. Iterative techniques for approximating fixed points of nonexpansive multi-valued mappings have been investigated by various authors using the Mann iteration scheme or the Ishikawa iteration scheme (see [11], [14], [16] and so on).

We assume throughout this paper that $(X, \|.\|)$ is a Banach space and K is a nonempty subset of X. The set K is called proximinal if for each $x \in X$, there exists some $y \in K$ such that $d(x, y) = d(x, K)$, where $d(x, K) = inf\,\{d(x, y) : y \in K\}$. In the sequel, the notations $\mathcal{P}_{px}(K), \mathcal{P}_{cb}(K), \mathcal{P}_{cp}(K)$ and $\mathcal{P}(K)$ will denote the families of nonempty proximinal subsets, closed and bounded subsets, compact subsets and all subsets of K , respectively. A point $y \in K$ is said to be a fixed point of $T : K \to \mathcal{P}(K)$ if $y \in T(y)$. The set of fixed points of T will be denote by $F(T)$. Let $H(,)$ be the Hausdorff distance on $\mathcal{P}_{cb}(K)$ is defined by

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$H(A,B) = max\{sup_{x\in A} d(x,B), sup_{y\in B} d(y,A), \forall A,B \in \mathcal{P}_{cb}(K), x \in A, y \in B\}$.

Let $T : K \to \mathcal{P}(K)$ be a multivalued mapping. An element $p \in K$ is said to be a fixed point of T, if $p \in T(p)$. The set of fixed points of T will be denoted by $F(T)$. A multivalued mapping $T : K \to \mathcal{P}(K)$ is said to be nonexpansive, if $H(Tx,Ty) \leq \|x-y\|$, for all $x,y \in K$, quasi-nonexpansive, if $F(T) \neq \emptyset$ and $H(Tx,Tp)) \leq \|x-p\|$, for all $x \in K$, and all $p \in F(T)$. It is well known that if K is a nonempty closed, bounded and convex subset of a uniformly convex Banach space X, then a multivalued nonexpansive mapping $T : K \to \mathcal{P}(K)$ has a fixed point has a fixed point [7]. Shahzad and Zegeye [14] presented the set $P_T(x) = \{y \in Tx : d(x,Tx) = \|x-y\|\}$ for a multivalued mapping, $T : K \to \mathcal{P}(K)$ and showed that Mann and the Ishikawa iteration processes for multi-valued mappings are well defined. They proved the convergence of these iteration processes for multivalued mappings in a uniformly convex Banach space. In 2011, Abkar and Eslamian [1] extended the notion of condition (C) to the case of multi-valued mappings. In 2012, Abkar and Eslamian [2] introduced an iterative process for a finite family of generalized nonexpansive multivalued mappings and proved Δ-convergence and strong convergence theorems in CAT(0) spaces.

In this paper, we introduce multi-valued version iterative scheme presented in [4] for multi-valued mappings as follows: for arbitrary $x_1 \in K$ construct a sequence $\{x_n\}$ by

$$\begin{cases} v_n = (1-c_n)x_n + c_n\tau_n, \\ s_n = (1-b_n)\tau_n + b_n\eta_n, \\ w_n \in P_T(s_n), \\ x_{n+1} = (1-a_n)t_n + a_nu_n, \forall n \in \mathbb{N}, \end{cases} \tag{1.1}$$

where $\{a_n\}, \{b_n\}$ and $\{c_n\} \in (0,1)$, $\tau_n \in P_T(x_n), \eta_n \in P_T(z_n), t_n \in P_T(w_n), u_n \in P_T(\eta_n)$.

One can find in the literature that there are important studies about generalized nonexpansive mappings that are weaker nonexpansive mappings and stronger than quasi-nonexpansive mappings. For instance, in 2008, Suzuki [15] defined a class of generalized nonexpansive mappings on a nonempty subset K of a Banach space X. Such type of mappings was called the class of mappings satisfying the condition (C) (also referred as Suzuki generalized nonexpansive mapping), which properly includes the class of nonexpansive mappings. Another one of the generalized nonexpansive mappings , in 2011, García-Falset et al. [6] introduced two new conditions on single-valued mappings, are called condition (E)(also referred as Garsia-Falset generalized nonexpansive mapping) and (C_λ) which are weaker than nonexpansive and stronger than quasi-nonexpansive.

A single-valued mapping $T : K \to X$ satisfies condition (E_μ) on K, if there exists $\mu \geq 1$ such that for all $x,y \in K$,

$$\|x - Ty\| \leq \mu\|x - Tx\| + \|x - y\|.$$

Moreover, it is said that T satisfies condition (E) on K, whenever T satisfies condition (E_μ), for some $\mu \geq 1$. It is obvious that if $T : K \to X$ is nonexpansive, then it satisfies condition (E_1) and from Lemma 7 in [15] we know that if $T : K \to K$ satisfies condition (C) on K, then T satisfies condition (E_3) (see [6]). Proposition 1 in [6], we know also that if $T : K \to X$ a mapping which satisfies condition (E) on K has some fixed point, then T is quasi-nonexpansive. The converse is not true (see example in [6]). Thus the class of Garcia-Falset generalized nonexpansive mappings exceeds the class of Suzuki generalized nonexpansive mappings

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1. Abkar, M. Eslamian, *A fixed point theorem for generalized nonexpansive multivalued mappings*, Fixed Point Theory 12(2) (2011), 241–246.

(and therefore the class of nonexpansive mappings), but still remains stronger than quasi-nonexpansiveness.

Motivated by the above, we prove some weak and strong convergence results using (1.1) iteration process for multi-valued Garsia-Falset generalized nonexpansive mappings(generalized nonexpansive mappings satisfying condition (E)) in uniformly convex Banach spaces. Moreover, we present an illustrative numerical example of approximating fixed point of multi-valued generalized nonexpansive mappings satisfying condition (E) considering the iteration process (1.1).

Now we recall some notations to be used in main results:

A Banach space X is said to satisfy *Opial's condition* [10] if, for each sequence $\{x_n\}$ in X, the condition $x_n \to x$ converges weakly as $n \to \infty$ and for all $y \in X$ with $y \neq x$ imply that

$$\limsup_{n\to\infty}\|x_n - x\| < \limsup_{n\to\infty}\|x_n - y\|.$$

In the following we shall give some preliminaries on the concepts of asymptotic radius and asymptotic center which are due to [3].

Let $\{x_n\}$ be a bounded sequence in a Banach space X. Then

(1) *The asymptotic radius of $\{x_n\}$ at point $x \in X$ is the number*

$$r(x, \{x_n\}) = \limsup_{n\to\infty} \|x_n - x\|.$$

(2) *The asymptotic radius of $\{x_n\}$ relative to K* is defined by

$$r(K, \{x_n\}) = \inf\{r(x, \{u_n\}) : x \in K\}.$$

(3) *The asymptotic center of $\{x_n\}$ relative to K* is the set

$$A(K, \{x_n\}) = \{x \in K : r(x, \{x_n\}) = r(K, \{x_n\})\}.$$

It is well known that, in uniformly convex Banach space, $A(K, \{x_n\})$ consists of exactly one-point.

Lemma 1.1. *([12]) Suppose that X is a uniformly convex Banach space and $0 < k \leq t_n \leq m < 1$ for all $n \in \mathbb{N}$. Let $\{x_n\}$ and $\{x_n\}$ be two sequence of X such that $\limsup_{n\to\infty}\|x_n\| \leq \xi$, $\limsup_{n\to\infty}\|y_n\| \leq \xi$ and $\limsup_{n\to\infty}\|t_n x_n + (1-t_n)y_n\| = \xi$ hold for $\xi \geq 0$. Then $\lim_{n\to\infty}\|x_n - y_n\| = 0$.*

Definition 1.2. Let $T : K \to \mathcal{P}_{cb}(K)$. A sequence $\{x_n\}$ in K is called an approximate fixed point sequence (or a.f.p.s) for T provided that $d(x_n, Tx_n) \to 0$ as $n \to \infty$.

Definition 1.3. A multivalued mapping $T : K \to \mathcal{P}(K)$ is called demiclosed at $y \in K$ if for any sequence $\{x_n\}$ in K weakly convergent to x and $y_n \in Tx_n$ strongly convergent to y, we have $y \in Tx$.

The following is the multi-valued version of condition (I) of Senter and Dotson [13].

Definition 1.4. A multivalued mapping $T : K \to \mathcal{P}(K)$ is said to satisfy *condition(I)*, if there is a nondecreasing function $\varphi : [0, \infty) \to [0, \infty)$ with $\varphi(0) = 0$ and $\varphi(\xi) > 0$ for all $\xi \in (0, \infty)$ such that $d(x, Tx) \geq \varphi(d(x, F(T)))$ for all $x \in K$.

Ref

3. M. Edelstein, *Fixed point theorems in uniformly convex Banach spaces*, Proc. Amer. Math. Soc., 44(2) (1974), 369–374.

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Lemma 1.5. *([16]) Let $T : K \to \mathcal{P}_{px}(K)$ and $P_T(x) = \{y \in Tx : d(x, Tx) = \|x - y\|\}$. Then the following are equivalent.*

(1) $x \in F(T)$.

(2) $P_T(x) = \{x\}$.

(3) $x \in F(P_T)$.

Moreover, $F(T) = F(P_T)$.

Now we give the definition of multi-valued generalized nonexpansive mapping satisfying condition (E):

Definition 1.6. Let K be a nonempty subset of a Banach space X. A mapping $T : K \to \mathcal{P}_{cb}(K)$ is called a multi-valued generalized nonexpansive mapping satisfying condition (E) if there exists an $\mu \geq 1$ such that for each $x, y \in K$,

$$d(x, Ty) \leq \mu d(x, Tx) + d(x, y)$$

We say that $T : K \to \mathcal{P}_{cb}(K)$ satisfies condition (E) on K whenever T satisfies (E_μ) for some $\mu \geq 1$.

Every multi-valued nonexpansive mapping satisfies condition (E_1) (see [2]). Moreover, if $z \in K$ is a fixed point of the mapping $T : K \to \mathcal{P}_{cb}(K)$, and this mapping satisfies condition (E_μ) on K, then for all $x \in K$, $d(z, Tx) \leq \|z - x\|$. In other words, T is a quasi-nonexpansive mapping.

Proposition 1.7. [5] *Let $T : K \to \mathcal{P}_{cb}(K)$ be a mapping satisfying condition $(C_{1/2})$. Then, T satisfies condition (E_3).*

II. Convergence of Multi-Valued Generalized Nonexpansive Mappings

In this section, we prove weak and strong convergence theorems for (1.1) iterative scheme of multi-valued generalized nonexpansive mappings satisfying condition (E) in uniformly convex Banach space.

Lemma 2.1. *Let K be a nonempty closed convex subset of a uniformly convex Banach space X. Let $T : K \to \mathcal{P}_{px}(K)$ be a multi-valued mapping such that $F(T) \neq \emptyset$ and P_T is a Garsia-Falset generalized nonexpansive mapping. Let $\{x_n\}$ be a sequence generated by (1.1). Then $\lim_{n\to\infty} \|x_n - p\|$ exists for all $p \in F(T)$.*

Proof. Let $p \in F(T)$. By Lemma 1.5, $P_T(p) = \{p\}$ and $F(T) = F(P_T)$. Since P_T is a Garsia-Falset generalized nonexpansive mapping, then P_T is a quasi-nonexpansive mapping. Now, for any $p \in F(T)$, we have

$$\begin{aligned} H(P_T(v_n), P_T(p)) &\leq \|v_n - p\|, \\ H(P_T(s_n), P_T(p)) &\leq \|s_n - p\|, \\ H(P_T(w_n), P_T(p)) &\leq \|w_n - p\|, \\ H(P_T(\tau_n), P_T(p)) &\leq \|\tau_n - p\|, \\ H(P_T(\eta_n), P_T(p)) &\leq \|\eta_n - p\|. \end{aligned}$$

Next by (1.1), we have

$$\begin{aligned} \|\tau_n - p\| &\leq d(\tau_n, P_T(p)) \\ &\leq H(P_T(x_n), P_T(p)) \\ &\leq \|x_n - p\|. \end{aligned} \quad (2.1)$$

Ref

2. A. Abkar, M. Eslamain, *Convergence theorems for a finite family of generalized nonexpansive multivalued mappings in CAT(0) spaces*, Nonlinear Anal.,75 (2012) 1895–1903.

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By (2.1), we have

$$\begin{aligned} \|v_n - p\| &= \|(1-c_n)x_n + c_n\tau_n - p\| \\ &\leq (1-c_n)\|x_n - p\| + c_n\|\tau_n - p\| \\ &\leq (1-c_n)\|x_n - p\| + c_n d(\tau_n, P_T(p)) \\ &\leq (1-c_n)\|x_n - p\| + c_n H(P_T(x_n), P_T(p)) \\ &\leq (1-c_n)\|x_n - p\| + c_n\|x_n - p\| = \|x_n - p\| \end{aligned} \tag{2.2}$$

and also we have

$$\begin{aligned} \|\eta_n - p\| &\leq d(\eta_n, P_T(p)) \\ &\leq H(P_T(v_n), P_T(p)) \\ &\leq \|v_n - p\|. \end{aligned} \tag{2.3}$$

By (2.1), (2.2) and (2.3), we have

$$\begin{aligned} \|s_n - p\| &= \|(1-b_n)\tau_n + b_n\eta_n - p\| \\ &\leq (1-b_n)\|x_n - p\| + b_n\|v_n - p\| = \|x_n - p\|. \end{aligned} \tag{2.4}$$

By (2.4), we have

$$\begin{aligned} \|w_n - p\| &\leq d(w_n, P_T(p)) \\ &\leq H(P_T(s_n), P_T(p)) \\ &\leq \|s_n - p\| \leq \|x_n - p\|. \end{aligned} \tag{2.5}$$

By (2.1)-(2.5), we have

$$\begin{aligned} \|x_{n+1} - p\| &= \|(1-a_n)t_n + a_n u_n - p\| \\ &\leq (1-a_n)\|t_n - p\| + a_n\|u_n - p\| \\ &\leq (1-a_n)d(t_n, P_T(p)) + a_n d(u_n, P_T(p)) \\ &\leq (1-a_n)H(P_T(w_n), P_T(p)) + a_n H(P_T(\eta_n), P_T(p)) \\ &\leq (1-a_n)\|w_n - p\| + b_n\|\eta_n - p\| \\ &\leq (1-a_n)\|s_n - p\| + b_n\|v_n - p\| \\ &\leq (1-a_n)\|x_n - p\| + a_n\|x_n - p\| = \|x_n - p\|. \end{aligned} \tag{2.6}$$

This implies that $\{\|x_n - p\|\}$ is bounded and non-increasing for all $p \in F(T)$. It follows that $\lim_{n\to\infty} \|x_n - p\|$ exists.

Theorem 2.2. *Let K be a nonempty closed convex subset of a uniformly convex Banach space X. Let $T : K \to \mathcal{P}_{px}(K)$ be a multi-valued mapping and P_T is a generalized nonexpansive mapping satisfying condition (E). Let $\{x_n\}$ be a sequence generated by (1.1). Then $F(T) \neq \emptyset$ if and only if $\{x_n\}$ is bounded and $\lim_{n\to\infty} \|x_n - \tau_n\| = 0$.*

Proof. Suppose $F(T) \neq \emptyset$ and let $p \in F(T)$. By Lemma 2.1, $\lim_{n\to\infty} \|x_n - p\|$ exists and $\{x_n\}$ is bounded. Put

$$\lim_{n\to\infty} \|x_n - p\| = \xi. \tag{2.7}$$

From (2.2)-(2.5), we have

$$\limsup_{n\to\infty} \|v_n - p\| \leq \limsup_{n\to\infty} \|x_n - p\| \leq \xi,$$

$$\limsup_{n\to\infty}\|\tau_n - p\| \le \limsup_{n\to\infty}\|x_n - p\| \le \xi. \tag{2.8}$$

Also

$$\limsup_{n\to\infty}\|\eta_n - p\| \le \limsup_{n\to\infty}\|x_n - p\| \le \xi,$$

$$\limsup_{n\to\infty}\|s_n - p\| \le \limsup_{n\to\infty}\|x_n - p\| \le \xi$$

and

$$\limsup_{n\to\infty}\|w_n - p\| \le \limsup_{n\to\infty}\|x_n - p\| \le \xi.$$

Also we have the following inequalities

$$\|u_n - p\| \le H(P_T(\eta_n), P_T(p)) \le \|\eta_n - p\|$$

and

$$\|t_n - p\| \le H(P_T(w_n), P_T(p)) \le \|w_n - p\|.$$

On taking $\limsup_{n\to\infty}$ on both sides of the all above inequalities, we obtain that

$$\limsup_{n\to\infty}\|t_n - p\| \le \xi, \tag{2.9}$$

$$\limsup_{n\to\infty}\|u_n - p\| \le \xi \tag{2.10}$$

and so

$$\begin{aligned}\xi &= \limsup_{n\to\infty}\|x_{n+1} - p\| \\ &= \limsup_{n\to\infty}\|(1-a_n)t_n + a_n u_n - p\|\end{aligned}$$

By Lemma 1.1, we have

$$\limsup_{n\to\infty}\|t_n - u_n\| = 0. \tag{2.11}$$

Now

$$\begin{aligned}\|x_{n+1} - p\| &= \|((1-a_n)t_n + a_n u_n - p\| \\ &= \|(t_n - p) + a_n(u_n - t_n)\| \\ &\le \|t_n - p\| + a_n\|u_n - t_n\|.\end{aligned}$$

Making $n \longrightarrow \infty$ and from (2.11) we get

$$\xi = \limsup_{n\to\infty}\|x_{n+1} - p\| \le \limsup_{n\to\infty}\|t_n - p\|.$$

So by from (2.9) we have

$$\limsup_{n\to\infty}\|t_n - p\| = \xi.$$

Then

$$\|t_n - p\| \le \|t_n - u_n\| + \|u_n - p\|$$

Making $n \longrightarrow \infty$ and from (2.11), we get

$$\xi \le \limsup_{n\to\infty}\|u_n - p\|.$$

Hence together with (2.10) we have

$$\xi = \lim_{n\to\infty}\|u_n - p\|.$$

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Thus

$$\begin{aligned}
\xi &= \lim_{n\to\infty}\|u_n - p\| \le \lim_{n\to\infty} H(P_T(\eta_n), P_T(p)) \\
&\le \lim_{n\to\infty}\|\eta_n - p\| \le \lim_{n\to\infty} H(P_T(v_n), P_T(p)) \\
&\le \lim_{n\to\infty}\|v_n - p\| \\
&\le \lim_{n\to\infty}\|(1-c_n)x_n + c_n\tau_n - p\| \\
&\le \lim_{n\to\infty}(1-c_n)\|x_n - p\| + c_n\|\tau_n - p\| \\
&\le \xi.
\end{aligned}$$

Consequently

$$\lim_{n\to\infty}\|(1-c_n)(x_n - p) + c_n(\tau_n - p)\| = \xi. \tag{2.12}$$

Thus from (2.7), (2.8), (2.12) and by Lemma 1.1 we have

$$\lim_{n\to\infty}\|x_n - \tau_n\| = 0$$

which implies that $\lim_{n\to\infty} d(x_n, P_T(x_n)) = 0$.

Conversely, suppose that $\{x_n\}$ is bounded and $\lim_{n\to\infty} d(x_n, P_T(x_n)) = 0$. Let $p \in A(K, \{x_n\})$. Then we have

$$d(x_n, P_T(p)) \le \mu d(x_n, P_T(x_n)) + \|x_n - p\|$$

Using the definition of asymptotic center we have

$$\begin{aligned}
r(Tp, \{x_n\}) &= \limsup_{n\to\infty} d(x_n, P_T(p)) \\
&\le \mu \limsup_{n\to\infty} d(P_T(x_n), x_n) + \limsup_{n\to\infty}\|x_n - p\| \\
&= \limsup_{n\to\infty}\|x_n - p\| = r(p, \{x_n\}).
\end{aligned}$$

This implies that for $Tp = p \in A(K, \{x_n\})$. Since X is uniformly Banach space, $A(K, \{x_n\})$ is consists of a unique element. Thus, we have $Tp = p$. Hence $F(T) \neq \emptyset$.

In the next result, we prove our strong convergence theorems as follows.

Theorem 2.3. *Let K be a nonempty compact convex subset of a uniformly convex Banach space X. Let $T : K \to \mathcal{P}_{px}(K)$ be a multi-valued mapping such that $F(T) \neq \emptyset$ and P_T is a generalized nonexpansive mapping satisfying condition (E). Let $\{x_n\}$ be a sequence generated by (1.1). Then $\{x_n\}$ converges strongly to a fixed point of T.*

Proof. $F(T) \neq \emptyset$, so by Theorem 2.2, we have $\lim_{n\to\infty} d(x_n, P_T(x_n)) = 0$. Since K is compact, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_k} \longrightarrow q$ as $k \to \infty$ for some $q \in K$. Because P_T is a generalized nonexpansive mapping satisfying condition (E), one can find some real constant $\mu \ge 1$, such that

$$d(x_{n_k}, P_T(q)) \le \mu d(x_{n_k}, P_T(x_{n_k})) + \|x_{n_k} - q\|.$$

As $F(T) = F(P_T)$, on taking limit as $k \to \infty$, we get $q \in P_T(q)$ i.e. $q \in F(T)$. So $\{x_n\}$ converges strongly to a fixed point of T. □

The proof of the following result is elementary and hence omitted.

Theorem 2.4. *Let K be a nonempty closed convex subset of a uniformly convex Banach space X. Let $T : K \to \mathcal{P}_{px}(K)$ be a multi-valued mapping such that P_T is a generalized nonexpansive mapping satisfying condition (E). $\{x_n\}$ be a sequence generated by (1.1). If $F(T) \neq \emptyset$ and $\lim_{n\to\infty} d(x_n, F(T)) = 0$, then $\{x_n\}$ converges strongly to a fixed point of T.*

Theorem 2.5. *Let K be a nonempty closed convex subset of a uniformly convex Banach space X. Let $T : K \to \mathcal{P}_{px}(K)$ be a multi-valued mapping satisfying condition (I) such that $F(T) \neq \emptyset$. $\{x_n\}$ be a sequence generated by (1.1). If P_T is a generalized nonexpansive mapping satisfying condition (E), then $\{x_n\}$ converges strongly to a fixed point of T.*

Proof. By Lemma 2.1, we have $\lim_{n\to\infty} \|x_n - p\|$ exists and for all $p \in F(T)$. Put $\xi = \lim_{n\to\infty} \|x_n - p\|$ for some $\xi \geq 0$. If $\xi = 0$ then the result follows. Suppose that $\xi > 0$. Then

$$\lim_{n\to\infty} \|x_{n+1} - p\| \leq \lim_{n\to\infty} \|x_n - p\|.$$

It follows that

$$\lim_{n\to\infty} d(x_{n+1}, F(T)) \leq \lim_{n\to\infty} d(x_n, F(T)).$$

$\lim_{n\to\infty} d(x_n, F(T))$ exists. We show that it follows $\lim_{n\to\infty} d(x_n, F(T)) = 0$. From Theorem 2.2 $\lim_{n\to\infty} d(x_n, P_T(x_n)) = 0$. As $F(T) = F(P_T)$, by Theorem 2.2 and condition (I) we have $\lim_{n\to\infty} \varphi(d(x_n, F(T)) \leq \lim_{n\to\infty} d(x_n, P_T(x_n)) = 0$. That is, $\lim_{n\to\infty} \varphi(d(x_n, F(T))) = 0$. Since $\varphi : [0, \infty) \to [0, \infty)$ is a nondecreasing function satisfying $\varphi(0) = 0$ and $\varphi(\xi) > 0$ for all $\xi \in (0, \infty)$, we have $\lim_{n\to\infty} d(x_n, F(T)) = 0$. All the conditions of Theorem 2.4 are satisfied, therefore by its conclusion $\{x_n\}$ converges strongly to a fixed point of T. The proof is completed. □

Finally, we prove the weak convergence of the iterative scheme (1.1) for multi-valued generalized nonexpansive mappings satisfying condition (E) in a uniformly convex Banach space satisfying Opial's condition.

Theorem 2.6. *Let X be a real uniformly convex Banach space satisfying Opial's condition and K be a nonempty closed convex subset of X. Let $T : K \to \mathcal{P}_{px}(K)$ be a multi-valued mapping such that $F(T) \neq \emptyset$. Suppose P_T is a generalized nonexpansive mapping satisfying condition (E) and $I - P_T$ is demi-closed with respect to zero. Then $\{x_n\}$ defined by (1.1) converges weakly to a fixed point of T.*

Proof. Let $p \in F(T) = F(P_T)$.By Lemma 2.1, the sequence $\{x_n\}$ is bounded and $\lim_{n\to\infty} \|x_n - p\|$ exists for all $p \in F(T)$. Since X is uniformly convex, X is reflexive. By the reflexiveness of X , there exists a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $\{x_{n_j}\}$ converges weakly to some $\omega_1 \in K$. Since $I - P_T$ is demi-closed with respect to zero, $\omega_1 \in F(P_T) = F(T)$. We prove that ω_1 is the unique weak limit of $\{x_n\}$. Let one can find another weakly convergent subsequence $\{x_{n_j}\}$ of $\{x_n\}$ with weak limit say $\omega_2 \in K$ and $\omega_2 \neq \omega_1$. Again $\omega_1 \in F(P_T) = F(T)$. From the Opial's property and Lemma 2.1, we obtain

$$\begin{aligned} \lim_{n\to\infty} \|x_n - \omega_1\| &= \lim_{j\to\infty} \|x_{n_j} - \omega_1\| < \lim_{j\to\infty} \|x_{n_j} - \omega_2\| = \lim_{n\to\infty} \|x_n - \omega_2\| \\ &= \lim_{k\to\infty} \|x_{n_k} - \omega_2\| < \lim_{k\to\infty} \|x_{n_k} - \omega_1\| = \lim_{n\to\infty} \|x_n - \omega_1\|, \end{aligned}$$

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which is a contradiction. So, $\omega_1 = \omega_2$. Therefore $\{x_n\}$ converges weakly to a fixed point of T. This completes the proof. $\square$

III. Example

Example 3.1. *Let $K = [0, \infty) \subset \mathbb{R}$ endowed with usual norm in $\mathbb{R}$ and $T : K \to \mathcal{P}(K)$ be defined by*

$$Tx = \begin{cases} (0), & 0 \leq x < \frac{1}{400} \\ [0, \frac{3x}{4}], & \frac{1}{400} \leq x \leq 1 \end{cases}$$

If $x \in [0, \frac{1}{400})$, then $P_T(x) = 0$. For $x \in [\frac{1}{400}, 1]$, then $P_T(x) = \{\frac{3x}{4}\}$. We show that P_T is generalized nonexpansive mapping satisfying condition $(E_{\mu=4})$ with $F(T)$. We consider the following cases:

Case I: *Let $x \in [0, \frac{1}{400})$ and $y \in [0, \frac{1}{400})$. We have*

$$d(x, P_T(y)) = |x| \leq \mu|x| \leq \mu d(x, P_T(x)) + |x - y|.$$

Case II: *Let $x \in [\frac{1}{400}, 1]$ and $y \in [\frac{1}{400}, 1]$. We have*

$$\begin{aligned} d(x, P_T(y)) &\leq d(x, P_T(x)) + H(P_T(x), P_T(y)) \\ &= d(x, P_T(x)) + |\frac{3x}{4} - \frac{3y}{4}| \\ &\leq d(x, P_T(x)) + \frac{3}{4}|x - y| \leq d(x, P_T(x)) + |x - y| \\ &\leq \mu d(x, P_T(x)) + |x - y| \end{aligned}$$

Case III: *Let $x \in [\frac{1}{400}, 1]$ and $y \in [0, \frac{1}{400})$. One has*

$$d(x, P_T(y)) = |x| = \frac{4|x|}{4} = \mu d(x, P_T(x)) \leq \mu d(x, P_T(x)) + |x - y|$$

Thus, P_T is generalized nonexpansive mapping satisfying condition $(E_{\mu=4})$ with $p = 0$ fixed point.

Finally, let us prove that T does not satisfy condition (C). Indeed, if we take $x = \frac{1}{1200}, y = \frac{1}{400}$ then

$$\frac{1}{2}d(x, P_T(x)) = \frac{1}{2}|\frac{1}{1200} - 0| = 0.000416 < 0.001666 = |x - y|.$$

$$H(P_T(x), P_T(y)) = |\frac{3}{4} \times \frac{1}{400} - 0| = 0.001875 > 0.001666 = |x - y|.$$

Thus P_T does not satisfy Suzuki's condition (C).

Let $a_n = b_n = c_n = 0.75$ for all $n \in \mathbb{N}$ and be $x_1 = 0.5$. We compute that the sequence $\{x_n\}$ generated by iterative scheme (1.1) converge to fixed point 0 of the multi-valued generalized nonexpansive mapping satisfying condition (E) defined in Example 3.1 which is shown by the Figure 1.

IV. Conclusions

We study the convergence of (1.1)-iteration process to fixed for the multi-valued generalized nonexpansive mapping satisfying condition (E) in uniformly convex Banach space. Moreover, we give an illustrative numerical example that is multi-valued generalized nonexpansive mapping satisfying condition (E) but is not Suzuki generalized nonexpansive mapping, as in Example 3.1 of this paper.

Notes

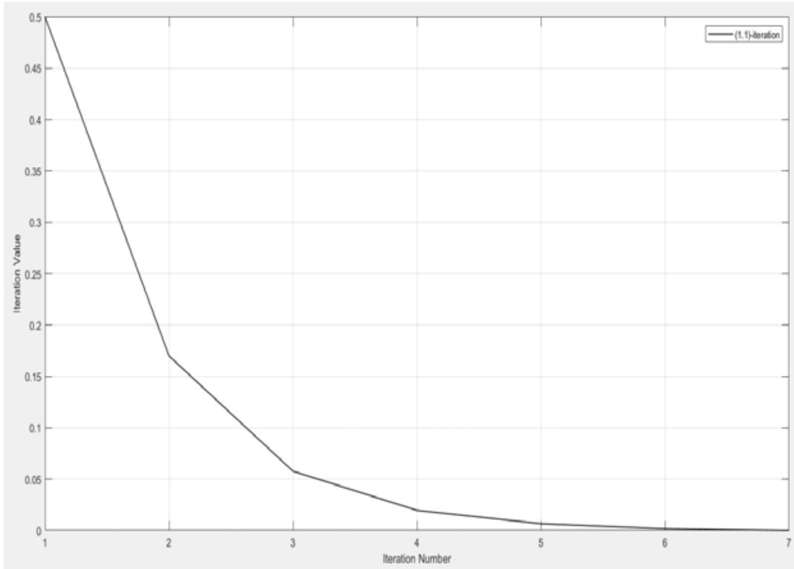

Figure 1: Convergence of (1.1)-iteration to the fixed point 0 of the multi-valued generalized nonexpansive mapping satisfying condition (E) defined in Example 3.1.

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Comparison of Intermediate Lithuanian Mathematics Programs with the Franch Mathematics Program

By Birutė Ragalytė & Alma Paukštienė

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Summary- The article compares interwar Lithuanian mathematics programs with French mathematics programs. It shows how Klein's ideas influenced the content of the programs.

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Birutė Ragalytė [α] & Alma Paukštienė [σ]

Summary- The article compares interwar Lithuanian mathematics programs with French mathematics programs. It shows how Klein's ideas influenced the content of the programs.

Keywords: mathematics programs, klein's ideas, renewal of interwar lithuanian mathematics education.

I. Introduction

The most important event that influenced mathematics curricula in Western Europe was 1905. The convention of the Society of German Naturalists and Doctors was held in the city of Merane. The initiator of this convention was the famous German mathematician Felix Klein. At the convention, a draft of the German mathematics curriculum for general education was adopted, which is now called the Merane curriculum. According to this syllabus, mathematics had to be taught using functions. First, the concept of function was introduced in algebra and trigonometry. Areas of geometric figures, volumes of bodies are suggested to be interpreted as functions of their dimensions. Merane's program envisages introducing the concepts of derivative and integral of a function in secondary school.

The idea to reorganize mathematics programs in independent Lithuania arose immediately after regaining independence. A major influence in the development of mathematics programs was France.

Sincov D.M.'s publication "Sbornik program i instrukcij po prepodavaniju matematiki v zapadnaj Evrope"[4] contains not only the programs and methodological instructions of the foreign countries France, Denmark, Italy, Austria, the German federal states of Württemberg and Baden, but also indicates the number of weekly hours, for studying the subject of mathematics. Using the material presented in this publication, a comparison of the number of lessons devoted to teaching mathematics was made in various foreign countries and in Lithuania in the 20th century. at the beginning.

The first mathematics curriculum of Independent Lithuania for primary school was prepared in 1919, published in the periodical in 1921, in the journal "Science and Life". The program is very concise. Most often, the subject that must be covered during one academic year is described in one or more sentences [3]). P. Mašiotas notes in the

Ref

4. Sincov D. M., Sbornik program i instrukcij po prepodavaniju matematiki v zapadnoj Evrope.- Maskva, 1914.

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article "Mathematics programs in construction" [2] that the programs are defined too narrowly. Not every teacher who receives such a program is able to name the subjects and draw up a detailed program himself. In this article, he states that "only very well-established programs can be poorly taught." [2]

There was no common plan for secondary and higher schools, and we had to wait longer for it. According to A. Ažubalis, matters of arithmetic teaching in senior classes are discussed for the first time in S. Balčytis' article "On the mathematics teaching plan in middle school" [1].

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II. Comparison of the Mathematics Program of the First Lithuanian Primary School with the One in 1912. French Primary School Mathematics Program

Before we start comparing math programs, let's look at the French curriculum. Teaching starts from primary grades. After them, preparatory classes begin. Interestingly, they start numbering from the ninth grade in descending order. Preparatory is only ninth grade. Elementary grades are eighth and seventh grade. Cycle I begin after these classes. This cycle consists of the sixth, fifth, fourth and third grades. Cycle I consist of two sections A and B. The training of the second cycle lasts 3 years. Cycle II is divided into several types of schools.

We will compare the first mathematics program of the Lithuanian primary school, which was prepared in 1919. with the French Elementary Mathematical Syllabus (1912).

Although the first Lithuanian primary school mathematics program was prepared in 1919, it was published in a periodical only in 1921.

In 1921 the primary school program in Lithuania is not much different from 1912. French primary school mathematics programs. However, there are differences, we will analyze them.

1st class. In Lithuania, the arithmetic course introduces reading, addition, and subtraction up to 20, and teaches how to count with whole tens up to 100. In the French program, already in the tenth grade (in the program, the classes are presented in descending order, the classes were also numbered) actions were performed with numbers up to 1000. Both programs introduce basic metric units of measurement: steps, meters, centimeters, liters, grams. As in the French program, as well as the Lithuanian one, it is taught to estimate sizes. The program includes exercises for guessing length, distance, width, weight and more.

If in the Lithuanian program the fractions æ and ß are introduced in the 1st chapter, then in France - in the 9th grade (in the 2 class). Intuition in evaluating things continues to develop.

At a similar level, the program presents the complexity of calculations: addition, subtraction, multiplication and division of single-digit and multi-digit numbers.

2 class. In Lithuania, a part of the geometry course is introduced, which teaches measurement of straight lines, drawing, division into equal parts, measurement and calculation of the area of a square.

In the French program, operations with decimal fractions are introduced first, followed by operations with simple fractions. The Lithuanian program examines separate composition, subtraction, multiplication of fractions and division of whole numbers of simple fractions with denominators 2, 4, 8, 16, 5, 10, 100, 3, 9. The

Lithuanian program examines decimal fractions after learning part of the simple fractions training course.

3 class. In Lithuanian primary school, actions up to 1000 are provided. It should be noted that memory calculation exercises are also provided up to 1000, learning to determine the length and width of figures by heart.

The part of the geometry course, which includes measuring, drawing, dividing straight lines, equalizing angles, measuring angles with a ruler, and measuring and calculating the areas of squares, is already included in the Lithuanian elementary program of 3 class. In the corresponding class in the French program, only basic geometric figures and their models are introduced.

Notes

In the eighth grade of France (corresponding to the 3rd grade of Lithuania), a lot of attention is paid to the metric system.

4 class. (Respectively in the 7th grade) the French program provides for teaching to work with decimal fractions, then simple fractions. In geometry, measuring the surfaces of such figures as: cube, pyramid, prism, cylinder.

In Lithuania, 4 class also provides for teaching time tasks, calendar knowledge, simple fractions. It should be noted that simple fractions are taught first, then decimals. Both square and cubic measurements are entered. Geometry training includes measuring and calculating the area of parallelograms, triangles, polygons, rectangular pole, right prism, cylinder, surface and volume of a cylinder. Learn to plan with an ecker.

We can say that in the primary school program in 1919 more complex topics are provided than in the analogous French primary school curriculum. The analyzed programs have many similarities.

III. Comparison of French (1912) and Lithuanian (1929) Mathematics Programs

In the 1st grade of secondary school, the Lithuanian program provides for learning calculation with random numbers, solving the simplest time problems, getting to know decimal fractions and the simplest actions with them. The program covers the following topics: area and volume measurements, volumes of rectangular boxes and cubes, drawing rectangles and squares in their actual sizes using a scale. In the examined Lithuanian mathematics program, it is constantly reminded that tasks must be taken from the environment, from social life, knowledge of the region and other branches of science.

In the French mathematics curriculum, simple fractions are introduced first, followed by decimal fractions. But only after familiarizing with simple fractions, learning to name them and perform actions with them, decimal fractions and their actions are introduced. It is also mentioned here that when solving problems, especially those related to fractions, concrete examples should be used. The teacher is advised not to present any theory. The main goal is to learn to understand the meaning of each action.

In the French mathematics curriculum, the introductory course prepares students for learning simple fractions. It starts with dividing numbers by divisors, then it is planned to teach how to find the largest divisor and the smallest multiple. Next in the program are simple fractions and operations with them. Although operations with decimals were already taught in the 1st grade, after learning operations with simple fractions, we return to the teaching of decimal fractions. Taught topics ”Approximated data” and ”Determining the results of an approximated data”. Problems about prisms, coils, pyramids, cones, parallelograms, triangles, trapezoids, polygons, circles, circles are already solved in the 2nd grade of secondary school.

In the Lithuanian program in class III, it is planned to solve tasks encountered in practice: calculation of profit and loss, calculation of discounts and interest, concentration of milk fat and other liquids.

Comparing the Lithuanian program with the French one, we can notice a lot of similarities. The topics are very similar, only the order of the topics is different. The influence of F. Klein's ideas can be felt, as students are trained to understand the concept of function from the 3rd grade, solve and create equations with one unknown. Students are "practiced" the concept of a function sequentially - starting with the creation of tables of the size of the phenomenon.

Unlike the Lithuanian program, ordinary fractions are taught. In the fourth grade (respectively in the III Lithuanian grade) the signs of division by 2, 5, 9, 3 are studied, the greatest common divisor and the least common multiple are searched for, even though the concept of simple fraction was already introduced in the sixth grade, operations with fractions with the same denominators. However, only after 2 years, the introduction of the concept of the lowest common multiple leads to a return to the teaching of simple fractions.

When entering the concepts of positive and negative numbers, the Lithuanian and French programs are similar. Concepts are explained with concrete examples. In both programs, once the concept of positive and negative numbers is introduced, actions with polynomials and monomials are considered, and equations with one unknown are solved.

Examining the geometry course, we can also find many similarities in the mathematics programs of Lithuania and France. However, it should be noted that the first theorems, properties of triangles, parallelogram angles, sides and diagonals are proved in both programs when introducing a systematic planimetry course. The examined Lithuanian mathematics program does not specify which statements will be proved. The French math syllabus specifies which statements will be presented without proof.

When examining drawing tasks, the Lithuanian program provides for the solution of standard tasks: drawing a line through a given point, drawing a line parallel to a given point, raising, and lowering a perpendicular, dividing segments and angles into 2, 4, 8, etc. part, drawing angles 900, 450, 600, 300, etc. with the help of a ruler and a protractor. The French program does not specify exactly which drawing problems are intended to be solved, it only says that we solve simple examples and problems by drawing.

Next, we will compare the 1929 a mathematics program for higher schools with Greek and with enhanced foreign language teaching, with a French program for schools with Latin and Greek teaching.

The content of the programs is somewhat different, but in each program, there is a sense of application of functions in solving tasks. Both programs emphasize computation with approximate numbers and estimation of error.

The French curriculum emphasizes that the purpose of the first two grades (i.e., the last grades in the French curriculum) is to prepare students for learning physics, where most physical processes are written in equations, the quantities of which have a functional dependence. Also, students must be able to perform actions with fractions, decimal numbers, use the metric system of measurements.

Notes

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In the Lithuanian program, when solving quadratic equations, not only real but also imaginary roots are found. The French program does not provide for finding complex solutions of the quadratic equation.

In both the Lithuanian and French programs, the concept of symmetrical points and symmetry is introduced when examining similar triangles and the areas of the simplest figures. Only this is done in the Lithuanian program in the IV th grade of the secondary school, and in the French one - in the Ist grade, completing the geometry course.

Notes

In the French program, only studies of circles inscribed and circumscribed about a triangle are provided. From this point of view, the Lithuanian program is broader. Not only circles inscribed and circumscribed into triangles are considered, but also circles inscribed and circumscribed into regular polygons. The teaching of regular polygons is not included in the compulsory curriculum, but it is in the optional (optional) curriculum, although this curriculum does not say anything about what is planned to be taught. In the Lithuanian curriculum, the concept of infinitesimal Ness and limit is provided in the VI class, and in the French only in the optional curriculum. in Lithuania in 1929 a mathematics program was created for several types of schools (with Latin and commercial schools, with reinforced teaching of mathematics and science, with Greek and reinforced teaching of foreign languages). Also in France, not only a program for schools with Latin and Greek, but also a program for schools with Latin and exact sciences was created. In the Lithuanian program, the teaching of logarithms is provided for in all three programs, and in the French one, only in schools with Latin and exact sciences. In France, the main focus is on students learning to use logarithmic tables, the teacher is given the right to present the most general features of the theory, which is based on the teaching of progressions or the study of degree indicators. The Lithuanian mathematics program also teaches how to use logarithm tables. But it is not limited to that. The use of logarithms to find roots of numbers and to calculate more complex formulas, logarithms of trigonometric functions and expressions, properties of decimal logarithms and laws of practical use are examined. In both programs, immediately after learning logarithms, the topic ”Problems of complex fractions” is covered.

In the Lithuanian mathematics curriculum (the 1929 mathematics curriculum for high schools with Latin and commercial schools), the teaching of geometry in grade V is very similar to the teaching of geometry in the second (penultimate) grade of a French school with Latin and exact sciences. The topic ”Circle. Diameters. Bows. Properties of strings.” Analogously, this topic is also presented in the French program: ”Circle. Intersection of a circle and a straight line. Tangent. Bows and Strings”. Other topics of both programs are also presented very similarly. The topic of the Lithuanian program ”Relation between two sections and methods of finding it. Finding the approximate ratio of two segments. Proportionality of sections” corresponds to the theme of the French program ”Proportional lengths. Points that divide line segments in each ratio. The Concept of Harmonic Division’. Another topic in the program is ”Similar Triangles”. However, we can already find more differences here. The French mathematics curriculum describes this topic very succinctly - ”Similar Triangles”. The Lithuanian program under consideration also examines a separate case of similar triangles - right-angled similar triangles and their signs of similarity. Trigonometric functions: sine, cosine, tangent and cotangent are entered in both programs, but the Lithuanian program emphasizes that these functions are considered only for acute angles. Similar regular polygons and the ratios of their perimeters and areas, as well as

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the calculation of the area of a regular polygon, are covered in both programs. When examining regular polygons, the concept of the number is introduced (in both programs). The length of the circle is considered as the limit of the perimeters of the inscribed and defined polygon (in both programs).

Another group of geometry topics is ”Position of lines and curves in space. Planes”. Both programs deal with the topics ”Perpendicularity and parallelism of lines and planes. Double-walled corners. Perpendicular planes”. It should be noted here that symmetry is mentioned in many topics in the French mathematics curriculum. Symmetry is mentioned both when studying planes and when studying geometric figures.

The sequence of topics remains the same in both programs. Both programs explore topics about geometric bodies: prisms, coils, pyramids, cones, spheres. The surfaces and volumes of these geometric bodies are calculated. In the study of the topic ”Sphere”, plane sections of the sphere, large spheres and the characteristics of their arcs are studied. It should be noted that the topic surface of the ball and its parts, volumes of the sections of the ball are presented only in the Lithuanian mathematics program (these topics are not included in the French mathematics program).

In the French mathematics curriculum, a separate course ”Trigonometry” is distinguished. In the Lithuanian mathematics program, trigonometry is not singled out separately. It is taught during the lessons of the ”Geometry” subject. In both programs, trigonometric function interrelationship formulas, argument sum and difference formulas, double and half argument trigonometric functions are studied. We could single out the essential differences between these programs: the Lithuanian program contains problems of practical application of trigonometry, and the French one - the theory of projections.

In the Lithuanian school, in the VIII grade and the first (last) grade, the canonical calculus course of the French general mathematics program is part of the higher analytical geometry course. Both programs deal with quadratic equations in two variables and their geometric representation. Such curves as ellipses, hyperbolas, parabolas are examined. The properties of these curves, the search for tangents and normal lines are examined.

We can compare the Algebra VII and VIII grade course with the French math program for a special grade. In grade VII, the Lithuanian mathematics program starts with compound theory and Newton's binomial formula. These topics are also covered in a special class in the French mathematics curriculum. In the French mathematics program, a special class introduces complex numbers, Moivre's formula, and after a significant part of the mathematics course, introduces the concept of imaginary roots of an equation and finding those roots.

In the eighth grade, the Lithuanian mathematics program includes the course ”Analysis of infinitesimals”. This course examines the theory of limits, continuity of functions, basic laws of differentiation of functions, finding extrema of a function of one argument, integration, finding areas of curvilinear figures.

Examining the mathematics program of the French supplementary class, we can find from the previously listed topics of the ”Analysis of infinitesimals” course: continuity of a function, finding limits, finding the minimum and maximum value of a function, using the integral of a function to find the area of a curved trapezoid, integration. Differentiation of functions is not an additional class in the French curriculum, but it is taught in a previous math course.

Notes

Also, in the French additional mathematics class, the Analytical Geometry course deals with second-order curves. Their examination was also provided for in the previous program in the last grade, only at a much lower level (in the new program, these curves are examined much more extensively).

1912 the French mathematics program was significantly influenced by F. Klein's ideas. Comparing France in 1912 and Lithuania in 1929 mathematics programs have many similar topics, only the order of the topics is slightly different. In the analyzed programs, the application of functions for problem solving is felt.

Notes

IV. Conclusions

1. After analyzing the French mathematics program (1912), we can say that the program is strongly influenced by F. Klein's ideas.
2. Lithuanian math program for primary school (1921) has many advantages in comparison with French math program for primary school (1912).

Similarities:

- Elementary mathematical counts.
- Metric system and knowledge of initiative geometry.

Differences:

- The teaching method of the presentation of fractions differs in Lithuanian program common fractions are introduced higher than decimal fractions, whereas in French math program firstly decimal fractions and operations with them are introduced and only then common fractions.
- Lithuanian math program for primary school provides more difficult themes than analogous program for primary school in France.

3. In Lithuanian (1929) and French (1912) math programs:

- Pupils from the III form are prepared to understand the concept of function and functions used for the solution of tasks;
- The introducing of systematic plane geometry course provides the proof of the first theorems, triangle properties, properties of parallelogram angles, sides, and diagonals.
- The solution of percent tasks.
- Differentiation, integration.
- Binominal theorem.
- In geometry content is very similar in both programs, the order of the introduction of themes is the same.
- There are parts of the course of higher analytical geometry.

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Career Financial

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Financial

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Organize seminar/conference

Associates are authorized to organize symposium/seminar/conference on behalf of Global Journal Incorporation (USA). They can also participate in the same organized by another institution as representative of Global Journal. In both the cases, it is mandatory for him to discuss with us and obtain our consent. Additionally, they get free research conferences (and others) alerts.

Career Credibility Financial

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Exclusive Financial

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Financial

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Associate	Fellow	Research Group	Basic
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Certificate, LoR and Momento 2 discounted publishing/year **Gradation** of Research 10 research contacts/day 1 **GB** Cloud Storage **GJ** Community Access	**Certificate,** LoR and Momento **Unlimited** discounted publishing/year **Gradation** of Research **Unlimited** research contacts/day 5 **GB** Cloud Storage **Online Presense** Assistance **GJ** Community Access	**Certificates,** LoRs and Momentos **Unlimited** free publishing/year **Gradation** of Research **Unlimited** research contacts/day **Unlimited** Cloud Storage **Online Presense** Assistance **GJ** Community Access	**GJ** Community Access

Preferred Author Guidelines

We accept the manuscript submissions in any standard (generic) format.

We typeset manuscripts using advanced typesetting tools like Adobe In Design, CorelDraw, TeXnicCenter, and TeXStudio. We usually recommend authors submit their research using any standard format they are comfortable with, and let Global Journals do the rest.

Alternatively, you can download our basic template from https://globaljournals.org/Template.zip

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3. Ensure corresponding author's email address and postal address are accurate and reachable.
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Acknowledgments

Contributors to the research other than authors credited should be mentioned in Acknowledgments. The source of funding for the research can be included. Suppliers of resources may be mentioned along with their addresses.

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Authors can submit papers and articles in an acceptable file format: MS Word (doc, docx), LaTeX (.tex, .zip or .rar including all of your files), Adobe PDF (.pdf), rich text format (.rtf), simple text document (.txt), Open Document Text (.odt), and Apple Pages (.pages). Our professional layout editors will format the entire paper according to our official guidelines. This is one of the highlights of publishing with Global Journals—authors should not be concerned about the formatting of their paper. Global Journals accepts articles and manuscripts in every major language, be it Spanish, Chinese, Japanese, Portuguese, Russian, French, German, Dutch, Italian, Greek, or any other national language, but the title, subtitle, and abstract should be in English. This will facilitate indexing and the pre-peer review process.

The following is the official style and template developed for publication of a research paper. Authors are not required to follow this style during the submission of the paper. It is just for reference purposes.

Manuscript Style Instruction (Optional)

- Microsoft Word Document Setting Instructions.
- Font type of all text should be Swis721 Lt BT.
- Page size: 8.27" x 11'", left margin: 0.65, right margin: 0.65, bottom margin: 0.75.
- Paper title should be in one column of font size 24.
- Author name in font size of 11 in one column.
- Abstract: font size 9 with the word "Abstract" in bold italics.
- Main text: font size 10 with two justified columns.
- Two columns with equal column width of 3.38 and spacing of 0.2.
- First character must be three lines drop-capped.
- The paragraph before spacing of 1 pt and after of 0 pt.
- Line spacing of 1 pt.
- Large images must be in one column.
- The names of first main headings (Heading 1) must be in Roman font, capital letters, and font size of 10.
- The names of second main headings (Heading 2) must not include numbers and must be in italics with a font size of 10.

Structure and Format of Manuscript

The recommended size of an original research paper is under 15,000 words and review papers under 7,000 words. Research articles should be less than 10,000 words. Research papers are usually longer than review papers. Review papers are reports of significant research (typically less than 7,000 words, including tables, figures, and references)

A research paper must include:

a) A title which should be relevant to the theme of the paper.
b) A summary, known as an abstract (less than 150 words), containing the major results and conclusions.
c) Up to 10 keywords that precisely identify the paper's subject, purpose, and focus.
d) An introduction, giving fundamental background objectives.
e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition, sources of information must be given, and numerical methods must be specified by reference.
f) Results which should be presented concisely by well-designed tables and figures.
g) Suitable statistical data should also be given.
h) All data must have been gathered with attention to numerical detail in the planning stage.

Design has been recognized to be essential to experiments for a considerable time, and the editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned unrefereed.

i) Discussion should cover implications and consequences and not just recapitulate the results; conclusions should also be summarized.
j) There should be brief acknowledgments.
k) There ought to be references in the conventional format. Global Journals recommends APA format.

Authors should carefully consider the preparation of papers to ensure that they communicate effectively. Papers are much more likely to be accepted if they are carefully designed and laid out, contain few or no errors, are summarizing, and follow instructions. They will also be published with much fewer delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and suggestions to improve brevity.

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It is necessary that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

All manuscripts submitted to Global Journals should include:

Title

The title page must carry an informative title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) where the work was carried out.

Author details

The full postal address of any related author(s) must be specified.

Abstract

The abstract is the foundation of the research paper. It should be clear and concise and must contain the objective of the paper and inferences drawn. It is advised to not include big mathematical equations or complicated jargon.

Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Keywords

A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy: planning of a list of possible keywords and phrases to try.

Choice of the main keywords is the first tool of writing a research paper. Research paper writing is an art. Keyword search should be as strategic as possible.

One should start brainstorming lists of potential keywords before even beginning searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in a research paper?" Then consider synonyms for the important words.

It may take the discovery of only one important paper to steer in the right keyword direction because, in most databases, the keywords under which a research paper is abstracted are listed with the paper.

Numerical Methods

Numerical methods used should be transparent and, where appropriate, supported by references.

Abbreviations

Authors must list all the abbreviations used in the paper at the end of the paper or in a separate table before using them.

Formulas and equations

Authors are advised to submit any mathematical equation using either MathJax, KaTeX, or LaTeX, or in a very high-quality image.

Tables, Figures, and Figure Legends

Tables: Tables should be cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g., Table 4, a self-explanatory caption, and be on a separate sheet. Authors must submit tables in an editable format and not as images. References to these tables (if any) must be mentioned accurately.

Figures

Figures are supposed to be submitted as separate files. Always include a citation in the text for each figure using Arabic numbers, e.g., Fig. 4. Artwork must be submitted online in vector electronic form or by emailing it.

Preparation of Eletronic Figures for Publication

Although low-quality images are sufficient for review purposes, print publication requires high-quality images to prevent the final product being blurred or fuzzy. Submit (possibly by e-mail) EPS (line art) or TIFF (halftone/ photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Avoid using pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings). Please give the data for figures in black and white or submit a Color Work Agreement form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

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Tips for Writing a Good Quality Science Frontier Research Paper

Techniques for writing a good quality Science Frontier Research paper:

1. Choosing the topic: In most cases, the topic is selected by the interests of the author, but it can also be suggested by the guides. You can have several topics, and then judge which you are most comfortable with. This may be done by asking several questions of yourself, like "Will I be able to carry out a search in this area? Will I find all necessary resources to accomplish the search? Will I be able to find all information in this field area?" If the answer to this type of question is "yes," then you ought to choose that topic. In most cases, you may have to conduct surveys and visit several places. Also, you might have to do a lot of work to find all the rises and falls of the various data on that subject. Sometimes, detailed information plays a vital role, instead of short information. Evaluators are human: The first thing to remember is that evaluators are also human beings. They are not only meant for rejecting a paper. They are here to evaluate your paper. So present your best aspect.

2. Think like evaluators: If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

3. Ask your guides: If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

4. Use of computer is recommended: As you are doing research in the field of science frontier then this point is quite obvious. Use right software: Always use good quality software packages. If you are not capable of judging good software, then you can lose the quality of your paper unknowingly. There are various programs available to help you which you can get through the internet.

5. Use the internet for help: An excellent start for your paper is using Google. It is a wondrous search engine, where you can have your doubts resolved. You may also read some answers for the frequent question of how to write your research paper or find a model research paper. You can download books from the internet. If you have all the required books, place importance on reading, selecting, and analyzing the specified information. Then sketch out your research paper. Use big pictures: You may use encyclopedias like Wikipedia to get pictures with the best resolution. At Global Journals, you should strictly follow here.

6. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

7. Revise what you wrote: When you write anything, always read it, summarize it, and then finalize it.

8. Make every effort: Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

9. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several unnecessary diagrams will degrade the quality of your paper by creating a hodgepodge. So always try to include diagrams which were made by you to improve the readability of your paper. Use of direct quotes: When you do research relevant to literature, history, or current affairs, then use of quotes becomes essential, but if the study is relevant to science, use of quotes is not preferable.

10. Use proper verb tense: Use proper verb tenses in your paper. Use past tense to present those events that have happened. Use present tense to indicate events that are going on. Use future tense to indicate events that will happen in the future. Use of wrong tenses will confuse the evaluator. Avoid sentences that are incomplete.

11. Pick a good study spot: Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

12. Know what you know: Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

13. Use good grammar: Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice.

Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

14. Arrangement of information: Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. Multitasking in research is not good: Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. Never copy others' work: Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. Go to seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. Refresh your mind after intervals: Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

20. Think technically: Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

21. Adding unnecessary information: Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

22. Report concluded results: Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

The introduction: This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear: Adhere to recommended page limits.

Mistakes to avoid:

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

Title page:

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

  - Single section and succinct.
  - An outline of the job done is always written in past tense.
  - Concentrate on shortening results—limit background information to a verdict or two.
  - Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

The following approach can create a valuable beginning:

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

Materials may be reported in part of a section or else they may be recognized along with your measures.

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- Give details of all of your remarks as much as possible, focusing on mechanisms.
- Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

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Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
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