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Mathematics and Decision Science

Highlights

- Analog Computer on GPU
- Exploration of Essential Knowledge
- Pressure-Less Gas System
- Oriented Finite Element Programming

Discovering Thoughts, Inventing Future

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Quantum Computer as Analog Computer on GPU

By Alexander Soiguine
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Abstract- The geometric algebra lift of conventional quantum mechanics qubits is the game-changing quantum leap forward potentially kicking from the quantum computing market big fishes (IBM, Microsoft, Google, dozens of smaller ones) investing billions in elaborating quantum computing devices. It brings into reality a kind of physical field spreading through the whole three-dimensional space and values of the time parameter. The fields can be modified instantly in all points of space and time values. All measured observable values are simultaneously available all together, not through looking one by one. In this way the new type of quantum computer appeared to be a kind of analog computer keeping and instantly processing information by and on sets of objects possessing an infinite number of degrees of freedom. As practical implementation, the multithread GPUs with the CUDA language functionality allow creating of software simulating that kind of fields processing numbers of space/time discrete points only restricted by the GPU threads capacity.

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Ref

1. B. Ulmann, Analog Computing, Oldenbourg: De Gruyter , 2022.

Quantum Computer as Analog Computer on GPU

Alexander Soiguine[1]

Abstract- The geometric algebra lift of conventional quantum mechanics qubits is the game-changing quantum leap forward potentially kicking from the quantum computing market big fishes (IBM, Microsoft, Google, dozens of smaller ones) investing billions in elaborating quantum computing devices. It brings into reality a kind of physical field spreading through the whole three-dimensional space and values of the time parameter. The fields can be modified instantly in all points of space and time values. All measured observable values are simultaneously available all together, not through looking one by one. In this way the new type of quantum computer appeared to be a kind of analog computer keeping and instantly processing information by and on sets of objects possessing an infinite number of degrees of freedom. As practical implementation, the multithread GPUs with the CUDA language functionality allow creating of software simulating that kind of fields processing numbers of space/time discrete points only restricted by the GPU threads capacity.

I. Maxwell Equation in Geometric Algebra to Simulate Computing Device of Analog Computer

An analog computer is generally a type of computing device that uses the continuous variation aspect of physical phenomena to model the problem being solved[1]. One special type of physical phenomenon to model problems is considered below.

The circular polarized electromagnetic waves following from the solution of Maxwell equations in free space done in geometric algebra terms,[2], [3],are the electromagnetic fields of the form:

$$F = F_0 exp[I_S(\omega t - k \cdot r)] \tag{1.1}$$

which should be the solution of

$$(\partial_t + \nabla)F = 0 \tag{1.2}$$

Solution of (1.2) must be the sum of a vector (electric field e) and bivector (magnetic field $I_3 h$):

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$$F = e + I_3 h,$$

with some initial conditions:

$$e + I_3 h|_{t=0,\vec{r}=0} = F_0 = e|_{t=0,\vec{r}=0} + I_3 h|_{t=0,\vec{r}=0} = e_0 + I_3 h_0$$

For a given plane S in (1.1),the solution of the three-dimensional Maxwell equation (1.2) has two options:

- $F_+ = (e_0 + I_3 h_0) exp[I_S(\omega t - k_+ \cdot r)]$, with $\hat{k}_+ = I_3 I_S, \hat{e}\hat{h}\hat{k}_+ = I_3$, and the triple$\{\hat{e}, \hat{h}, \hat{k}_+\}$ is right hand screw oriented, that's rotation of $\hat{e}$to $\hat{h}$ by $\pi/2$ gives movement of *right hand screw* in the direction of $k_+ = |k| I_3 I_S$;
- $F_- = (e_0 + I_3 h_0) exp[I_S(\omega t - k_- \cdot r)]$, with $\hat{k}_- = -I_3 I_S, \hat{e}\hat{h}\hat{k}_- = -I_3$, and the triple $\{\hat{e}, \hat{h}, \hat{k}_-\}$ is left hand screw oriented, that's the rotation of $\hat{e}$ to $\hat{h}$ by $\pi/2$ gives movement of *left-hand screw* in the direction of $k_- = -|k| I_3 I_S$ or, equivalently, movement of *right-hand screw* in the opposite direction, $-k_-$;

where e_0 and h_0, initial values of $e$and h, are arbitrary mutually orthogonal vectors of equal length, lying on the plane$S$. Vectors $k_\pm = \pm|k_\pm| I_3 I_S$ are normal to that plane. The length of the "wave vectors" $|k_\pm|$ is equal to angular frequency ω.

Maxwell's equation (1.2) is a linear one. Then, any linear combination of F_+ and F_- saving the structure of (1.1) will also be a solution.

Let's write:

$$\begin{cases} F_+ = (e_0 + I_3 h_0) exp[I_S \omega(t - (I_3 I_S) \cdot r)] = (e_0 + I_3 h_0) exp[I_S \omega t] exp\big[-I_S[(I_3 I_S) \cdot r]\big] \\ F_- = (e_0 + I_3 h_0) exp[I_S \omega(t + (I_3 I_S) \cdot r)] = (e_0 + I_3 h_0) exp[I_S \omega t] exp\big[I_S[(I_3 I_S) \cdot r]\big] \end{cases} \quad (1.3)$$

Then, for arbitrary (real[2]) scalars λ and μ:

$$\lambda F_+ + \mu F_- = (e_0 + I_3 h_0) e^{I_S \omega t} \left(\lambda e^{-I_S[(I_3 I_S)\cdot r]} + \mu e^{I_S[(I_3 I_S)\cdot r]}\right) \quad (1.4)$$

is also solution of (1.2). The item in the second parenthesis is a weighted linear combination of two states (wave functions, g-qubits[4], [5]) with the same phase in the same plane but the opposite sense of orientation. The states are strictly coupled, entangled if you prefer, because the bivector plane should be the same for both, no matter what happens with that plane.

Arbitrary linear combination (1.4) can be rewritten as:

$$\lambda e^{I^+_{Plane}\, \varphi^+} + \mu e^{I^-_{Plane}\, \varphi^-} \quad (1.5),$$

with

$$\varphi^\pm = \cos^{-1}\left(\frac{1}{\sqrt{2}} \cos \omega (t \mp [(I_3 I_S) \cdot r])\right),$$

[2] Remember, in the current theory scalars are real ones. "Complex" scalars have no sense.

Ref

4. A. Soiguine, Geometric Phase in Geometric Algebra Qubit Formalism, Saarbrucken: LAMBERT Academic Publishing, 2015.

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$$I_{Plane}^{\pm} = I_S \frac{sin\omega(t \mp [(I_3 I_S)\cdot r])}{\sqrt{1+sin^2\omega(t \mp [(I_3 I_S)\cdot r])}} + I_{B_0}\frac{cos\omega(t \mp [(I_3 I_S)\cdot r])}{\sqrt{1+sin^2\omega(t \mp [(I_3 I_S)\cdot r])}}$$

$$+ I_{E_0}\frac{sin\omega(t \mp [(I_3 I_S)\cdot r])}{\sqrt{1+sin^2\omega(t \mp [(I_3 I_S)\cdot r])}}$$

The triple of unit value basis orthonormal bivectors $\{I_S, I_{B_0}, I_{E_0}\}$ comprises the I_S bivector, dual to the propagation direction vector; I_{B_0} is dual to the initial vector of magnetic field; I_{E_0} is dual to the initial vector of electric field. The expression (1.5) is a linear combination of two geometric algebra states, g-qubits.

Linear combination of the two equally weighted solutions of the Maxwell equation F_+ and F_-, $\lambda F_+ + \mu F_-$ with $\lambda = \mu = 1$ reads:

$$\lambda F_+ + \mu F_-|_{\lambda=\mu=1} = 2\cos\omega[(I_3 I_S)\cdot r]\left(\frac{1}{\sqrt{2}}\cos\omega t + I_S\frac{1}{\sqrt{2}}\sin\omega t + I_{B_0}\frac{1}{\sqrt{2}}\cos\omega t + I_{E_0}\frac{1}{\sqrt{2}}\sin\omega t\right) \quad (1.6)$$

where $\cos\varphi = \frac{1}{\sqrt{2}}\cos\omega t$ and $\sin\varphi = \frac{1}{\sqrt{2}}\sqrt{1+(\sin\omega t)^2}$. It can be written in standard exponential form $\cos\varphi + \sin\varphi\, I_B = e^{I_B\varphi}$.[3]

I will call such kind of g-qubits *spreons* (or sprefields.) They spread over the entire three-dimensional space for all values of time and, particularly, instantly change under Clifford translations over the whole three-dimensional space for all values of time, along with the results of measurement of any observable.

II. CUDA GPU Simulation of the Analog Modeling Computer

In classical mechanics, states are identified by vectors in three-dimensional space [6]. *Infinitesimal transformation of a state, vector, in classical mechanics is:*

$$\vec{X}(t+\Delta t) \approx \vec{X}(t) + \frac{\partial}{\partial t}\vec{X}(t)\Delta t$$

$$\frac{\vec{X}(t+\Delta t) - \vec{X}(t)}{\Delta t} \approx \frac{\partial}{\partial t}\vec{X}(t)$$

In our torsion mechanics [7] states, wave functions, are identified by points on three-sphere $\mathbb{S}^3$.A state[4]there is, see[5], Sec.2.5:

$$e^{I_S(t)\varphi(t)},$$

where I_S is a unit value bivector.

[3] Good to remember that the two solutions F_+ and F_- differ only by the sign of $I_3 I_S$, which is caused by the orientation of I_S that, in its turn, defines if the triple $\{\hat{E}, \hat{H}, \pm I_3 I_S\}$ is right-hand screw or left-hand screw oriented.

[4] "State" and "wave function" are synonyms in the current formalism and particular one is used when it is necessary to stress the meaning.

Ref

7. A. Soiguine, The Torsion Mechanics, LAMBERT Academic Publishing, 2023.

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If we take a Hamiltonian $H(t) = \alpha + I_3(\beta B_1 + \gamma B_2 + \delta B_3)$ in some orthogonal bivector basis, see[2], then for infinitesimal transformation we have:

$$e^{I_S(t_0+\Delta t)\varphi(t_0+\Delta t)} = e^{-I_H(t_0)|H(t_0)|\Delta t} e^{I_S(t_0)\varphi(t_0)}$$

and

$$\lim_{\Delta t \to 0} \frac{\Delta e^{I_S(t_0)\varphi(t_0)}}{\Delta t} = \lim_{\Delta t \to 0} \frac{e^{I_S(t_0+\Delta t)\varphi(t_0+\Delta t)} - e^{I_S(t_0)\varphi(t_0)}}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{(1 - I_H(t_0)|H(t_0)|\Delta t) e^{I_S(t_0)\varphi(t_0)} - e^{I_S(t_0)\varphi(t_0)}}{\Delta t}$$

$$= -I_H(t_0)|H(t_0)| e^{I_S(t_0)\varphi(t_0)}$$

We received the Schrodinger equation for the state $e^{I_S(t)\varphi(t)}$. That means that the Schrodinger equation governs the evolution of operators, states, which act on observables.

While in classical mechanics, measurement of observable $\vec{x}(t)$(action by a state) is a shift by a state:

$$\vec{x}(t) \to \vec{x}(t) + \vec{X}(t),$$

in the torsion mechanics it is:

$$C \to e^{-I_S(t)\varphi(t)} C e^{I_S(t)\varphi(t)},$$

that's rotation in the plane of bivector I_S by angle φ.
Take the general case of observable as a g-qubit: $C = C_0 + C_1 B_1 + C_2 B_2 + C_3 B_3$.
Its measurement by a state $\alpha + \beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3$is [4]:

$$C_0 + C_1 B_1 + C_2 B_2 + C_3 B_3 \xrightarrow{\alpha+\beta_1 B_1+\beta_2 B_2+\beta_3 B_3} C_0:$$

$$+ \big(C_1[(\alpha^2 + \beta_1^2) - (\beta_2^2 + \beta_3^2)] + 2C_2(\beta_1\beta_2 - \alpha\beta_3) + 2C_3(\alpha\beta_2 + \beta_1\beta_3)\big) B_1$$

$$+ \big(2C_1(\alpha\beta_3 + \beta_1\beta_2) + C_2[(\alpha^2 + \beta_2^2) - (\beta_1^2 + \beta_3^2)] + 2C_3(\beta_2\beta_3 - \alpha\beta_1)\big) B_2$$

$$+ (2C_1(\beta_1\beta_3 - \alpha\beta_2) + 2C_2(\alpha\beta_1 + \beta_2\beta_3) + C_3[(\alpha^2 + \beta_3^2) - (\beta_1^2 + \beta_2^2)]) B_3$$

When the state is (1.6) we have:

$B_1 = I_S$, $B_2 = I_{B_0}$, $B_3 = I_{E_0}$, $\alpha = 2\cos\omega[(I_3 I_S)\cdot r]\frac{1}{\sqrt{2}}\cos\omega t$, $\beta_1 = 2\cos\omega[(I_3 I_S)\cdot r]\frac{1}{\sqrt{2}}\sin\omega t$, $\beta_2 = 2\cos\omega[(I_3 I_S)\cdot r]\frac{1}{\sqrt{2}}\cos\omega t$, $\beta_3 = 2\cos\omega[(I_3 I_S)\cdot r]\frac{1}{\sqrt{2}}\sin\omega t$

Ref

2. A. Soiguine, "Instantaneous Spreading of the g-Qubit Fields," *Journal of Applied Mathematics and Physics*, vol. 7, pp. 591-602, 2019.

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And the measurement result is G_3^+ element spreading through the three-dimensional space for all values of time parameter t:

$$4cos^2\omega[(I_3 I_S)\cdot r]\left[C_0 + C_3 I_S + (C_1 \sin 2\omega t + C_2 \cos 2\omega t)I_{B_0} + (C_2 \sin 2\omega t - C_1 \cos 2\omega t)I_{E_0}\right] \quad (2.1)$$

Notes

Geometrically, that means that the measured observable is rotated by $\frac{\pi}{2}$ in the I_{B_0} plane, such that the $C_3 I_S$component becomes orthogonal to plane I_S and remains unchanged. Two other components became orthogonal to I_{B_0} and I_{E_0} and continue rotating in I_S with angular velocity $2\omega t$. The factor $4cos^2\omega[(I_3 I_S)\cdot r]$ defines the dependency of that transformed values through all points of the three-dimensional space.

The hardware creating sprefields may require special implementation as a photonic/laser device that does not exist yet. Instead, we have a very convenient equivalent simulation scheme where the amount of simultaneously available space/time points of observable measured values is only restricted by the overall available Nvidia GPU number of threads.

The case of measuring $C = C_0 + C_1 B_1 + C_2 B_2 + C_3 B_3$by the sprefield is processed by using the following kernel function in the CUDA code:

```
__global__ void quant Kernel(float4* output, int dimx, int dimy, int dimz, float t)
{

floatC1 = 1.0//optional components of the observable
floatC2 = 1.0;
floatC3 = 1.0;
float omega = 12560000.0;                // variant of angular velocity in the sprefield
float tstep = 1.0f;

float factor = 0.0;

    int qidx = threadIdx.x + blockIdx.x * blockDim.x;
int qidy = threadIdx.y + blockIdx.y * blockDim.y;
int qidz = threadIdx.z + blockIdx.z * blockDim.z;

size˙t oidx = qidx + qidy*dimx + qidz*dimx*dimy;

  output[oidx][0] = oidx*tstep;
  factor =4*(cosf(omega * output[oidx][0])) * (cosf(omega * output[oidx][0]));
output[oidx][0] += factor * C3;
output[oidx][1] = oidx*tstep;
  output[oidx][1] += factor * (C1 sin(2 * omega * t) + C2 cos(2 * omega * t));
  output[oidx][2] = oidx*tstep;
  output[oidx][2] += factor * (C2 sin(2 * omega * t) −C1 cos(2 * omega * t));
output[oidx][3] =factor;
}
```


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More flexibility can be achieved by scattering the sprefield wave function before applying it to observables.

Arbitrary Clifford translation $e^{I_{B_C}\gamma} = \cos\gamma + \sin\gamma(\gamma_1 I_S + \gamma_2 I_{B_0} + \gamma_3 I_{E_0})$ acting on spreons (1.6) gives:

$$2\cos\omega[(I_3 I_S)\cdot r]\Big[\frac{1}{\sqrt{2}}(\cos\gamma\cos\omega t - \gamma_1\sin\gamma\sin\omega t - \gamma_2\sin\gamma\cos\omega t - \gamma_3\sin\gamma\sin\omega t) +$$
$$\frac{1}{\sqrt{2}}(\cos\gamma\sin\omega t + \gamma_1\sin\gamma\cos\omega t - \gamma_2\sin\gamma\sin\omega t + \gamma_3\sin\gamma\cos\omega t)I_S +$$
$$\frac{1}{\sqrt{2}}(\cos\gamma\cos\omega t + \gamma_1\sin\gamma\sin\omega t + \gamma_2\sin\gamma\cos\omega t - \gamma_3\sin\gamma\sin\omega t)I_{B_0} +$$
$$\frac{1}{\sqrt{2}}(\cos\gamma\sin\omega t - \gamma_1\sin\gamma\cos\omega t + \gamma_2\sin\gamma\sin\omega t + \gamma_3\sin\gamma\cos\omega t)I_{E_0}\Big] \quad (2.2)$$

This result is defined for all values of t and r, in other words, the effect of Clifford translation instantly spreads through the whole three dimensions for all values of the time.

The instant of time when the Clifford translation was applied makes no difference for the state (2.2) because it is simultaneously redefined for all values of t. The values of measurements $O(C_0, C_1, C_2, C_3, I_S, I_{B_0}, I_{E_0}, \gamma, \gamma_1, \gamma_2, \gamma_3, \omega, t, r)$ also get instantly changed for all values of time of measurement, even if the Clifford translation was applied later than the measurement. That is an obvious demonstration that the suggested theory allows indefinite event casual order. In that way, the very notion of the concept of cause and effect, ordered by time value increasing, disappears.

The result of the measurement in general case is a bit tedious. Let us take as an example the bivector components with $\gamma_2 = 1$, $\gamma_1 = \gamma_3 = 0$.

In that case, the result of measurement of $C_0 + C_1 I_S + C_2 I_{B_0} + C_3 I_{E_0}$ can be calculated as its measurement by $\cos\gamma + \sin\gamma I_{B_0}$:

$$C_0 + C_1 I_S + C_2 I_{B_0} + C_3 I_{E_0} \xrightarrow{\cos\gamma + \sin\gamma I_{B_0}} C_0 + (C_1\cos 2\gamma + C_3\sin 2\gamma)I_S + C_2 I_{B_0} + (C_3\cos 2\gamma - C_1\sin 2\gamma)I_{E_0}$$

followed by measurement by

$$2\cos\omega([(I_3 I_S)\cdot\vec{r}])\left(\frac{1}{\sqrt{2}}\cos\omega t + \frac{1}{\sqrt{2}}I_S\sin\omega t + \frac{1}{\sqrt{2}}I_{B_0}\cos\omega t + \frac{1}{\sqrt{2}}I_{E_0}\sin\omega t\right) \text{ that gives:}$$

$$4(\cos\omega\,[(I_3 I_S)\cdot\vec{r}])^2\big[C_0 + (C_3\cos 2\gamma - C_1\sin 2\gamma)(\cos 2\gamma\, I_S - \sin 2\gamma\, I_{E_0})$$
$$+ \big((C_1\cos 2\gamma + C_3\sin 2\gamma)\sin 2\omega t + C_2\cos 2\omega t\big)I_{B_0}$$
$$+ (C_2\sin 2\omega t - (C_1\cos 2\gamma + C_3\sin 2\gamma)\cos 2\omega t)(\cos 2\gamma\, I_{E_0} + \sin 2\gamma\, I_S)\big]$$

If we take the new orthonormal bivector basis:

$$\{\cos 2\gamma\, I_S - \sin 2\gamma\, I_{E_0}, \sin 2\gamma\, I_S + \cos 2\gamma\, I_{E_0}, \quad I_{B_0}\} \equiv \{I_{1\gamma}, I_{2\gamma}, I_{B_0}\}$$

the result of the measurement reads:

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$$4(cos\,\omega\,[(I_3 I_S)\cdot\vec{r}])^2\big[C_0+(C_3\cos 2\gamma-C_1\sin 2\gamma)I_{1\gamma}$$

$$+\big((C_1\cos 2\gamma+C_3\sin 2\gamma)\sin 2\omega t+C_2\cos 2\omega t\big)I_{B_0}$$

$$+\,(C_2\sin 2\omega t-(C_1\cos 2\gamma+C_3\sin 2\gamma)\cos 2\omega t)I_{2\gamma}\big]$$

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that has constant value in plane $I_{1\gamma}$ plus rotation in planes I_{B_0} and $I_{2\gamma}$ with angular velocity 2ω.

In that way, we can get an$\vec{r}$-dependent variety of constant components of the results of measurements:

$$4(cos\,\omega\,[(I_3 I_S)\cdot\vec{r}])^2\big[C_0+(C_3\cos 2\gamma-C_1\sin 2\gamma)\big(\cos 2\gamma\, I_S-\sin 2\gamma\, I_{E_0}\big)\big]$$

III. Examples of Applications of the GPU Simulating Quantum Computer

a) *Circulation of flow of a fluid*

Assume that fluid in 3D is a field of point-dependent values of vector field $\vec{v}(P)$.Consider a volume element τ containing within it a point P, and denote the bounding surface of τ by σ. Then, the flux of $\vec{v}$ over σper unit volume is

$$\frac{\int \vec{v}\cdot\vec{n}d\sigma}{\tau}$$

where the integral is taken over the surface σ and $\vec{n}$ is the exterior unit normal to σ. Similarly, we can define:

$$curl\,\vec{v}(P)=\lim_{\tau\to 0}\frac{\int \vec{v}\times\vec{n}d\sigma}{\tau}$$

Let C is simple closed curve bounding a plane area A. At a given point P of A construct unit normal $\vec{v}$ so directed that $\vec{v}$ points in the direction of an advancing right-hand screw when C is traversed in the positive sense.

Take τ as a cylinder with base A and a small height parallel to$\vec{v}$. The cylinder surface is σ. The definition of $curl\,\vec{v}(P)$yields:

$$\vec{v}\cdot curl\,\vec{v}(P)=\lim_{\tau\to 0}\frac{\int \vec{v}\cdot\vec{v}\times\vec{n}d\sigma}{\tau}=\lim_{A\to 0}\frac{\int \vec{v}\cdot d\vec{r}}{A}$$

where the second integral is along the curve C, and $d\vec{r}$ is the differential of the position vector on C.

The line integral$\int \vec{v}\cdot d\vec{r}$ is called the circulation of $\vec{v}$ along C.

If $\vec{v}$ represents the velocity of a fluid, then $\vec{v}\cdot d\vec{r}$ takes account of the tangential component of velocity $\vec{v}$ and, a fluid particle moving with this velocity circulates along C. A particle moving with velocity $\vec{v}\cdot\vec{n}$ normal to C, on the other hand, crosses C. That

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is, it flows either into or out of the region bounded by C. Hence $\vec{v} \cdot curl\ \vec{v}(P)$ provides a measure of the circulation per unit area at point P.

Suppose a fluid is given in a three-dimensional region by continuously differential vector field $\vec{v}(P)$, the fluid velocity. At every point P we have a value $curl\ \vec{v}(P)$that can be identified by $C = C_0 + C_1 B_1 + C_2 B_2 + C_3 B_3 \equiv Re^{I_S(t)\varphi(t)}$with geometrically known entities[2]-[5]. The $curl\ \vec{v}(P)$ is the infinitesimal volume density of the net vector circulation, that is magnitude and spatial orientation of the field around the point.

Let us apply the spreon (1.6) to the $curl\ \vec{v}(P)$ at some point P. According to (2.1), the result is:

$$4Rcos^2\omega[(I_3 I_S)\cdot r]\big[C_0 + C_3 I_S + (C_1 \sin 2\omega t + C_2 \cos 2\omega t) I_{B_0} + (C_2 \sin 2\omega t - C_1 \cos 2\omega t) I_{E_0}\big]$$

In the current scheme any number of test observables can be placed into the continuum of the $(t,\ \vec{r})$ dependent values of the spreon state, thus fetching out any amount of values $O\big(C_0, C_1, C_2, C_3, I_S, I_{B_0}, I_{E_0}, \gamma, \gamma_1, \gamma_2, \gamma_3, \omega, t, r\big)$spread over three-dimensions and at all instants of time not generally following cause/effect ordering[8].

While the Schrodinger equation governs infinitesimal transformations of a wave function by Clifford translations, a finite Clifford translation moves a wave function along a big circle of $\mathbb{S}^3$ by any Clifford parameter.

In G_3^+multiplication is:

$$g_1 g_2 = (\alpha_1 + I_{S_1}\beta_1)(\alpha_2 + I_{S_2}\beta_2) = \alpha_1\alpha_2 + I_{S_1}\alpha_2\beta_1 + I_{S_2}\alpha_1\beta_2 + I_{S_1} I_{S_2}\beta_1\beta_2$$

It is not commutative due to the not commutative product of bivectors $I_{S_1} I_{S_2}$. Indeed, taking vectors to which I_{S_1} and I_{S_2} are dual: $s_1 = -I_3 I_{S_1}$, $s_2 = -I_3 I_{S_2}$, we have:

$$I_{S_1} I_{S_2} = -s_1 \cdot s_2 - I_3(s_1 \times s_2)$$

Then:

$$g_1 g_2 = \alpha_1\alpha_2 - (s_1 \cdot s_2)\beta_1\beta_2 + I_{S_1}\alpha_2\beta_1 + I_{S_2}\alpha_1\beta_2 - I_3(s_1 \times s_2)\beta_1\beta_2$$

and

$$g_2 g_1 = \alpha_1\alpha_2 - (s_1 \cdot s_2)\beta_1\beta_2 + I_{S_1}\alpha_2\beta_1 + I_{S_2}\alpha_1\beta_2 + I_3(s_1 \times s_2)\beta_1\beta_2$$

I the case when both elements are of exponent form:

$$e^{I_{S_1}\varphi_1} = \alpha_1 + I_{S_1}\beta_1 = \alpha_1 + \beta_1 b_1^1 B_1 + \beta_1 b_1^2 B_2 + \beta_1 b_1^3 B_3$$

$$e^{I_{S_2}\varphi_2} = \alpha_2 + I_{S_2}\beta_2 = \alpha_2 + \beta_2 b_2^1 B_1 + \beta_2 b_2^2 B_2 + \beta_2 b_2^3 B_3,$$

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Ref

2. A. Soiguine, "Instantaneous Spreading of the g-Qubit Fields," *Journal of Applied Mathematics and Physics*, vol. 7, pp. 591-602, 2019.

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with

$$(\alpha_1)^2 + (\beta_1)^2((b_1^1)^2 + (b_1^2)^2 + (b_1^3)^2) = (\alpha_1)^2 + (\beta_1)^2 = 1$$

$$(\alpha_2)^2 + (\beta_2)^2((b_2^1)^2 + (b_2^2)^2 + (b_2^3)^2) = (\alpha_2)^2 + (\beta_2)^2 = 1,$$

as in the case of a wave function and Clifford translation, we get:

$$e^{I_{S_2}\varphi_2} e^{I_{S_1}\varphi_1} = \cos\varphi_1 \cos\varphi_2 + (s_1 \cdot s_2) \sin\varphi_1 \sin\varphi_2 + I_3 s_2 \cos\varphi_1 \sin\varphi_2 + I_3 s_1 \cos\varphi_2 \sin\varphi_1 - I_3(s_2 \times s_1) \sin\varphi_1 \sin\varphi_2$$

Then it follows that two wave functions are, in any case, connected by the Clifford translation:

$$e^{I_{S_2}\varphi_2} = (e^{I_{S_2}\varphi_2} e^{-I_{S_1}\varphi_1}) e^{I_{S_1}\varphi_1} \equiv Cl(S_2, \varphi_2, S_1, \varphi_1) e^{I_{S_1}\varphi_1},$$

where

$$Cl(S_2, \varphi_2, S_1, \varphi_1) \equiv e^{I_{S_2}\varphi_2} e^{-I_{S_1}\varphi_1} = \cos\varphi_1 \cos\varphi_2 + (s_1 \cdot s_2) \sin\varphi_1 \sin\varphi_2 + I_3 s_2 \cos\varphi_1 \sin\varphi_2 + I_3 s_1 \cos\varphi_2 \sin\varphi_1 + I_3(s_2 \times s_1) \sin\varphi_1 \sin\varphi_2.$$

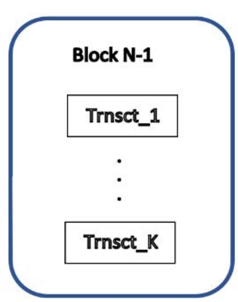
From knowing Clifford translation connecting any two wave functions as points on $\mathbb{S}^3$, *it follows that the* result of measurement of any observable C by wave function $e^{I_{S_1}\varphi_1}$, for example, $e^{-I_{S_1}\varphi_1} C\, e^{I_{S_1}\varphi_1} \equiv C(S_1, \varphi_1)$, immediately gives the result of (not made) measurement by $e^{I_{S_2}\varphi_2}$:

$$e^{-I_{S_2}\varphi_2} C\, e^{I_{S_2}\varphi_2} = e^{-I_{S_2}\varphi_2} e^{I_{S_1}\varphi_1} e^{-I_{S_1}\varphi_1} C e^{I_{S_1}\varphi_1} e^{-I_{S_1}\varphi_1} e^{I_{S_2}\varphi_2} = e^{-I_{S_2}\varphi_2} e^{I_{S_1}\varphi_1}\, C(S_1, \varphi_1) e^{-I_{S_1}\varphi_1} e^{I_{S_2}\varphi_2}$$
$$= Cl(S_2, -\varphi_2, S_1, -\varphi_1) C(S_1, \varphi_1) \overline{Cl(S_2, -\varphi_2, S_1, -\varphi_1)}$$

When assuming observables are also identified by points on $\mathbb{S}^3$ *and thus are connected by formulas as the above one, we get that measurements of any amount of observables by arbitrary set of wave functions are simultaneously available.*

b) Application to block chain schemes

Let us briefly consider the basics of how the block chain scheme generally works in the suggested spreon formalism.



Take a block of transactions stored at some value of time:

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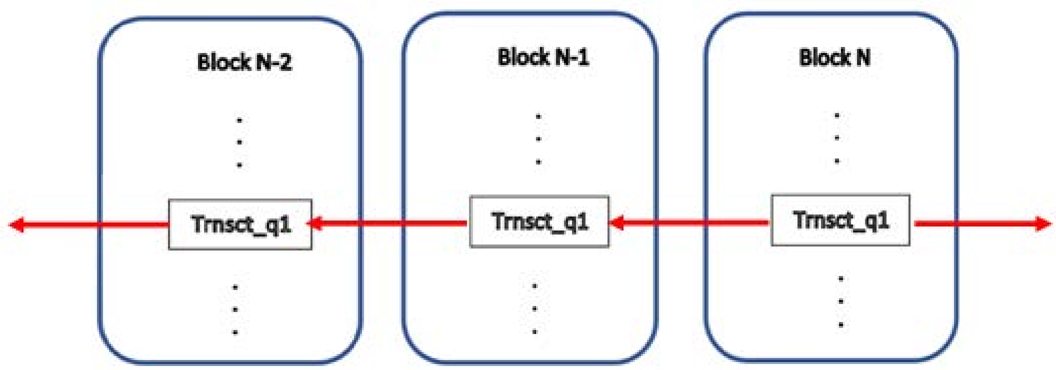
The next block is assumed to include transaction Trnsct˙q1 of the part of the measurement result of the observable $C_0 + C_1 B_1 + C_2 B_2 + C_3 B_3$ by the state, that is Clifford translation by $\cos\gamma + I_{B_0}\sin\gamma$ of the spreon

$$2\cos\omega([(I_3 I_S)\cdot\vec{r}])\left(\frac{1}{\sqrt{2}}\cos\omega t + \frac{1}{\sqrt{2}}I_S\sin\omega t + \frac{1}{\sqrt{2}}I_{B_0}\cos\omega t + \frac{1}{\sqrt{2}}I_{E_0}\sin\omega t\right),$$

namely, the part which exists and is constant for all values of time before the transaction and after the transaction. The transaction is a set of elements of G_3^+. The hypothetical quantum channel should transfer sequences of such elements, comprised of factors $4(\cos\omega\,[(I_3 I_S)\cdot\vec{r_n}])^2$ with predefined values of $\vec{r_n}$ sufficient for enough information about the result of measurement; scalar values C_3 and C_1 of the observable; basis bivectors I_S and I_{E_0}; scalar γ used in creating of a state which acted on the observable, along with the length of wave vector $|k| = \omega$.

Since Trnsct_q1 exists for all values of time it will be automatically included in all previous blocks and in all blocks transacted in the future.



When in future transactions of observable measurement result is transferred, new Trnsct_q2 is placed in all blocks in the same way as above for Trnsct_q1.

IV. Conclusions

In the suggested theory, all measured observable values get available all together, not through looking one by one. In this way, quantum computer appeared to be a kind of analog computer, keeping and instantly processing information by and on sets of objects possessing an infinite number of degrees of freedom. The multithread GPUs with the CUDA language functionality allow to simultaneously calculate observable measurement values at a number of space/time discrete points, forward and backward in time, the number only restricted by the GPU threads capacity. That eliminates the challenging hardware problem of creating vast and stable arrays of qubits, the basis of quantum computing in conventional approaches.

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Exploration of Essential Knowledge in Chinese Senior Middle School Mathematics

By Wang Mingshan, Chen Guanjun, Li Xiuping & Yan Erru

Abstract- Essential knowledge refers to the basic and universal knowledge that students have accumulated in their long-term learning; The principle of exploration is to rely on the curriculum standards for breadth and the textbooks for depth. In China, the vast majority of essential mathematical knowledge is presented in the form of knowledge points in the curriculum standards. The necessary knowledge of marginality, in terms of breadth, should be intersection for the connection between junior middle school and senior middle school. The marked content should be based on the needs of the university and the wide use of high school, and infiltration knowledge should be defined as "entering the dictionary"; In terms of depth, what is clearly provided in the textbook is essential knowledge. For those who appear in other forms of the same textbook, the "Triple degree " are used to define essential knowledge. For those who penetrate the same textbook or have different textbook settings, they are defined in a unioned manner. Generalization and inclusion are the overall methods for determining essential knowledge.

Keywords: essential knowledge, triple degree, entering the dictionary, intersection, unionedmanner.

GJSFR-F Classification: LCC: QA11.2

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Exploration of Essential Knowledge in Chinese Senior Middle School Mathematics

中国高中数学必备知识探究[1]

Wang Mingshan α, Chen Guanjun σ, Li Xiuping ρ & Yan Erru ω

Abstract- Essential knowledge refers to the basic and universal knowledge that students have accumulated in their long-term learning; The principle of exploration is to rely on the curriculum standards for breadth and the textbooks for depth. In China, the vast majority of essential mathematical knowledge is presented in the form of knowledge points in the curriculum standards. The necessary knowledge of marginality, in terms of breadth, should be intersection for the connection between junior middle school and senior middle school. The marked content should be based on the needs of the university and the wide use of high school, and infiltration knowledge should be defined as "entering the dictionary"; In terms of depth, what is clearly provided in the textbook is essential knowledge. For those who appear in other forms of the same textbook, the "Triple degree " are used to define essential knowledge. For those who penetrate the same textbook or have different textbook settings, they are defined in a unioned manner. Generalization and inclusion are the overall methods for determining essential knowledge.

Keywords: *essential knowledge, triple degree, entering the dictionary, intersection, unionedmanner.*

摘要: **必备知识是指学生长期学习的知识储备中的基础性、通用性知识；探究原则是广度依课标，深度靠教材。在中国，绝大多数数学必备知识，在课标中以知识点形式呈现。边缘性的必备知识，广度上，初高中衔接内容取交，打*号内容以大学之需和高中用广推定，渗透类知识以“入典”来界定；深度上，教材中明确给出的，属于必备知识，同一教材其他形式出现者，以“三定度”来界定必备知识，同一教材渗透者或不同教材设置者，以并的方式界定。通用、入典是判断必备知识的总方法。**

关键词: 必备知识,三定度,入典,交,并

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[1]本文是江苏省基础教育前瞻性教学改革试验项目“基于发展数学核心素养的高中数学写作实践研究”（项目编号：2020JSQZ0147）阶段成果延伸课题之一.

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如今的高考数学命题,已推进到“素养立意”，而且从顶层设计上，概括为“核心价值、学科素养、关键能力、必备知识”[1]；这当中的前三者，已经具体到学科层面给出了架构[2]，唯有必备知识，除了理论上给出了一个总的定义之外，还没有在学科层面给出一个具体清晰的界定。操作层面，诸多口号和理念，最初起点和最终落点不外乎是必备知识和学科思维；具体到数学上，作为表达的数学写作离不开规范，规范说到底也是必备知识的识别问题；同时作为高中最为关注的高考，在试题命制加入排查二级结论环节和试题分析报告的宗旨变为“教学引领”的大背景下，哪些二级结论可以计入必备知识必将备受关注。因此，很有必要对必备知识进行一些必要的探究。

I. 必备知识的内含

a) *必备知识的含义*

必备知识的含义，在我国可追溯到“可以满足一般从事专业技术人员要求的知识”[3]，在此意义下，陆续出现了各行业的必备知识含义；高中教育方面的必备知识是指“即将进入高等学校的学习者在面对与学科相关的生活实践或学习探索问题情境时，高质量地认识问题、分析问题、解决问题所必须具备的知识。是由人文社会科学和自然科学各学科的基本事实、基本概念、基本技术与基本原理组成的基本知识体系”[4]，通俗地说，指的是：学生长期学习的知识储备中的基础性、通用性知识[5]。

再具体到数学上，必备知识由概念、公式、命题、表示四大系统组成。概念由核心概念及一层层导出概念构成[6]，公式由原始公式与法则及导出公式构成，命题由基本事实(公理及原理)、定理、推论组成，表示则是在口语表达的基础上，由文字语言、图形语言和符号语言所组成。

b) *必备知识的探究原则*

高中数学必备知识，主要根据《普通高中数学课程标准》(以下简称课标)和普通高中数学教材来界定。具体的，一般从广度与深度两个方向展开：所谓知识的广度，是指与该知识并列的所有知识的数量；深度是指该知识的种类关系或范畴层次；广度对课标的依赖程度较大，而深度又总是以教材为基础。正因如此，数学必备知识的探究，广度依课标，深度靠教材，作为探究的原则。

这里，我们也仅就课标中所列的必修和选择性必修内容进行必要的探究。

II. 高中数学必备知识的广度探究

如前所述，课标中已经给出了绝大多数数学必备知识，而且以各知识点的形式来呈现；但这里存在的主要问题是，一些边缘性知识，究竟算不算必备知识？具体表现在：一是初、高中衔接内容的知识，二是课标中给出的打*号的知识。

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a) 初、高中衔接的知识

初、高中衔接的知识，是否属于必备知识，主要根据通用性原则来判断。这里的通用性，指的是高中必备，初中涉猎，具体可根据诸多观点中取公共者，亦即取交，来作为必备知识。

由于课标分别在2020年有所修订或调整，所以通过对2020年6月之后，通过对衔接教材知识点统计[7]、具体实施的师生反馈建议[8]，总结得出，当下初、高中数学衔接的必备知识有：

i. *代数部分：*立方和公式和立方差公式，分解因式；多项式的除法(建议用竖式计算、等式表达)。

ii. *几何部分：*轨迹含义(建议重点放在由静到动的思维感知上)。

*b) 课标中打*号的内容*

课标中打*号的内容，界定为选学内容，不作为高考要求，但不能因为高考不考便一定不教、不学，[1]中还特别将这种“考什么、教什么、学什么”强调为弊端；李尚志先生对此通俗地说：“中学教材，凡是打星号的，都比不打星号的更重要，更有用”“星号就是用来分流的”那么这些内容究竟算不算必备知识？这一直是高中争论颇丰的问题。

既然高中是为大学培养新生服务的，所以要本着大学之需的原则展开。具体仍然是对大学普遍认为需补[9]和高中打*号但用途较广[10]的这些内容取交来展开。

这些内容有：反函数、复数的三角形式、数学归纳法、圆与椭圆的参数方程，它们可以计入必备知识。

III. 高中数学必备知识的深度探究

知识深度，问题的核心是“哪些结论可以直接应用”？外显是用这些结论解决问题的型式与步骤长短，其中与必备知识直接联系的是可直接应用的结论。
教材中明确给出的概念、基本事实、定理、公式，可以直接应用，当然作为必备知识。

这里还有两个突出问题：一是同一教材，没有以必备知识的形式出现，诸如在例题、练习题或习题中出现，怎么办？二是在“一纲多本”的政策之下，总有一些结论，可能在这个教材中以必备知识的形式出现，在另外教材中，则不以必备知识的形式出现，教材设置有些许差异，这些究竟是否计入必备知识？

a) 同一教材的必备知识界定

i. 一般性界定

对于同一教材知识出现的形式问题，一个可行处理原则是：将所有的数学结论公理层化，梳理成原理层(不加证明就承认正确并应用的结论)→定理层→一级推论层→二级推论层→三级推论层→……，如表1：

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表1： 必备知识公理化梳理一览表

类别	原理层→定理层→一级推论层→二级推论层→……
概念	核心概念→导出概念→次导出概念→再次导出概念→⋯
公式	原式公式→导出公式→次导出公式→再次导出公式→⋯
公理体系	基本事实(公理和原理)→定理→一级推论→二级推论→三级推论→⋯
表达	口语→文字语言→符号语言或图形语言

Notes

*如：*针对高考，命题以原理层为起点，可直接应用的结论到一级推论；针对希望杯数学竞赛一试，命题以定理层为起点，二级推论层之前的结论都可直接应用；同理，针对省级“奥赛”，命题以一级推论层为起点，可以直接应用的结论到三级推论层；⋯⋯以此类推，可以看出，各种从命题到可直接应用的结论，都是经过了三层，故称
“三定度”[12]。

这一方法，可以界定同一教材中绝大多数的必备知识。

ii. *个别渗透类的知识*

个别知识，在教材中没有明确给定，但渗透其中。渗透的方式，有的是前有示范后沿袭再用，有的是在旁以批注的方式呈现，还有的以阅读或思考的形式存在，其中写入普及型典籍(再版达十次之上的工具书)[13][14][15]中的，说明应用较为广泛，具体到高中数学解题中往往作为模型加以应用，可以当作必备知识加以处理。这些知识有：

1) 集合元素个数的容斥原理。
2) 否定一个命题，坚持“任意与存在”、“并且与或者”、“是与非”三点互换。
3) ⇔表达术语“充要条件”“当且仅当”“必要且只要”“等价”“等价于”。
4) 归纳、类比的含义。
5) 函数 $y=f(x)$ 是偶函数，则 $f(x)=f(-x)=f(|x|)$； $y=f(x)$

   是奇函数，且在原点有定义，则 $f(0)$

   =0；若某一区间上的奇函数存在最大值和最小值，则最大值和最小值的和为零。
6) 方程的重根计一个零点。
7) 函数的初等变换结论，见图2

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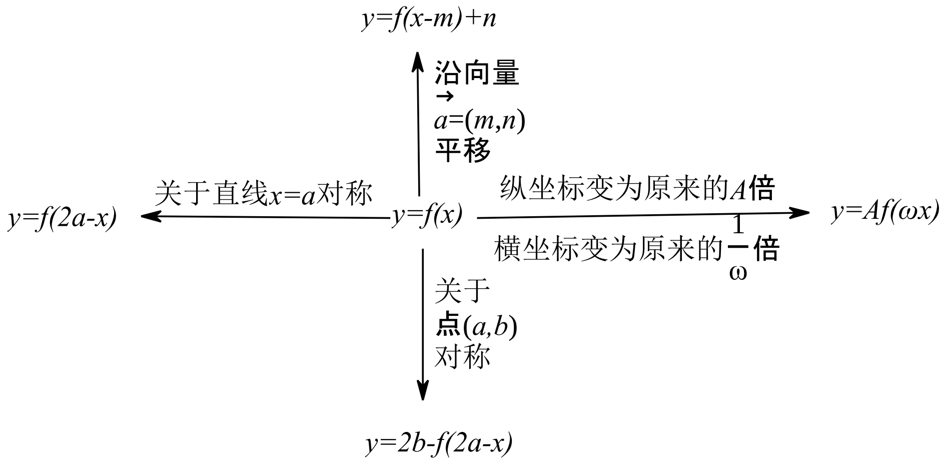

图2: **函数初等变换结论图**

8) 对钩函数 $y=x+\dfrac{k^2}{x}$ 的图象。

9) 两个函数复合函数 “同增异减”单调性规律。

10) 可导函数 $y=f(x)$ 在 $x=x_0$ 处的切线方程是: $y=f'(x_0)(x-x_0)+f(x_0)$

11) 数列$\{a_n\}$是等差数列， $m+n=s+t$， 则 $a_m+a_n=a_s+a_t$； 数列$\{b_n\}$是等比数列，

$$m+n=s+t\text{，则 } b_m b_n=b_s b_t\text{。}$$

12) 等差数列$\{a_n\}$的前 n 项和 S_n 与 n 的二次式关系：$-a_n$ ″是公差为 d 的等差数列，当且仅当 S_n 能写成 $\dfrac{d}{2}n^2+bn$ 形式。

13) $1^2+2^2+3^2+...+n^2=\dfrac{1}{6}n(n+1)(2n+1)$， $1^3+2^3+3^3+...+n^3=\left[\dfrac{n(n+1)}{2}\right]^2$

14) $\sum_{k=0}^{n} C_n^k=2^n$

15) 函数 $y=f(x)$ 是周期为T的函数，则 $\dfrac{T}{|\omega|}$ 是函数 $y=f(\omega x+\phi)$ 的一个周期。

16) 已知△ ABC 的三边分别为 a,b,c， 则其面积为 $S=\sqrt{p(p-a)(p-b)(p-c)}$， 其中 p 为半周长 $\dfrac{a+b+c}{2}$

17) 点 A,B,M 的坐标分别为 $(x_1,y_1),(x_2,y_2),(x_0,y_0)$，$O$为平面内任意一点，则

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$$\overrightarrow{OM}=\frac{1}{2}(\overrightarrow{OA}+\overrightarrow{OB})\Leftrightarrow M\text{ 为 }AB\text{ 中点}\Leftrightarrow\begin{cases}x_0=\dfrac{x_1+x_2}{2}\\ y_0=\dfrac{y_1+y_2}{2}\end{cases}$$

18) 圆锥曲线的统一定义(到定点距离与到定直线距离的比为常数的点的轨迹)。

19) 相关点法求轨迹方程：一般的，一个点随另一个点的变动而变动，这两个点互称对方的相关点。根据互为相关点间的坐标可以互相表示，用之来求曲线方程的方法，称相关点法，其要点是：求谁设谁表相关，代入化简一气连，注意条件勿增减。如：曲线 $f(x,y)=0$ 关于直线 $y=b$、$y=x+b$、$y=-x+b$ 对称的曲线方程分别为 $f(x,2b-y)=0$、$f(y-b,x+b)=0$、$f(b-y,b-x)=0$。

20) 平面直角坐标系内，方程 $f_1(x,y)+\lambda f_2(x,y)=0$ 对应的曲线过 $f_1(x,y)=0$ 和 $f_2(x,y)=0$ 对应曲线的交点；由此延伸出点差法或称差分法的运算技巧。

b) 教材设置差异问题

关于教材设置的些许差异，首先需要肯定的是，教材编写者，对课标是有一定深度研读的，而且也有相当的教学经验，在这种意义下，其中的些许差异是一种正常的现象；其次，对于其中的必备知识，本着“并”“包”的原则去认识和界定；再次，需要将这种原则明确、可操作化。

这里的“并”，指的是各大教材中，只要有一个以必备知识形式出现，就认定为必备知识,只有都不以必备知识形式出现，才不界定为必备知识；“包”指的是，各大教材中，都有设置，但设置不完全相同，需要将其进行必要地整合。

目前国内在用的高中数学教材，有人教A版、人教B版、沪教版、北师大版、苏教版、湘教版、鄂教版，七大教材。这样，这些设置差异的知识，主要体现为三类：需要增设的定义、公式、表示，此其一；需要增设的结论，内含需要整合的内容，此其二；蕴含的思想方法，此其三。

i. *集合、简易逻辑与不等式*

1) 集合的表示法有列举法、描述法、图示法和符号简记法。

2) 推不出 $\not\Rightarrow$ 符号；逆命题、猜想、判定、性质、充分不必要条件、必要不充分条件、既不充分又不必要条件的含义；综合法、分析法、反证法。

3) 不等式性质的整合：

性质1：$a>b\Leftrightarrow b<a$

性质2：$a>b,b>c\Rightarrow a>c$，可连记为 $a>b>c$

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性质3： $a > b \Rightarrow a + c > b + c$

推论1： $a + b > c \Rightarrow a > c - b$

推论2： $\left.\begin{matrix} a > b \\ c > d \end{matrix}\right\} \Rightarrow a + c > b + d$

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性质4： $\left.\begin{matrix} a > b \\ c > 0 \end{matrix}\right\} \Rightarrow ac > bc$， $\left.\begin{matrix} a > b \\ c < 0 \end{matrix}\right\} \Rightarrow ac < bc$

推论1： $\left.\begin{matrix} a > b \geq 0 \\ c > d \geq 0 \end{matrix}\right\} \Rightarrow ac > bd$

推论2： $a > b \geq 0 \Rightarrow a^n > b^n (n \in N, n \geq 2)$

推论3： $a > b \geq 0 \Rightarrow \sqrt[n]{a} > \sqrt[n]{b} (n \in N, n \geq 2)$

推论4： $a > b \underset{b>0}{\Leftrightarrow} \frac{a}{b} > 1$(作商比较得依据)

推论5： $\left.\begin{matrix} a > b \\ ab > 0 \end{matrix}\right\} \Rightarrow \frac{1}{a} < \frac{1}{b}$(倒数比较得依据)

性质5： $|a + b| \leq |a| + |b|$，等号成立当且仅当 $ab \geq 0$

4) 基本不等式为： $a, b \in R$， $a^2 + b^2 \geq 2ab$，等号成立当且仅当 $a = b$； a, b 为非负实数，则 $\frac{a+b}{2} \geq \sqrt{ab}$，等号成立当且仅当 $a = b$

ii. *函数和导数*

1) 含有绝对值的不等式的解法——零点分段法。
2) 建立函数模型的方法有分步法、图表法、归纳法、特值推广法(操作要诀是：分步图表归纳，特值推广简化；四步设列解答，范围注意不差)。
3) 反函数的概念、表达、求法以及与原函数图象的关系。
4) $\log_a 1 = 0$， $\log_a a = 1$。
5) 导数运算法则的证明。
6) 可导函数 $f(x)$，在区间 I 上， $f'(x) \geqslant (\leqslant) 0$，等号成立当且仅当 x 取区间 I 上可列个值，则 $f(x)$ 在区间 I 上单调递增(减)；反之也成立。

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iii. *数列、三角*

1) 已知数列$\{a_n\}$前n项和为S_n，则$a_n=\begin{cases}S_1, n=1\\ S_n, n\ge 2\end{cases}$

2) 对任意正整数n，$a_{n+1}>a_n$称数列$\{a_n\}$单调递增，$a_{n+1}<a_n$称数列$\{a_n\}$单调递减。

3) 等差数列定义式为$a_{n+1}-a_n=d$；等比数列定义式为$\frac{b_{n+1}}{b_n}=q$

4) 数列$\{a_n\}$等差，当且仅当a_n可写成$dn+b$形式；数列$\{b_n\}$等比，当且仅当b_n可写成bq^n $(bq\neq 0)$形式。

5) 在弧度制下，扇形面积公式$S=\frac{1}{2}lr=\frac{1}{2}\alpha r^2$。

6) $\frac{k}{2}\pi\pm\alpha$ $(k\in \mathrm{Z})$三角诱导公式要诀为“奇变偶不变，符号看象限”。

7) $y=\sin x$、$y=\cos x$、$y=\tan x$的对称中心分别为$(k\pi, 0)$、$(\frac{\pi}{2}+k\pi, 0)$、$(\frac{k}{2}\pi, 0)$，其中$k\in Z$，正弦、余弦的对称中心也是其相应的零点，正余弦函数在对称轴处函数值取最值。

8) 三角恒等变换中，$A\sin x+B\cos x$化为$\sqrt{A^2+B^2}\sin(x+\phi)$的辅助角方法。

9) 正弦定理$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$(其中$R$为$\triangle ABC$外接圆半径)；若$\triangle ABC$的内角$A$、$B$、$C$所对的边分别为$a$、$b$、$c$，则此三角形的面积$S=\frac{1}{2}ab\sin C=\frac{1}{2}bc\sin A=\frac{1}{2}ca\sin B$

10) $y=A\sin(\omega x+\varphi)$中各参数的物理意义。

iv. *解析几何*

1) $\vec{a}$方向上的单位向量是$\frac{\vec{a}}{|\vec{a}|}$，与$\vec{a}$共线的单位是$\pm\frac{\vec{a}}{|\vec{a}|}$

2) 直线的方向向量及法向量的含义。

3) 复数共轭的性质：$\overline{\overline{z}}=z$，$\overline{z_1\pm z_2}=\overline{z_1}\pm\overline{z_2}$，$\overline{z_1 z_2}=\overline{z_1}\,\overline{z_2}$，$\overline{\left(\frac{z_1}{z_2}\right)}=\frac{\overline{z_1}}{\overline{z_2}}$

4) 复数模的性质：$|z|=|\overline{z}|$，$z\overline{z}=|z|^2$，$|z_1z_2|=|z_1||z_2|$，$\left|\frac{z_1}{z_2}\right|=\frac{|z_1|}{|z_2|}$

5) 实系数一元二次方程$ax^2+bx+c=0$在$\triangle=b^2\text{-}4ac<0$情况下两个解为$x=\frac{-b\pm\sqrt{-\Delta}i}{2a}$

6) 直线的点法式方程。

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7) 方程法判断平面内两直线的位置关系以及垂直结论。

8) 曲线与方程的含义以及直译法(简记为：建、设、显、代、化)求法。

9) 圆锥曲线的直观含义：与圆锥轴成不同角的平面截圆锥面得到的曲线

10) 直线截曲线的弦长公式：$|P_1P_2|=\sqrt{1+k^2}\,|x_2-x_1|=\sqrt{1+\dfrac{1}{k^2}}\,|y_2-y_1|$

11) 双曲线及其渐近线方程统一性：方程 $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=\lambda$

在λ≠0时表示双曲线方程，在λ＝0时表示其渐近线方程。

v. *空间几何*

1) 异面直线判定定理：平面内一点与平面外一点的连线，与平面内不过这个点的直线是异面直线。

2) 等角定理的重新整合：一个角的两边和另一个角的两边分别平行，且方向相同，那么这两个角相等

*推论1：*一个角的两边和另一个角的两边分别平行，那么这两个角相等或互补。

*推论2：*一个角的两边和另一个角的两边所在直线对应平行，那么它们各自所成的锐角或直角相等。

3) 面面平行的性质定理

*性质1：*两个平面相互平行，一个平面内的任意一条直线平行于另一平面。

*性质2：*一个平面被两个平行平面所截，交线平行。

4) 空间存在唯一性定理整合：

① 过已知直线外一点，有且仅有一条直线与已知直线平行。

② 过一点有且仅有一条直线垂直于已知平面。

③ 过已知平面外一点，有且仅有一个平面与已知平面平行。

5) 三垂线定理及其逆定理。

6) 法向量法求二面角平面角“同等异补”整合：平面α、β的法向量分别为$\vec{m}$、$\vec{n}$，在两个半平面α、β内分别取不在棱l上的一点A、B，如果$\vec{m}\cdot\overrightarrow{AB}$与$\vec{n}\cdot\overrightarrow{AB}$同号，则$\left(\vec{m},\vec{n}\right)$为二面角α-$l$-β的平面角；否则$\vec{m}\cdot\overrightarrow{AB}$与$\vec{n}\cdot\overrightarrow{AB}$异号，则$\left(\vec{m},\vec{n}\right)$为二面角α-$l$-β的平面角的补角[16]。

vi. *统计和概率*

1) 和号Σ的规定及性质：$\sum_{k=1}^{n}a_k=a_1+a_2+a_3+\ldots+a_n$，$\sum_{k=1}^{n}(a_k+b_k)=\sum_{k=1}^{n}a_k+\sum_{k=1}^{n}b_k$，

$$\sum_{k=1}^{n}\lambda a_k=\lambda\sum_{k=1}^{n}a_k$$

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2）均值方差的线性性质：若 $y_k = ax_k + b$ (k=1,2,3,⋯,n)，则 $\overline{y} = a\overline{x} + b$， $S_y{}^2 = a^2 S_x{}^2$，

$\sigma_y = |a| \sigma_x$；类似地，若 $\eta = a\xi + b$，则E(η)= $aE(\xi) + b$,D(η)= $a^2 D(\xi)$

3）重要分布的特征值：ξ ～B(n,p)，则E(ξ)=np,D(ξ)=np(1-p)； η ～H(N,M,n),则E(η)= $n\dfrac{M}{N}$，D(η)= $n \cdot \dfrac{M}{N} \cdot \dfrac{N-M}{N} \cdot \dfrac{N-n}{N-1}$

4）排列、组合应用题基本模式有：直接、优限、并元、插孔、选排。

诚然，必备知识本身是随着课标、教材而变化的，所以，我们这里探究和罗列的必备知识，对应具有相对稳定性；如果将蕴含其中的方法和思想也纳入必备知识的范畴，它也是在不断拓展当中，但总体上，可以以通用、入典作为判断是不是必备知识的总方法。

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ANOBJECTORIENTEDFINITEELEMENTPROGRAMMINGAPPROACHTOANALYZESTRUCTURALDISCONTINUITIESOFABRACKETPLATESECTION

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Kazi Naimul Hoque [α] & Md. Shahidul Islam [α]

Abstract- Structural discontinuities, arising from abrupt changes in the cross-sectional shape of structural sections, pose challenges in various scenarios within structural sections. Analyzing stress distributions in these situations often necessitates resource-intensive and time-consuming methods. This paper presents an object-oriented programming (OOP) approach for analyzing two-dimensional plane stress problems, focusing on a bracket plate section. The developed program accurately estimates stress values at four integration points and offers significant advantages in terms of efficiency and accuracy. Validation of the program's results against commercial Finite Element Analysis (FEA) software demonstrates minimal error percentages, attesting to its precision. Additionally, a mesh viewer program enhances visualization capabilities, accommodating both quadrilateral and triangular meshes. This study employs OOP techniques to address structural discontinuity challenges, contributing to a deeper understanding of structural issues and enabling more effective analysis and solutions.

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Nomenclature

Q	global displacement vector	σ_Y	yield stress
F	global load vector	σ_x	normal stress along x-direction
q	element displacement vector	σ_y	normal stress along y-direction
ξ-, η	natural coordinates	τ_{xy}	shear stress
D	stress strain displacement matrix	σ_{misses}	von-Mises stress
B	element strain displacement matrix	U_X	displacement along x-direction
K	element stiffness matrix	U_y	displacement along y-direction
E	modulus of elasticity	υ	Poisson's ratio
Π	potential energy	σ	stress

I. Introduction

A structural discontinuity emerges whenever there is a sudden change in the cross-sectional shape of a structural section. This can occur in various scenarios, such as

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when there are openings in decks, hatches, and machinery, access points on sidewalls, cutouts in web plates, or holes in crankshafts, among other instances. These irregularities in the structure can alter how stress is distributed within it, leading to the introduction of stresses around the edges of holes or at the ends of cracks. Professionals from various fields, including hydrodynamic experts, mathematicians, offshore and ocean engineers, as well as naval architects, have conducted extensive research to understand the consequences of these structural discontinuities. Typically, assessing stress in these situations involves either analytical or experimental methods, which can be resource-intensive and time-consuming, especially when dealing with complex configurations of structural discontinuities. The intricate nature of these discontinuity configurations often makes it extremely challenging to find a suitable solution.

[3] Bea et al. (Bea et al., 1995) embarked on an extensive exploration into the significance of fatigue cracks and corrosion in tankers. [2] Baumann (Baumann, 1997), employing finite element modeling, scrutinized the structural integrity of a hull girder penetration and a short longitudinal bulkhead. It's worth noting that research addressing the influence of corrosion and fatigue cracking on ship hull structural integrity was relatively scarce, as highlighted by [19] Soares and Garbatov (Soares and Garbatov, 1998). [15] Niu (Niu, 1998) conducted an exhaustive examination of stress analysis within the context of an airframe structure. Given the substantial uncertainties associated with these failure modes, modeling the effects of corrosion and fatigue cracking on ship hull structural integrity presents a formidable challenge.

The investigation delved into the applicability of OOP techniques to enhance the efficient and comprehensible organization of data in numerical analysis codes. [1] Baugh and Rehak (Baugh and Rehak, 1990)addressed the excessively sequential and excessively restrictive nature of traditional algorithm implementation, also introducing a more flexible data flow structure. Early efforts to devise finite element software architecture were presented by [8] Forde et al., [21] Zimmermann et al., and [5] Dubois-Pelerin et al. (Forde et al., 1990; Zimmermann et al., 1992; Dubois-Pelerin et al., 1992), while [7] Fenves, [14] Miller, [4] Desjardins, and Fafard contributed to further advancements (Fenves, 1990; Miller, 1991; Desjardins and Fafard, 1992). The work of [12] Mackerle (Mackerle, 2000) brought object-oriented principles to the forefront of finite element programming, gaining considerable attention. [13] Martha (Martha, 2002) conducted an in-depth exploration of the object-oriented framework for finite element programming, demonstrating that fundamental concepts applied to the finite element method (FEM) yielded a highly modular, comprehensible, and extensible codebase. [9] [11] Hoque (Hoque, 2016) conducted a comprehensive study on the two-dimensional plane stress and plane strain analysis of structural discontinuities associated with ship structures. Multiple research studies [17] (Presuel-Moreno et al., 2022, 2018; [18] Presuel-Moreno and Hoque, 2019; [10] Hoque, 2020) investigated crack and corrosion propagation in marine structures constructed with marine materials. [20] Thohura and Islam (Thohura and Islam, 2013) employed the finite element method (FEM) to explore the impact of mesh quality on the stress concentration factor for plates with holes. [6] Edholm (Edholm, 2013), utilizing the Python programming language, executed finite element analysis to visualize quadrilateral and triangular mesh structures.

In this study, an OOP approach was employed. When tackling the development of extensive and intricate software systems, such as Finite Element Models (FEMs) designed to handle a multitude of element types, constitutive models, and analysis algorithms, OOP proves exceptionally advantageous. In this study, four integration

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1. Baugh, J.W., and Rehak, D.R. (1990): Computational abstractions for finite element programming, Technical Report R-89-182, Department of Civil Engineering, Carnegie Mellon University, Pittsburgh, USA.

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points, commonly referred to as gauss points, were employed to delve into stress aspects, including normal stress, shear stress, and von-Mises stress, specifically within a bracket section associated with structural components.

To investigate structural discontinuity issues, an OOP technique was employed to develop a finite element program for four-node quadrilateral elements. To validate this study's findings concerning stress distribution at gauss points for bracket section, a comparison was conducted with the results obtained from the commercial FEA software, referred to as SIMULIA ABAQUS.

Notes

II. Basic Formulation

The FEM stands as the predominant technique for discretizing problems in structural mechanics. Its fundamental premise revolves around dividing a region into distinct, non-overlapping components of simple geometric shapes known as finite elements, often abbreviated as elements. In the context of two-dimensional modeling, the most used elements are linear or quadratic triangles and quadrilaterals.

In two-dimensional scenarios, each node possesses the freedom to displace in two directions, resulting in two degrees of freedom for each node. Consequently, the displacement components of node "j" are designated as Q_{2j-1} for displacement in the x-direction and Q_{2j} for displacement in the y-direction. This configuration allows to represent the global displacement vector as follows:

$$\{Q\} = [Q_1, Q_2, \ldots\ldots\ldots\ldots, Q_N]^T \tag{1}$$

and global load vector,

$$\{F\} = [F_1, F_2, \ldots\ldots\ldots\ldots, F_N]^T \tag{2}$$

Here, N signifies the number of degrees of freedom (d.o.f.), a measure representing the nodes' ability to move in specified directions. In the context of a two-dimensional problem, where nodes are allowed to move both in the $\pm x$ and $\pm y$ directions, this leads to the assignment of two degrees of freedom to each individual node.

A general quadrilateral element is considered as shown in Figure 1(a), which has local nodes numbered as 1, 2, 3 and 4 in a counterclockwise fashion and (x_i, y_i) are the coordinates of node i. The vector $\{q\} = [q_1, q_2, \ldots\ldots q_8]^T$ represents the element displacement vector. To develop the shape functions, a master element is considered (Figure 1(b)) having a square shape and being defined in ξ-, η- coordinates (or natural coordinates). The Lagrange shape functions, where i = 1, 2, 3 and 4, are defined such that N_i is equal to unity at node i and is zero at other nodes. In particular:

$$N_1 = \begin{cases} 1, & at\ node\ 1 \\ 0, & at\ node\ 2,\ 3\ and\ 4 \end{cases} \tag{3}$$

The requirement that $N_1 = 0$ at nodes 2, 3 and 4 is equivalent to requiring that $N_1 = 0$ along edges $\xi = +1$ and $\eta = +1$ (Figure 1(b)).

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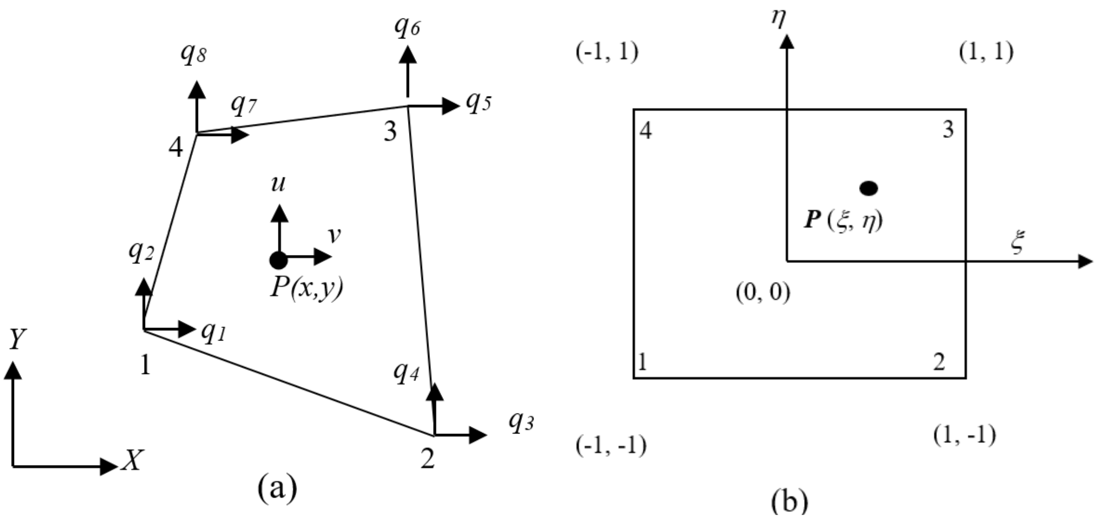

Figure 1: a) Four-node quadrilateral element; b) The quadrilateral element in ξ, η space (master element)

All the four shape functions can be expressed as follows:

$$N_1 = \frac{1}{4}(1-\xi)(1-\eta) \qquad N_2 = \frac{1}{4}(1+\xi)(1-\eta)$$

$$N_3 = \frac{1}{4}(1+\xi)(1+\eta) \qquad N_4 = \frac{1}{4}(1-\xi)(1+\eta) \tag{4}$$

When addressing the appropriate boundary conditions and formulating the equilibrium equations, the penalty approach was employed. As a result, the equilibrium equations can be derived by minimizing the potential energy Π with respect to Q, while subjecting to the boundary conditions. These boundary conditions typically take the form of:

$$Q_{p1} = a_1, Q_{p2} = a_2, \ldots\ldots\ldots, Q_{pr} = a_r \tag{5}$$

where, P_1, P_2, ……, P_r are denoted to be the degrees of freedom and r is judged to be the number of supports in the structure. The requirement that the potential energy Π takes on a minimum value is obtained from the equation.

$$\frac{d\Pi}{dQ_i} = 0 i = 2,3,\ldots\ldots,N \tag{6}$$

By applying these specified boundary conditions and utilizing Equation (6), the finite element equations can be presented in matrix form as follows:

$$[K]\{Q\} = \{F\} \tag{7}$$

where, $[K]$ and $\{F\}$ are the modified stiffness and load matrices. For matrix solving, conjugate gradient method was used.

Equation (7) can be resolved for the displacement vector $\{Q\}$ through Gaussian elimination. Given that the $[K]$ matrix is nonsingular, the boundary condition can be

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considered of specified properly. Once $\{Q\}$ has been determined, the element stress can be evaluated using the equation derived from Hooke's law,

$$\{\sigma\} = [E][B]\{q\} \tag{8}$$

where $[B]$ is the element strain-displacement matrix and $\{q\}$ is the element displacement vector for each element, which is extracted from $\{Q\}$, using element connectivity information.

Notes

The von-Mises stress offers a unified depiction of various stresses, encompassing normal stress in two directions and the corresponding shear stress, present at a particular point in case of a two-dimensional problem. When the von-Mises stress exceeds the material's yield strength, it results in yielding at that location. Likewise, if the von-Mises stress surpasses the ultimate strength, it leads to material rupture at that point. According to the failure criterion, the von-Mises stress (σ_{mises}) must be less than the material's yield stress (σ_Y). This criterion can be expressed in an inequality form as follows:

$$\sigma_{mises} \leq \sigma_Y$$

The von-Mises stress σ_{mises} is given by,

$$\sigma_{mises} = \sqrt{(\sigma_x + \sigma_y)^2 - 3(\sigma_x\sigma_y - \tau_{xy}^2)} \tag{9}$$

III. Mesh Generation

An integral phase of the finite element method in numerical computation entails the process of mesh generation. Mesh generation involves creating a polygonal or polyhedral mesh that closely approximates a given geometric domain. In this procedure, the task is to take a domain, which can be a polygon or polyhedron (in more intricate scenarios, even domains with curved boundaries), and partition it into simple "elements" that interconnect in clearly defined manners. While striving for a minimal number of elements, certain regions within the domain may require smaller elements to enhance computation accuracy. Additionally, all these elements should possess characteristics that render them "well shaped," though the precise definition of this term may vary depending on the specific context and often relates to constraints on element angles or aspect ratios. Figure 2 provides some illustrative examples of mesh generation.

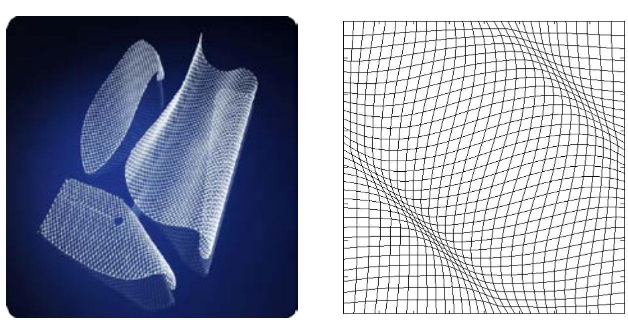

Figure 2: Some examples of mesh generation

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a) Structured mesh

A structured mesh, as depicted in Figure 3, is characterized by a uniform distribution of adjacent elements around all interior nodes. Typically, such meshes are produced using structured grid generators, resulting in meshes composed entirely of quadrilateral or hexahedral elements. The algorithms employed in this process often incorporate intricate iterative smoothing methods, aimed at aligning elements with domain boundaries or physical structures. In cases where complex boundaries are involved, "block structured" techniques can be employed, allowing users to divide the domain into topological blocks. Structured grid generators find prominent application in the field of Computational Fluid Dynamics (CFD), where precise element alignment is often a requirement imposed by the analysis code or essential for capturing specific physical phenomena.

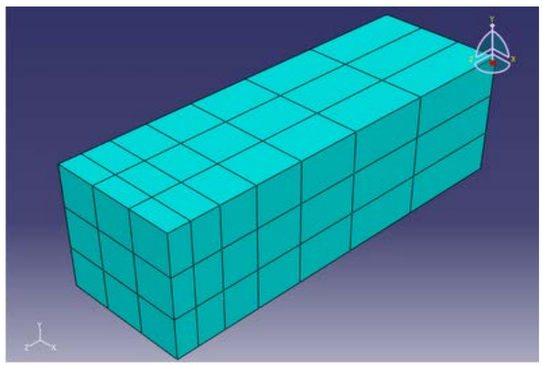

Figure 3: Structured mesh

b) Mesh generation techniques

In this section, techniques for generating meshes are considered, using triangular elements (with three nodes) and quadrilateral elements (with four nodes), specifically focusing on two-dimensional linear elements. While the analysis primarily employs a quadrilateral mesh, it's worth noting that triangles can be further subdivided into quadrilaterals, as illustrated in Figure 4. Consequently, understanding the generation of triangular elements holds significance and relevance, as it offers the possibility of transforming a triangular mesh into a quadrilateral one.

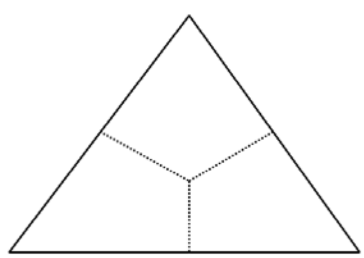

Figure 4: A triangular element is converted into three quadrilateral elements

IV. Object-Oriented Concepts

In recent years, there has been a growing interest in incorporating object-oriented concepts into finite element programming. One of the primary benefits of embracing an object-oriented approach is the simplicity and naturalness with which programs can be expanded, with minimal impact on existing code. This greatly enhances the reusability of code. Additionally, in comparison to traditional structured

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programming, the utilization of OOP facilitates a closer alignment between theoretical principles and their computer implementation. OOP proves especially advantageous when developing extensive and intricate programs, such as finite element systems designed to accommodate diverse element types, constitutive models, and analysis algorithms.

Nevertheless, to fully leverage the advantages of the object-oriented approach, it is imperative to have a comprehensive understanding of the methodology employed and to invest substantial effort in organizing the program effectively. The objective of this research is to introduce an ongoing initiative aimed at developing a finite element analysis system based on object-oriented programming, known as FEMOOP, which encompasses the system's overall architecture.

V. Finite Element Programming

Prior to delving into the organizational structure of the FEMOOP program, it is essential to grasp that nonlinear finite element analysis involves computations on three distinct levels: the structural level, the elemental level, and the integration point level.

At the structural (or global) level, a range of algorithms is applied to address the problem, including linear static, linearized buckling, linear dynamic, nonlinear path-following, and nonlinear dynamic analyses. These algorithms are executed using global vectors and matrices and are not reliant on the specific types of elements and materials employed in the analysis.

The elemental level primarily concentrates on the calculation of elemental vectors and matrices, such as the internal force vector and stiffness matrix. These calculations are crucial for assembling the global vectors and matrices that the analysis algorithms use. Importantly, the computation of these elemental vectors and matrices remains entirely independent of the chosen analysis algorithm.

The interconnection between the global and elemental levels functions in two directions. In the upward direction, global vectors and matrices are computed by amalgamating contributions from individual elements. In the downward direction, element displacements are extracted from the global displacement vector. These communication tasks are based on nodal degrees of freedom and element connectivity.

Finally, at the integration point level, the computation of stress vectors and tangent constitutive matrices occurs. These quantities are utilized in the computation of elemental vectors and matrices. However, they do not depend on the specific element formulation if the essential input data for stress computation, such as strain components, are provided.

Previous research has extensively investigated the convergence properties of two-dimensional solutions for elastic continuum problems using both quadrilateral and triangular elements. Those studies have identified several key factors that influence the convergence characteristics of finite element solutions. These factors encompass the fundamental shape of the elements, element distortion, polynomial order of the elements, completeness of polynomial functions, integration techniques, and material incompressibility.

The flow chart of the developed program is shown in Figure 5.

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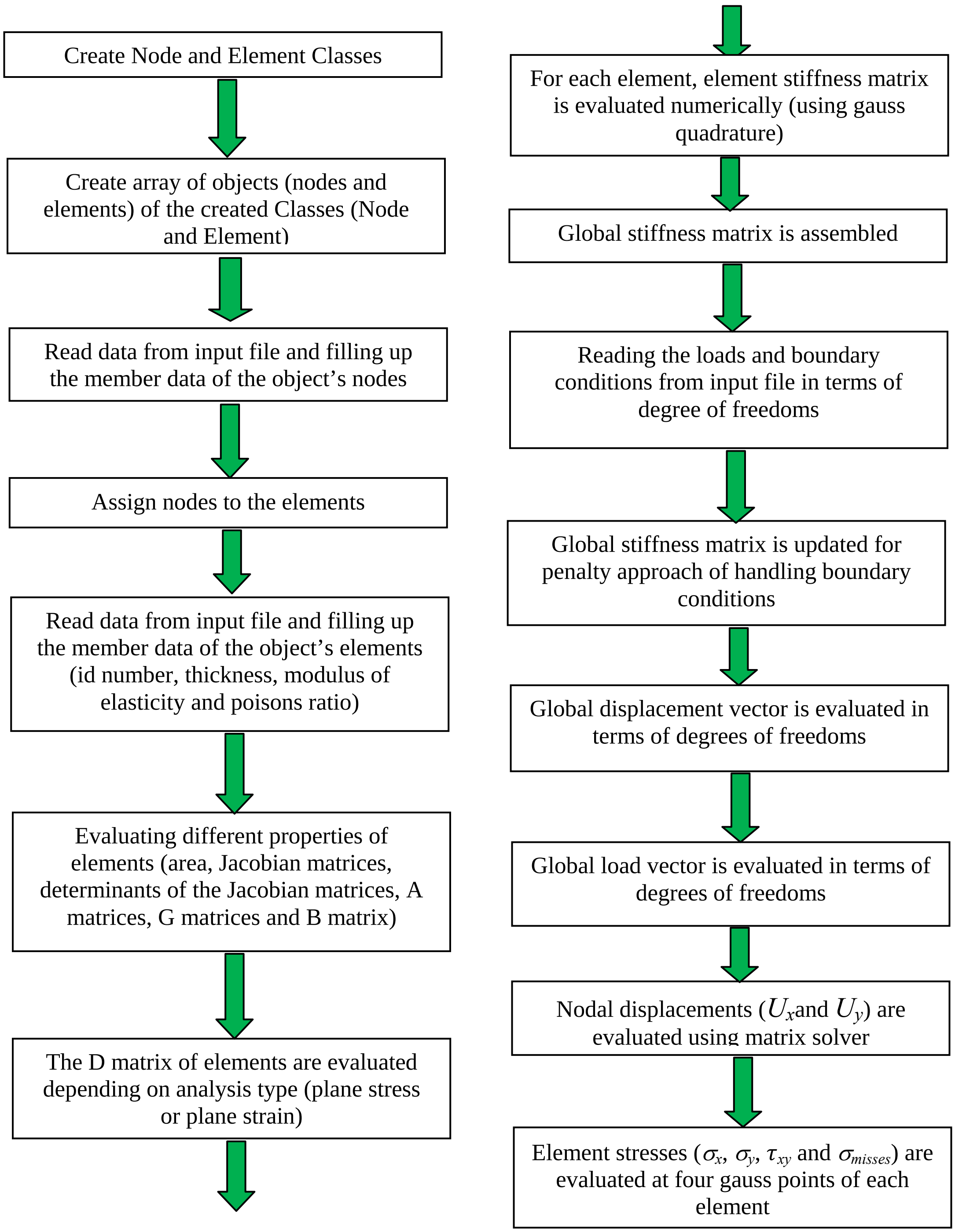

Figure 5: Flow chart of the developed program

In general, it is widely recognized that quadrilateral elements outperform simplex triangular elements. Quadrilateral elements are favored for two-dimensional meshes due to their higher accuracy and efficiency, while hexahedral elements are preferred for three-dimensional meshes. This preference is evident in structural analysis and extends to various engineering disciplines.

Nevertheless, it is also mentioned that triangular elements, especially those with higher-order displacement assumptions, can deliver acceptable accuracy and

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convergence characteristics. However, a notable drawback of triangular elements is the potential for mesh locking due to material incompressibility. This issue represents a significant limitation when employing triangular elements in certain applications.

A brief overview of the object-oriented code utilized in this research is presented here.

Notes

The code itself is structured around the creation of two distinctive classes, namely “Node” and “Element.” Within the ”Node” class, a spectrum of attributes, including coordinates, displacements, boundary constraints, and loads, is encapsulated, and the duty of reading and computing these attributes falls upon its member functions. Similarly, the “Element” class hosts an array of attributes, encompassing constituent nodes, element thickness, and material characteristics. Its member functions are thoughtfully tailored to handle the retrieval and computation of these specific properties.

Subsequently, the code gives rise to an array of objects, each derived from these meticulously crafted classes. This ensemble of objects harmoniously converges to enable the creation of the global stiffness matrix and global load vector, pivotal components in the realm of finite element analysis.

In addressing boundary conditions, the code adopts the penalty approach, skillfully integrating these constraints into the calculations. In this manner, the code progresses to evaluate nodal displacements and elemental stresses, completing its comprehensive analysis.

VI. Results and Discussion

In this study, the SIMULIA ABAQUS 13.3 version finite element analysis software is employed for various aspects of a comprehensive finite element analysis. This encompasses pre-processing tasks like modeling, the actual processing or finite element analysis, and post-processing activities, which involve generating reports, images, animations, and other output files. The finite element analysis software allows for pre-processing, post-processing, and monitoring the solver’s progress. Within this investigation, issues related to plane stress analysis, particularly concerning structural members, were explored. These investigations were conducted with appropriate boundary conditions and loadings using the finite element analysis software. To present the findings, images and a report file generated by the finite element analysis program were employed. The output file produced by the developed program was used to analyze two-dimensional plane stress problems. The primary focus lies on examining normal stresses, shear stress, and von-Mises stress at the four integration points, which represent the most vulnerable areas of structural components. Consequently, the stress data obtained from the developed program is compared with that from the finite element analysis software. The level of accuracy or deviation in these results was also quantified and presented.

A two-dimensional plane stress problem is shown in Figure6.In this problem, the objective is to fabricate a basic bracket section using a steel plate with a thickness of 20 mm. The plate is fixed at the two smaller holes on the left and subjected to a load applied at the larger hole on the right.

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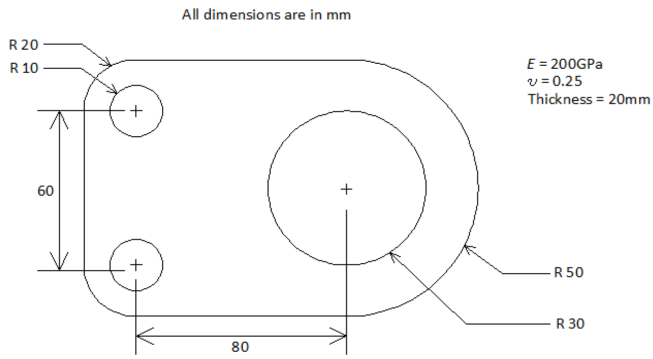
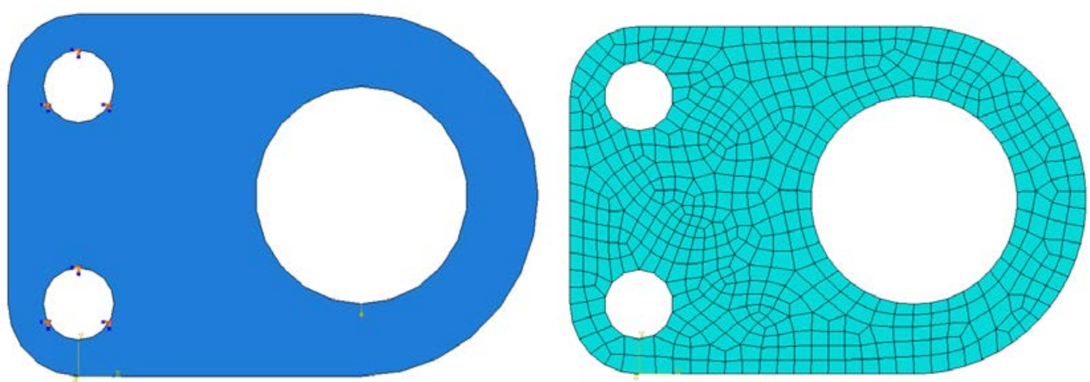


Figure 6: Bracket plate

The relevant parameters employed in this problem includes:

Concentrated force applied	$F = 1000$ N
Thickness of the plates	$t = 20$ mm
Modulus of Elasticity	$E = 200 \times 10^3$ N/mm^2
Poisson's ratio	$\upsilon = 0.25$
Smaller hole radius	$R_1 = 10$ mm
Larger hole radius	$R_2 = 30$ mm



(a) (b)

Figure 7: a) Application of boundary conditions and loadings (bracket plate); b) Quadrilateral mesh (bracket plate)

Figure 7(a) depicts the illustrated boundary conditions and applied loads. In this representation, the plate is securely fixed at the two small holes on the left, preventing any movement in both the x and y directions ($Ux = Uy = 0$). Additionally, a concentrated force of 1000 N is applied downwards at the lowest point within the larger hole on the right. For this model, which comprises 614 nodes and 539 quadrilateral elements, the quadrilateral mesh that has been generated is showcased in Figure 7(b).

The FEM analysis of the model in terms of σ_{mises} (von-Mises stress) is illustrated in Figure 8.It is observed that the von-Mises stress is maximum around the region of element no. 194 and 56, particularly near the node no. 27 which is shown in Figure 9.

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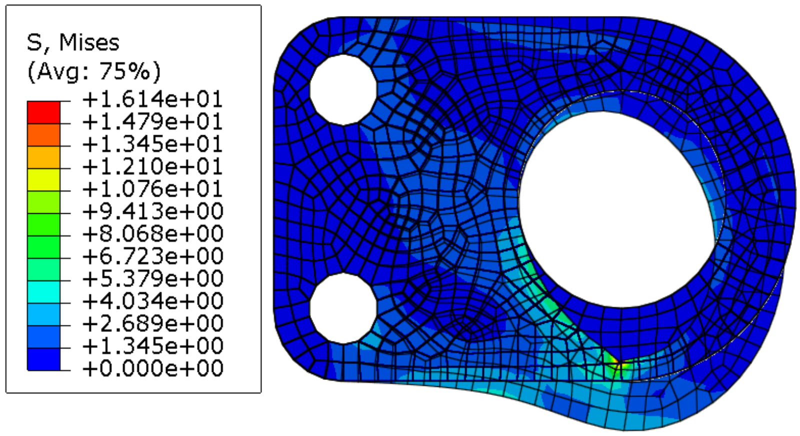

Figure 8: FEM analysis of bracket plate in terms of σ_{mises}

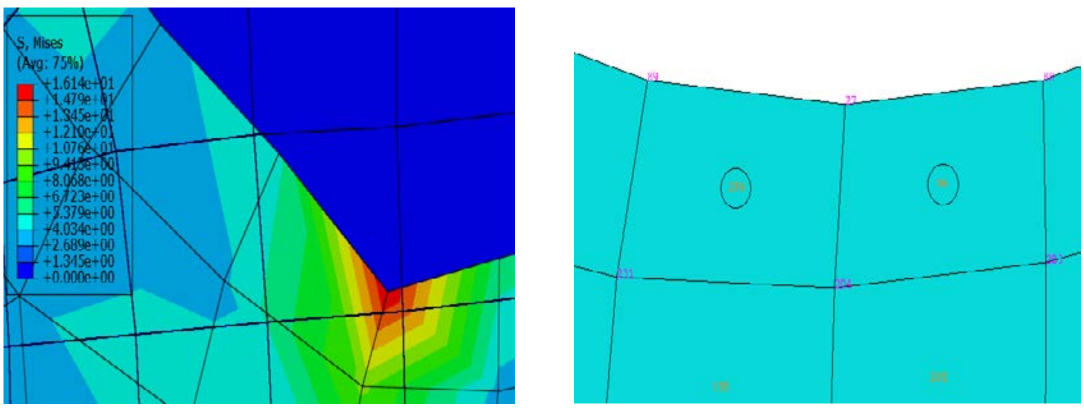

Figure 9: Location of most vulnerable region of bracket plate in terms of σ_{mises}

By employing the developed program designed for plane stress analysis, an output file has been generated for the bracket plate section, which includes stress values at the four integration points and displacement data at the nodal points. Figure 9 clearly illustrates that the maximum von-Mises stresses occur within two specific elements: element no. 194, comprising nodes 27, 89, 204, and 231, and element no. 56, composed of nodes 27, 88, 203, and 204. Comparison of σ_{misses}, σ_x, σ_y, and τ_{xy} for element no. 194 and 56 at the four integration points, and U_x and U_y for element no. 194 and 56 at their four nodal points obtained from the developed program and the analysis are shown in Table 1 to Table 6.

Table 1: Comparison of σ_{misses} for a bracket plate section

Element No.	Integration point (Node No.)	σ_{misses} from developed program (MPa)	σ_{misses} from analysis (MPa)	% of error
194	1 (204)	9.507	9.506	0.002
	2 (27)	12.100	12.100	0
	3 (89)	7.309	7.308	0.002
	4 (231)	3.696	3.691	0.005
56	1 (27)	12.526	12.525	0.002
	2 (204)	9.372	9.370	0.002
	3 (203)	2.387	2.381	0.007
	4 (88)	6.425	6.423	0.002

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Table 2: Comparison of σ_x for a bracket plate section

Element No.	Integration point (Node No.)	σ_x from developed program (MPa)	σ_x from analysis (MPa)	% of error
194	1 (204)	-4.089	-4.086	0.004
	2 (27)	-7.117	-7.117	0
	3 (89)	-4.884	-4.883	0.001
	4 (231)	-2.095	-2.091	0.005
56	1 (27)	-8.541	-8.541	0
	2 (204)	-5.227	-5.225	0.003
	3 (203)	-2.705	-2.700	0.006
	4 (88)	-5.916	-5.915	0.001

Table 3: Comparison of σ_y for a bracket plate section

Element No.	Integration point (Node No.)	σ_y from developed program (MPa)	σ_y from analysis (MPa)	% of error
194	1 (204)	-7.929	-7.927	0.003
	2 (27)	-8.169	-8.168	0.002
	3 (89)	-0.610	-0.603	0.008
	4 (231)	-0.748	-0.741	0.010
56	1 (27)	-10.766	-10.765	0.002
	2 (204)	-9.630	-9.629	0.002
	3 (203)	-1.155	-1.149	0.010
	4 (88)	-1.951	-1.948	0.005

Table 4: Comparison of τ_{xy} for a bracket plate section

Element No.	Integration point (Node No.)	τ_{xy} from developed program (MPa)	τ_{xy} from analysis (MPa)	% of error
194	1 (204)	-3.795	-3.795	0
	2 (27)	-5.390	-5.389	0.001
	3 (89)	-3.275	-3.272	0.004
	4 (231)	-1.851	-1.849	0.004
56	1 (27)	4.472	4.471	0.001
	2 (204)	2.457	2.456	0.002
	3 (203)	0.237	0.229	0.047
	4 (88)	2.162	2.159	0.006

Table 5: Comparison of U_x for a bracket plate section

Element No.	Node No.	U_x from developed program (mm)	U_x from analysis (mm)	% of error
194	204	-0.00032	-0.00032	0
	27	-0.00017	-0.00016	0.001
	89	0.00001	0.00001	0
	231	-0.00028	-0.00028	0
56	27	-0.00017	-0.00017	0
	204	-0.00032	-0.00032	0
	203	-0.00035	-0.00034	0.001
	88	-0.00032	-0.00031	0.001

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Table 6: Comparison of U_y for a bracket plate section

Element No.	Node No.	U_y from developed program (mm)	U_y from analysis (mm)	% of error
194	204	-0.00308	-0.00308	0
	27	-0.00325	-0.00325	0
	89	-0.00259	-0.00259	0
	231	-0.00254	-0.00253	0.001
56	27	-0.00325	-0.00324	0.001
	204	-0.00308	-0.00307	0.001
	203	-0.00320	-0.00310	0.001
	88	-0.00316	-0.00316	0

Notes

In the context of the bracket plate section, the regions of maximum stress vulnerability are identified as element no. 194, defined by nodes 27, 89, 204, and 231, and element no. 56, characterized by nodes 27, 88, 203, and 204. Consequently, a comparative analysis is conducted to assess the stress distributions at the four integration points and the displacements at the nodal points for both element no. 194 and element no. 56 within the above-mentioned model.

Table 1 displays the σ_{misses} values, indicating that the maximum tensile stress is located at integration point 1 for element no. 56. The calculated stress is 12.526 MP a from the developed program and 12.525 MPa from the analysis, resulting in near-identical values with a minimal deviation of only 0.002%. A comprehensive examination of all the values in Table 1 reveals consistently minimal error percentages in the σ_{misses} results. The findings regarding σ_{misses} from previous studies demonstrated a high degree of similarity (Hoque, 2016; Hoque and Islam, 2023).

When examining σ_x in Table 2, it becomes apparent that the maximum compressive stress is located at integration point 1 for element no. 56. This stress measures 8.541 MPa, which is identical when derived from both the developed program and the analysis, resulting in a perfect match with zero percentage of error. Upon reviewing all the other σ_x values in Table 2, it is evident that the results are consistently in close agreement, displaying only minimal error percentages.

Table 3 illustrates σ_y, where the maximum compressive stress is identified at integration point 1 for element no. 56. This stress registers at 10.766 MP a from the developed program and 10.765 MPa from the analysis, showcasing remarkably similar values with a minuscule deviation of only 0.002%. A thorough examination of all the other σ_y values in Table 3 reveals consistently close results with exceedingly small percentages of error.

In the case of τ_{xy}, as demonstrated in Table 4, an intriguing pattern are observed. Firstly, the maximum compressive stress occurs at integration point 2 for element no. 194, measuring 5.390 MP a in the developed program and 5.389 MP a in the analysis. These two values are remarkably similar, differing by a mere 0.001% margin. Secondly, the maximum tensile stress manifests at integration point 1 for element no. 56, with values of 4.472 MPa computed by the developed program and 4.471 MP a from the analysis. Impressively, these tensile stress values exhibit a near-identical match, showing only a minimal 0.001% disparity. Delving further into Table 4, it becomes evident that τ_{xy} values demonstrate a high degree of consistency, with errors consistently remaining

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relatively small. Previous research (Hoque, 2016; Hoque and Islam, 2023) revealed a remarkable level of similarity in their findings regarding τ_{xy}.

In the context of displacement analysis, an examination of U_x reveals interesting insights, as depicted in Table 5. The maximum displacement is pinpointed at node no. 203 within element no. 56, measuring 0.00035 mm according to the developed program and 0.00034 mm as per the analysis. These two values exhibit remarkable proximity, differing by just 0.001%, emphasizing their substantial agreement. Similarly, when investigating U_y, as presented in Table 6, the maximum displacement is found at node no. 27 of element no. 56. This displacement is quantified at 0.00325 mm through the developed program and 0.00324 mm through the analysis, showcasing a close match with a mere 0.001% variation. From the comprehensive comparison of all other values within Table 5 and Table 6, it becomes evident that the results for both U_x and U_y consistently maintain a relatively tight margin of error, underlining their close agreement.

The maximum von-Mises stress concentrations are notably located around element no. 194 and 56, particularly in the proximity of node no. 27, as depicted in Figure 9. Employing a developed program for plane stress analysis, an output file is generated, which details the stress values at the four integration points for a bracket plate section. Figure 9 vividly illustrates that the maximum von-Mises stresses manifest within two specific elements: element no. 194, encompassing nodes 27, 89, 204, and 231; and element no. 56, comprising nodes 27, 88, 203, and 204.Comparative data for σ_{mises}, σ_x, σ_y, and τ_{xy} for element no. 194 and 56 at the four integration points, as obtained from both the developed program and finite element analysis software, are presented in Table 1 through Table 4. For instance, when node no. 27 of element no. 56 is examined, the von-Mises stress registers at 12.526 MP a based on the developed program (theoretical). This von-Mises stress results from the combination of all stresses acting at that specific location, namely normal stresses in two-directions and shear stress. For node no. 27 of element no. 56, the normal stresses along the x and y directions, as well as the shear stress, are determined to be -8.541 MP a, -10.766 MP a, and 4.472 MP a, respectively. Utilizing these values for normal and shear stresses and applying Equation (9), a von-Mises stress of 12.526 MP a is obtained through theoretical calculations, which closely aligns with the analysis-derived von-Mises stress of 12.525 MP a, representing a mere 0.002% deviation. When comparing von-Mises stress values for other nodes (node no. 88, 203, 204) of element no. 56 and nodes (node no. 27, 89, 204, and 231) of element no. 194 between the developed program and analysis results, only minimal discrepancies are observed, typically ranging from 0% to 0.007%. Therefore, it can be said that the results from the developed program closely align with the analysis outcomes.

The material employed for the bracket plate section is steel, having a yield strength of 250 MP a. According to the von-Mises stress criterion, it is imperative that von-Mises stress does not exceed the material's yield strength, as surpassing this limit would lead to material yielding and failure. The von-Mises stress values obtained both from the developed program and the analysis, for the four nodes (node no. 27, 88, 203, and 204) of element no. 56 and the remaining four nodes (node no. 27, 89, 204, and 231) of element no. 194, all fall within the range of the material's yield strength. Consequently, it can be said that the bracket plate section will not yield or fail when subjected to a concentrated downward force acting at its lowest point (within the larger hole on the right side of the bracket plate section).

Notes

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To visualize the generated mesh for a bracket plate section, a specialized two-dimensional mesh viewer program has been developed using the Python programming language. The program provides a graphical representation of the mesh structure. In Figure 10, the outcomes of this endeavor can be observed, specifically focusing on the resultant quadrilateral mesh for a bracket plate section. These intricate mesh structures have been systematically generated, guided by the magnification factor meticulously calculated through the Python program.

Furthermore, the utilization of the developed mesh viewer program extends to the visualization of the triangular mesh within the bracket plate section. Figure 11 provides an illustrative representation of the resulting triangular mesh for this specific model, generated through the combined efforts of ABAQUS software and the developed mesh viewer program.

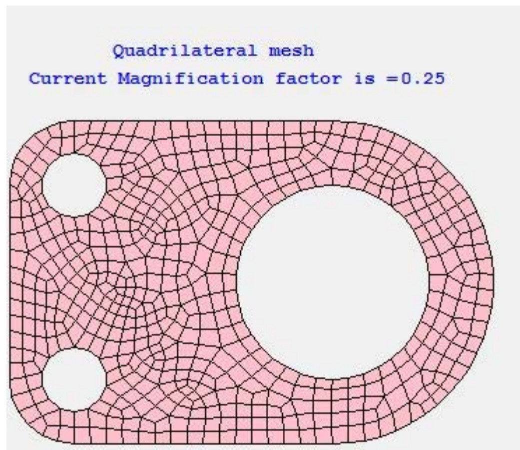

Figure 10: Quadrilateral mesh obtained by using the developed mesh viewer program for a bracket plate section

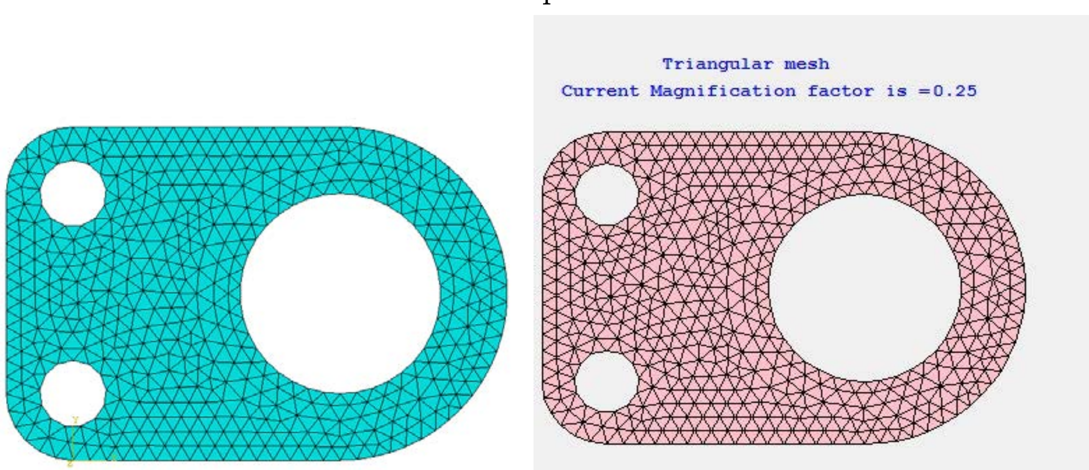

Figure 11: Triangular mesh for a bracket plate section via using the a) ABAQUS software; b) developed mesh viewer program

VII. Conclusions

The conclusions that can be made from this study are summarized below:

- An efficient and reliable object-oriented program has been developed to handle two-dimensional four-node quadrilateral elements. This program facilitates precise stress value calculations at the four integration points (gauss points) for bracket plate

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sections that include structural discontinuities. It serves as an invaluable tool for analyzing a diverse array of two-dimensional plane stress problems, greatly improving the efficiency and accuracy of the analysis process.

- The program developed for the assessment of the structural sections' most vulnerable areas has undergone successful validation of its analysis results. Upon meticulous comparison of stress values at each gauss point and displacement values at nodal points, it became evident that the program's analysis results displayed merely marginal percentages of error. This underscores the remarkable precision and reliability achieved by the developed program.
- The mesh viewer program that has been developed possesses the capability to visualize meshes of both quadrilateral and triangular configurations, with the added flexibility to adjust the magnification factor as required for different models associated with structural sections.

Notes

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A Forced Pressure-Less Gas System via Sticky Particles Turbulences

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Abstract- The turbulences in sticky particles dynamics are described by a class of Markov processes whose the derivatives (velocity fields of turbulence) are backward semi-martingales. The turbulence dynamics is connected to a forced (inhomogeneous) pressure-less gas system.

Keywords: burgers equation, conservation law, martingale, pressureless gas system, semi-martingale, sticky particle dynamics, turbulence.

GJSFR-F Classification: *MSC: 76D05*

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A Forced Pressure-Less Gas System via Sticky Particles Turbulences

Florent Nzissila [α] & Octave Moutsinga [σ]

Abstract- The turbulences in sticky particles dynamics are described by a class of Markov processes whose the derivatives (velocity fields of turbulence) are backward semi-martingales. The turbulence dynamics is connected to a forced (inhomogeneous) pressure-less gas system.

Keywords: burgers equation, conservation law, martingale, pressureless gas system, semi-martingale, sticky particle dynamics, turbulence.

I. Introduction

We are interested in the turbulence of one dimensional fluid flows in one dimensional sticky dynamics. In [1], the authors considered, in Eulerian coordinates, the velocity field u of fluid particles and a probability field μ representing their mass or charge distribution. The particles are supposed accelerated between two successive shock times; the dynamics is then governed by a force (measure) field ν. For suitable initial data $(\mu, u)|_{t=0} = (\mu_0, u_0)$ and by discrete approximations, they solved the forced pressureless gas system

$$\begin{cases} \partial_t(\mu) + \partial_x(u\mu) = 0 \\ \partial_t(u\mu) + \partial_x(u^2\mu) = \nu \\ \mu_t \to \mu_0, \quad u(\cdot,t)\mu_t \to u_0\mu_0 \ weakly \ \ as \ \ t \longrightarrow 0 \end{cases} \tag{1}$$

where the force ν is absolutely continuous, in the space states, with respect to (w.r.t.) μ.

In this paper, we consider non accelerated fluid particles, so the force of [1] is null and the solution of (1) is thus the one of [2, 8, 3]. In this work, we concentrate our attention on turbulences which generate, for (1), a *new force* whose the support is included in the set of shock (and *pure turbulence*) sites, in space-time.

Let us first recall the constructions of [2, 8, 3]. They all rely on the sticky particle dynamics which was introduced, at a discrete level, by Zeldovich [9] in order to explain the formation of large structures in the universe. That is a finite number of particles which move with constant velocities while they

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are not collided. All the shocks are inelastic following the conservation laws of mass and momentum.

At a continuous level, the initial state of particles is given by the support of a non negative measure μ_0. A particle starts from position x with velocity $u_0(x)$ and mass $\mu_0(\{x\})$. The particles move with constant velocities and masses while not collided. All the shocks are inelastic, following the conservation laws of mass and momentum. In their pioneering work, E et al [8] made this construction when the particles are every where in $\mathbb{R}$, u_0 is continuous and the mass of any interval $[a,b]$ is computed with a positive density f, i.e. $\mu_0([a,b]) = \int_a^b f(x)\mathrm{d}x$. At time t, a particle of position $x(t)$ has the mass $\mu(\{x(t)\},t)$ and the velocity $u(x(t),t)$, the momentum of any interval $[a(t),b(t)]$ is $\int_{a(t)}^{b(t)} u(x,t)\mu(\mathrm{d}x,t)$. The authors then solved (1) with $\nu \equiv 0$.

At the same time and independently, Brenier and Grenier [2] considered the case of particles confined in a in interval $[a,b]$, i.e. $\mu_0([a,b]^c) = 0$. By discretization of μ_0 and using discrete sticky particle dynamics, they solved the scalar conservation law $\partial_t M + \partial_x(A(M)) = 0$ by a weak solution (M,A), the unique which has some entropy condition. As a consequence, the Lebesgue-Stieltjes measure $\partial_x(A(M))$ is absolutely continuous w.r.t. $\partial_x M =: \mu(\cdot,t)$, of Randon-Nicodym derivative a function $u(\cdot,t)$. Then (μ,u) solves (1) with $\nu \equiv 0$.

In [3], Dermoune and Moutsinga constructed the sticky particles dynamics with an initial mass distribution μ_0, any probability measure, and a initial velocity function u_0, any continuous and locally integrable function such that $u_0(x) = o(x)$ as $x \to \infty$. The authors united and generalized previous works of [8, 2] with the arguments that the particles paths define a Markov process $t \mapsto X_t$ solution of the ODE

$$\mathrm{d}X_t = u(X_t,t)\mathrm{d}t, \tag{2}$$

and the velocity process $t \mapsto u(X_t,t)$ is a backward martingale. Moreover, $\mu(\cdot,t) = \mathrm{Law}(X_t)$.

In [6, 7], using suitable convex hulls, Moutsinga extended the construction when μ_0 is any non negative measure and u_0 has *no positive jump*. He gave the description of different kinds of clusters $[\alpha(x,t),\beta(x,t)]$, *i.e* the set of all the initial particles $y(0)$ which have the same position $y(t) = x$ at time t.

Following the preoccupation of Eyink and Drivas ([4]) about turbulences, Nzissila, Moutsinga and Eyi Obiang [5] defined a *turbulent interval* as a set $[a,b]$ of initial positions of sticky particles from which rise a turbulence. This means that for all $y \in [a,b]$, the interval $[a,b]$ is the widest among the intervals $[a',b'] \ni y$ which have the same position $y + \tau(y)u_0(y)$ at their common first shock time $\tau(y)$. The term "turbulence" (instead of "shock") is justified by the description of a degenerated turbulent interval $[a,b] = \{a\}$. In this case, at its *mathematical* first shock time $\tau(a)$, the particle a does not enter in a real shock but it begins a *coagulation* process; it enters in a *pure turbulence* without beginning by a real shock.

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2. Y. Brenier and E. Grenier. Sticky particles and scalar conservation laws. *Siam J. Numer. Anal.*, 35:2317–2328, 2008.

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At time of turbulence $\tau(a)$, the *turbulent interval* $[a, b]$ is part of a cluster $[\alpha, \beta]$ $(a, b \in [\alpha, \beta])$. The initial positions a, b, α, β are called *turbulent particles*. The motions of these particles are given by four backward Markov processes, respectively, Z^1, Z^2, Z^3 and Z^4 solutions of (2) and whose the velocity processes (the derivatives) are semi-martingales.

In this paper, we consider a process Z of more general form than in [5]. The gas system (1) is studied with a force generated at random turbulence time $\gamma = \tau(Z_0)$.

The paper is organized as follows. Section 2 is devoted to the sticky particles model. We recall its definition and the main properties used here. In section 3 we come back to the results of [5] according to the study of turbulence. These results were obtained when the support of μ_0 is an interval (i.e. there is no vacuum of matter). We generalize them to any type of support. The particularity, in presence of vacuum, is that traditional delta-shocks are transformed into *butterfly-shocks* (like in [3]). Section 4 is devoted to scalar conservations laws from the point of view of turbulent particles. First we give an entropy solution (N, A) with the same flux A as in [2], but with different initial data. Then, in subsection 4.1 we study the gas system. Considering the construction of [5], we define a process of more general form $t \mapsto Z_t = Z_t^1 \mathbb{1}_{A^1} + Z_t^2 \mathbb{1}_{A^2} + Z_t^3 \mathbb{1}_{A^3} + Z_t^4 \mathbb{1}_{A^4}$, with the help of any complete system of events A^1, A^2, A^3, A^4. A solution of (1), is given by $\mu(\cdot, t) := \mathrm{Law}(Z_t)$ and $u(Z_t, t) := \frac{\mathrm{d}Z_t}{\mathrm{d}t}$. The force ν is absolutely continuous w.r.t. the law of the couple (Z_γ, γ).

Although this solution is constructed from the sticky particles model, it does not have the properties of [1].

II. Flow and Velocity Field of Sticky Particles

a) *The sticky particle dynamics*

The definition of one dimensional sticky particle dynamics requires a mass distribution μ, any Radon measure (a measure finite on compact subsets) and a velocity function u, any real function such that the couple (μ, u) satisfies the *Negative Jump Condition* (NJC) defined in [6]. Precisely, consider the support $\mathcal{S} = \{x \in \mathbb{R} : \mu(x - \varepsilon, x + \varepsilon) > 0, \forall \varepsilon > 0\}$ of μ and the subsets $\mathcal{S}_- = \{x \in \mathbb{R} : \mu(x - \varepsilon, x) > 0\}$, $\mathcal{S}_+ = \{x \in \mathbb{R} : \mu(x, x + \varepsilon) > 0, \forall \varepsilon > 0\}$. Suppose that u is μ locally integrable and consider the generalized limits u^-, u^+ :

$$u^-(x) = \limsup_{\varepsilon \to 0} \frac{\int_{[x-\varepsilon,\, x)} u(\eta)\mu(d\eta)}{\mu[x - \varepsilon,\, x)}, \quad \forall x \in \mathcal{S}_-, \tag{3}$$

$$u^+(x) = \liminf_{\varepsilon \to 0} \frac{\int_{(x,\, x+\varepsilon]} u(\eta)\mu(d\eta)}{\mu(x,\, x + \varepsilon]}, \quad \forall x \in \mathcal{S}_+. \tag{4}$$

Ref

5. F. Eyi Obiang, O. Moutsinga, and F. Nzissila. Backward semi-martingale into burgers turbulence. *J. Math. Phys.*, 62:1–12, 2021.

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The *Negative Jump Condition* requires that

$$u^-(x) \geq u(x)\ \forall\, x \in \mathcal{S}_-, \quad u(x) \geq u^+(x)\ \forall\, x \in \mathcal{S}_+. \tag{5}$$

In the whole paper, we mainly use $\mu_0 = \lambda$, the Lebesgue measure. That's why we always suppose that the support $\mathcal{S} = \mathbb{R}$.

Considering particles of initial mass distribution μ_0 and of initial velocity function u_0, their sticky dynamics is defined in [7], when the couple $(\mu_0,\, u_0)$ satisfies (5) and $x^{-1}u(x) \to 0$ as $|x| \to +\infty$. The dynamics is characterized by a forward flow $(x, s, t) \mapsto \phi_{s,t}(x)$ defined on $\mathbb{R} \times \mathbb{R}_+ \times \mathbb{R}_+$.

b) Proposition (Forward flow)
For all x, s, t :

1. *$\phi_{s,s}(x) = x$ and $\phi_{s,t}(\cdot)$ is non-decreasing and continuous.*
2. *The value $\phi_{s,t}(x)$ is the position after supplementary time t of the particle which occupied the position x at time s. More precisely :*

$$\phi_{s,t}(\phi_{0,s}(y)) = \phi_{0,s+t}(y)\ , \quad \forall\, y. \tag{6}$$

3. *If $\phi_{0,t}^{-1}(\{x\}) =: [\alpha(x,0,t),\, \beta(x,0,t)]$ with $\alpha(x,0,t) < \beta(x,0,t)$, then*

$$x = \frac{\int_{[\alpha(x,0,t),\, \beta(x,0,t)]}(a + tu_0(a))\mathrm{d}\mu_0(a)}{\mu_0([\alpha(x,0,t),\, \beta(x,0,t)])}\ .$$

Else

$$x = \alpha(x,0,t) + tu_0(\alpha(x,0,t)) = \beta(x,0,t) + tu_0(\beta(x,0,t))\ .$$

4. *$\beta(x,0,t) + tu_0(\beta(x,0,t)) \leq x \leq \alpha(x,0,t) + tu_0(\alpha(x,0,t))$.*
If $\mu_0([\alpha(x,0,t),\, y]) > 0$ and $\mu_0(]y,\, \beta(x,0,t)]) > 0$, then

$$\frac{\int_{]y,\beta(x,0,t)]}(a + tu_0(a))\mathrm{d}\mu_0(a)}{\mu_0(]y, \beta(x,0,t)])} \leq x \leq \frac{\int_{[\alpha(x,0,t),y]}(a + tu_0(a))\mathrm{d}\mu_0(a)}{\mu_0([\alpha(x,0,t), y])}\ .$$

5. *The function $[0,t] \ni x \longmapsto \phi_{0,s}(\alpha(x,0,t))$ is concave. It is a straight line if and only if $x = \alpha(x,0,t) + tu_0(\alpha(x,0,t))$.*
The function $[0,t] \ni x \longmapsto \phi_{0,s}(\alpha(x,0,t))$ is convex. It is a straight line if and only if $x = \beta(x,0,t) + tu_0(\beta(x,0,t))$.
6. *For any compact subset $K = [a,b]\times[0,T]$, consider $A_T = \alpha(\phi_{s,T}(a), s, T)$, $B_T = \beta(\phi_{s,T}(b), s, T)$ and the probability $\mu_s^K = \frac{\mathrm{I}_{[A_T,B_T]}}{\mu_s([A_T,B_T])}\mu_s$. The sticky particle dynamics induced by (μ_s^K, u_s), during time interval $[0,T]$, is characterized by the restriction of the function $(y,t) \mapsto \phi_{s,t}(y)$ on $[A_T, B_T] \times [0,T]$.*

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7. O. Moutsinga. Burgers' equation and the sticky particles model. *J. Math. Phys*, 53:1–12, 2012.

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Notes

The latter means that the restriction of flow on a compact subset of space-time does not depend of the whole matter, but only on the restriction of the matter (distribution) on a compact subset of space states.

Remark that if $x = \alpha(x,0,t) + tu_0(\alpha(x,0,t)) = \beta(x,0,t) + tu_0(\beta(x,0,t))$, then the graphs $[0,t] \ni s \mapsto \phi_{0,s}(\alpha(x,0,t))$, $\phi_{0,s}(\beta(x,0,t))$ draw a delta-shock, well known in the literature (Figure 1). Otherwise, these graphs draw a kind of *butterfly-shock with foded wings* (Figure 2)

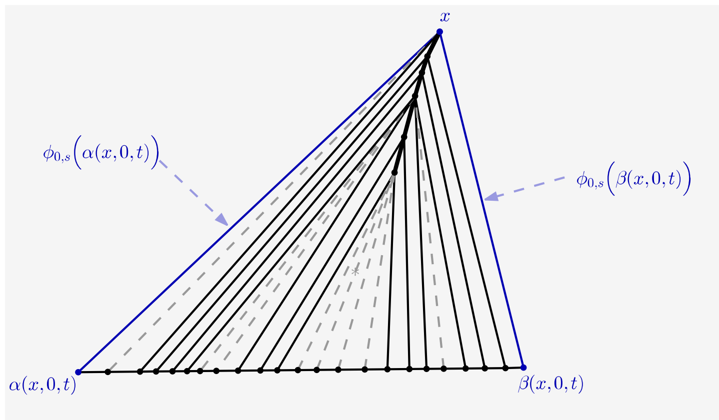

Figure 1: The blue line on the left (resp right) of the middle shock wave represents the trajectory of the particle which started from position $\alpha(x,0,t)$ (resp $\beta(x,0,t)$). It is trajectory $[0,t] \ni s \mapsto \phi_{0,s}(\alpha(x,0,t))$ (resp $[0,t] \ni s \mapsto \phi_{0,s}(\beta(x,0,t))$)

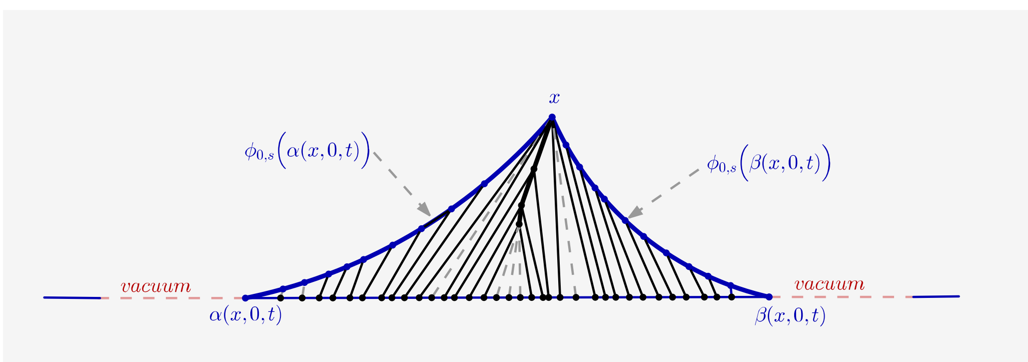

Figure 2: The blue curve on the left (resp right) represent the trajectory of particle which start at the position $\alpha(x,0,t)$ (resp $\beta(x,0,t)$) whch is the trajectory of $[0,t] \ni s \mapsto \phi_{0,s}(\alpha(x,0,t))$ (resp $[0,t] \ni s \mapsto \phi_{0,s}(\beta(x,0,t))$)

What about the velocity?

c) *Proposition (Flow derivative)*

1. *For all y, s, the function $t \mapsto \phi_{s,t}(y)$ has everywhere left hand derivatives. It has everywhere right hand derivatives, except when $\phi_{s,t}^{-1}(\phi_{s,t}(y)) =: [a,\, b]$ with $\mu_s([a,\, b]) = 0$ and $a < b$. Now and after, the notation $\dfrac{\partial}{\partial t}\phi_{s,t}(y)$ stands for the right hand derivative.*
2. *There exists a function $(x,t) \mapsto u_t(x)$ such that $\dfrac{\partial}{\partial t}\phi_{0,t}(y) = u_t(\phi_{0,t}(y))$ everywhere the right derivative exists.*

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3. *For any compact subset* $K = [a, b] \times [0, T]$, *consider* $A_T = \alpha(\phi_{s,T}(a), s, T)$, $B_T = \beta(\phi_{s,T}(a), s, T)$ *and the probability* $\mu_s^K = \frac{\mathrm{1}_{[A_T,B_T]}}{\mu_s([A_T,B_T])}\mu_s$. *If the right hand derivative exits for* $(x, y) \in K$, *the using the conditional expectation under* μ_s^K, *we have*

$$\frac{\partial}{\partial t}\phi_{s,t}(y) = \mathbb{E}_{\mu_s^K}\left[u_s|\phi_{s,t}(\cdot) = \phi_{s,t}(y)\right]. \tag{7}$$

We call *a cluster at time* t all interval of the type $[\alpha(x, 0, t),\, \beta(x, 0, t)]$. The last assertion of proposition 2.1 implies an important property on the velocity of a cluster.

d) Corollary

1. *If* $[\alpha(x, 0, t),\, \beta(x, 0, t)]$ *has positive mass, then*

$$u_t(x) = \frac{\int_{[\alpha(x,0,t),\,\beta(x,0,t)]} u_0(a)\mathrm{d}\mu_0(a)}{\mu_0([\alpha(x, 0, t),\, \beta(x, 0, t)])}\,.$$

If $\alpha(x, 0, t) = \beta(x, 0, t)$, *then* $u_t(x) = u_0(\alpha(x, 0, t))$.
Else $u_t(x)$ *is not (well) defined.*

2. $u_0(\beta(x, 0, t)) \le u_t(x) \le u_0(\alpha(x, 0, t))$.
If $\mu_0([\alpha(x, 0, t),\, y]) > 0$ *and* $\mu_0(]y,\, \beta(x, 0, t)]) > 0$, *then*

$$\frac{\int_{]y,\beta(x,0,t)]} u_0(a)\mathrm{d}\mu_0(a)}{\mu_0(]y, \beta(x, 0, t)])} \le u_t(x) \le \frac{\int_{[\alpha(x,0,t),y]} u_0(a)\mathrm{d}\mu_0(a)}{\mu_0([\alpha(x, 0, t), y])}\,.$$

3. *If* $\alpha(x, 0, t) \in \mathcal{S}_-$ *(resp.* $\beta(x, 0, t) \in \mathcal{S}_+$ *), then* $u_t^-(x) = u_0(\alpha(x, 0, t))$. *(resp.* $u_t^+(x) = u_0(\beta(x, 0, t))$*).*

4. *If* $u_0(\alpha(x, 0, t)) = u_t(x)$, *then* $\mu_0\left(]\alpha(x, 0, t),\, \beta(x, 0, t)]\right) = 0$ *and* $x = \alpha(x, 0, t) + tu_0(\alpha(x, 0, t)) = \beta(x, 0, t) + tu_0(\beta(x, 0, t))$.

5. *If* $u_0(\beta(x, 0, t)) = u_t(x)$, *then* $\mu_0\left([\alpha(x, 0, t),\, \beta(x, 0, t)[\right) = 0$ *and* $x = \alpha(x, 0, t) + tu_0(\alpha(x, 0, t)) = \beta(x, 0, t) + tu_0(\beta(x, 0, t))$.

6. *For all* $t \ge 0$, *we have* $u_t(x) = o(x)$ *as* $|x| \to +\infty$. *For all* $t > 0$, *if* $\alpha(x, 0, t) \in \mathcal{S}_-$ *(resp.* $\beta(x, 0, t) \in \mathcal{S}_+$ *), then* $\lim\limits_{\substack{y\to x\\ y<x}} u_t(y) = u_t^-(x) =$

$$u_0(\alpha(x, 0, t)) \text{ (resp. } \lim_{\substack{y\to x\\ y>x}} u_t(y) = u_t^+(x) = u_0(\beta(x, 0, t))).$$

e) Markov and martingale properties

Let $(\mu_0,\, u_0)$ be as in theorem 2.1. On abstract measure space $(\Omega,\, \mathcal{F},\, P)$ we define a measurable function $X_0 : \Omega \longrightarrow \mathbb{R}$ with image-measure $P \circ X_0^{-1} =$

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μ_0. In practice, $(\Omega, \mathcal{F}, P) = (\mathbb{R}, \mathcal{B}(\mathbb{R}), \mu_0)$ and X_0 is the identity function. For all $t \geq 0$, we set $X_t = \phi_{0,t}(X_0)$. As a consequence of theorem 2.1, we have the following :

f) Proposition (Markov and martingale property)

1. *$\forall s, t$, we have*

$$X_{s+t} = \phi_{s,t}(X_s) \tag{8}$$

2. *If u_0 is μ_0 integrable, then under the measure μ_0 (or P) :*

$$\frac{\mathrm{d}}{\mathrm{d}t} X_t = \mathbb{E}[u_0(X_0)|X_t] = u_t(X_t). \tag{9}$$

   Else, for any compact $K = [a, b] \times [0, t+s]$, if $\phi_{0,t+s}(a) \leq X_{t+s} \leq \phi_{0,t+s}(b)$, then under the conditional probability μ_0^K, we get (9).

3. *If u_0 is μ_0 integrable, then under the measure μ_0 (or P) :*

$$u_{t+s}(X_{t+s}) = \mathbb{E}[u_t(X_t)|\mathcal{F}_{t+s}]\,, \quad \textit{with} \quad \mathcal{F}_t = \sigma(X_u, u \geq t). \tag{10}$$

   Else, for any compact $K = [a, b] \times [0, t]$, $\phi_{0,t+s}(a) \leq X_{t+s} \leq \phi_{0,t+s}(b)$, then we get (10) under the probability μ_0^K (or under the conditional probability knowing $\alpha(\phi_{0,t+s}(a),\, 0,\, t+s) \leq X_0 \leq \beta(\phi_{0,t+s}(a),\, 0,\, t+s)$.

III. Turbulence

In this section, inspired by a preocupation from [4], we study the sticky particles dynamics from the point of view of turbulence. Generalizing the results of [5], we get a class of Markov processes solution (2). The velocities fields are backward semi-martingales.

a) Flow, delta-shock and butterfly-shock

In [5], was defined the first turbulence (or shock) time of the particle initial position a :

$$\tau(a) = \inf \left\{ t : u^-(\phi_{0,t}(a), t) \neq u^+(\phi_{0,t}(a), t) \right\}. \tag{11}$$

Let X_0 be of image-measure μ_0. Define $\gamma = \tau(X_0)$ and the cluster $[Z_0^3,\, Z_0^4] = [\alpha(X_\gamma,\, 0,\, \gamma),\, \beta(X_\gamma,\, 0,\, \gamma)]$ in which belongs X_0 at time γ. The turbulent interval $[Z_0^1,\, Z_0^2]$ is defined as the greatest interval containing X_0 on which τ is constant. It was shown in [5] that the velocities of these variables are semi-martingales, when $\mu_0 = \lambda$ the Lebesgue measure. The same result was obtained for the combination $Z_0^5 = Z_0^5 \mathbb{1}_A + Z_0^5 \mathbb{1}_{A^c}$, with the event A : " the particle enters in the shock from the left ". The interesting variable Z_0^5 was introduced [4] in order to study the Burgers turbulence.

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Our goal is to generalize the results of [5] to any non-negative measure μ_0 and any function u_0 with *negative jumps* (w.r.t. μ_0). For $i = 1, 2, 3, 4, 5$, we consider the process $t \longmapsto Z_t^i = \phi_{0,t}(Z_0^i)$. But one could have other preoccupations than the above event A of [5]. We are led to defined the process of more of more general form $t \longmapsto Z_t = Z_t^1 \mathbb{1}_{A^1} + Z_t^2 \mathbb{1}_{A^2} + Z_t^3 \mathbb{1}_{A^3} + Z_t^4 \mathbb{1}_{A^4}$, with the help of any partition A^1, A^2, A^3, A^4 of Ω, events of $\sigma(X_0)$. Following the implication the application of the Z_0^i's, we have fifteen $(2^4 - 1)$ types of processes . (If $A^i = \Omega$; then $Z = Z^i$).

i. *Proposition (Random butterfly-shock)*

1. *Let Z stand independently for Z^1, Z^2, Z^3 or Z^4.*

$$\forall\, t, s \geq 0, \quad Z_{s+t} = \phi_{s,t}(Z_s)\ , \quad \frac{\mathrm{d}}{\mathrm{d}t} Z_t = u(Z_t, t)\ .$$

2. $\tau(Z_0^1) = \tau(Z_0^2) = \tau(X_0) = \gamma$ *and*

$$\forall\, t \leq \gamma, \quad Z_t^1 = Z_0^1 + tu_0(Z_0^1) \leq X_t = X_0 + tu_0(X_0) \leq Z_t^1 = Z_0^2 + tu_0(Z_0^2)\ ,$$

$$\forall\, t \geq \gamma, \qquad X_t = Z_t^1 = Z_t^2 = Z_t^3 = Z_t^4\ .$$

3. $\tau(Z_0^3) \leq \gamma$ *and* $\tau(Z_0^4) \leq \gamma$.

$[0,\, \gamma] \ni\ t \longmapsto Z_t^3$ *is concave and* $[0,\, \gamma] \ni\ t \longmapsto Z_t^3$ *is convex.*

$$\forall\, t \leq \tau(Z_0^3), \quad Z_t^3 = Z_0^3 + tu_0(Z_0^3)$$

$$\forall\, t \leq \tau(Z_0^4), \qquad Z_t^4 = Z_0^4 + tu_0(Z_0^4);\, .$$

The segment $[Z_0^1,\, Z_0^2]$ and the paths $[0,\, \gamma] \ni\ t \longmapsto Z_t^1,\, Z_t^2$ draw a *prime delta-shock* (so called in [5] because of the first shock time of turbulence).

If $\tau(Z_0^3) = \tau(Z_0^4) = \gamma$, then the paths $[0,\, \gamma] \ni\ t \longmapsto Z_t^3,\, Z_t^4$ are linear; and the draw, with the segment $[Z_0^3,\, Z_0^4]$, *delta shock* (well known in the literature) (see figure 1).

If $\tau(Z_0^3) < \gamma$ (resp. $\tau(Z_0^4 < \gamma)$), then the path $[0,\, \gamma] \ni\longmapsto Z_t^3$ (resp. $[0,\, \gamma] \ni\longmapsto Z_t^3$) is linear; this can occur o,ly when $Z_0^3 \notin \mathcal{S}^-$ (resp. $Z_0^4 \notin \mathcal{S}^+$). If $\max\left(\tau(Z_0^3),\, \tau(Z_0^4)\right) < \gamma$, then the paths $[0,\, \gamma] \ni\longmapsto Z_t^3,\, Z_t^3$ draw, not a delta-shock, but *butterfly-shock with folded wings* (see Figure 3).

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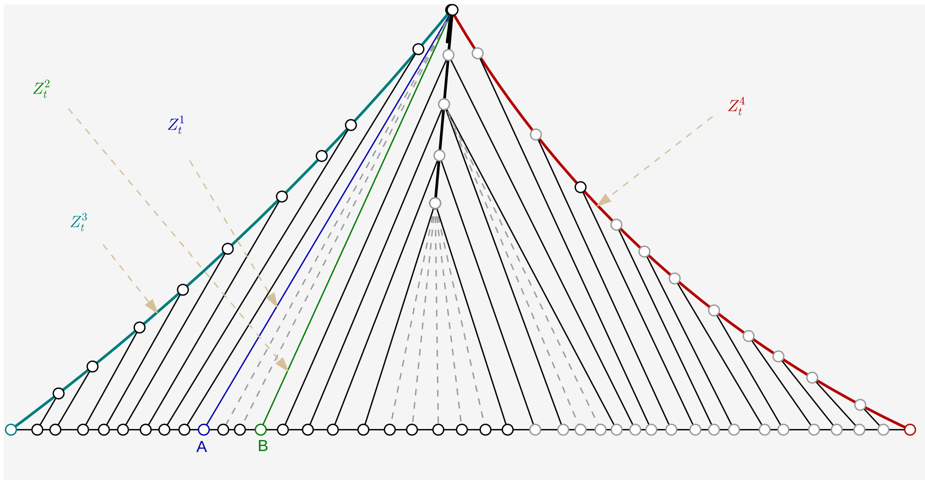

Figure 3: Delta-shock and butterfly-shock

b) *Velocity process as semi-martingale*

i. *Proposition*

1. *$t \longmapsto u(Z_t, t)\mathbb{1}_{t<\gamma}$ is bounded variational process adapted to the natural non increasing filtration $\mathcal{F}^X$ of X.*
2. *For all t, $u(Z_t, t) = [u(Z_t, t) - u_0(X_0)]\mathbb{1}_{t<\gamma} + M_t$, with $M_t = E\left[u_0(X_0)|\mathcal{F}_t^X\right]$. Hence, $t \longmapsto u(Z_t, t)$ is a backward càdlàg semi-martingale of $\mathcal{F}^X$.*
3. *If $\gamma = \tau(Z_0)$, then for all t, $u(Z_t, t) = [u_0(Z_0) - M_0]\mathbb{1}_{t<\gamma} + M_t$, with $M_t = E\left[u_0(X_0)|\mathcal{F}_t^Z\right]$. Hence, $t \longmapsto u(Z_t, t)$ is a backward càdlàg semi-martingale of $\mathcal{F}^Z$.*
4. *If γ is an optional time of $\mathcal{F}^Z$, then $t \longmapsto u(Z_t, t)$ is a backward càdlàg semi-martingale of the completed filtration $\overline{\mathcal{F}^Z}$. Moreover $t \longmapsto u(Z_t, t) - [u(Z_t, t) - M_\gamma^-]\mathbb{1}_{t<\gamma}$*

We recall that for any non increasing filtration $\mathcal{F}$, the filtration $\overline{\mathcal{F}}$ is defined by $\overline{\mathcal{F}_t} = \sigma\left(F_t \cup \mathcal{N}\right)$, where $\mathcal{N}$ is the set of negligible events of $\mathcal{F}_0$.

Before the proof, we recall some properties well known in the theory of stochastic processes.

c) *Lemma*

Let a process Z be adapted to a non increasing filtration $\mathcal{G} = (\mathcal{G}_t, t \geq 0)$. Let Γ be an optional time with respect to $\mathcal{G}$, i.e. for all $t \geq 0$, the event $\{\Gamma > t\} \in \mathcal{G}_t$. The following holds.

1. *The set $\mathcal{G}_\Gamma := \{A \in \mathcal{G}_0 \;:\; A \cap \{\Gamma > t\} \in \mathcal{G}_t\}$ is a sigma-algebra.*
2. *If all the paths of Z are either continuous on the right or on the left, then the r.v. $Z_\Gamma \mathbb{1}_{\Gamma<\infty}$ is $\mathcal{G}_\Gamma$ measurable.*
3. *Suppose that $\mathcal{G}$ is continuous on the right; that is, for all t, $\mathcal{G}_t = \sigma\left(\underset{s>t}{\cup}\mathcal{G}_s\right)$. If Z is a backward martingale with respect to $\mathcal{G}$, then for all*

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t, the right hand and left hand limits Z_{t^+}, Z_{t^-} exist a.s. Moreover, the process $t \mapsto Z_{(\Gamma\vee t)^+} - \Delta_\Gamma \mathbb{1}_{\Gamma>t}$ is a backward martingale with respect to the completed filtration $\overline{\mathcal{G}}$, with $\Delta_\Gamma = Z_{\Gamma^+} - Z_{\Gamma^-}$.

d) Lemma

1. *If a process Z is such that $Z_{s+t} = \phi_{s,t}(Z_s)$ for all $t, s \geq 0$, then $\tau(Z_0) =: \Gamma$ is an optional time with respect to the natural non increasing filtration $\mathcal{F}^Z$ of Z. Moreover, $\mathcal{F}^Z_0 = \mathcal{F}^Z_\Gamma$.*
2. *Suppose that $\{\Gamma \leq t\} \in \mathcal{F}^Z \cap \mathcal{F}^{Z'}$ for some $t \geq 0$. If $Z'_t \mathbb{1}_{\Gamma\leq t} = Z_t \mathbb{1}_{\Gamma\leq t}$, then $\mathrm{E}[F|Z'_t]\mathbb{1}_{\Gamma\leq t} = \mathrm{E}[F|Z_t]\mathbb{1}_{\Gamma\leq t}$ for all integrable r.v. F.*

The second assertion is satisfied by $(Z, Z') = (X, Z^1)$ and $(Z, Z') = (X, Z^2)$, with $\Gamma = \gamma$. Both Z^3 and Z^4 satisfy only the first assertion.

Proof. We begin with the first assertion. $u^-(\cdot, t), u^+(\cdot, t)$ are Borel functions and it is well known that if u is discontinuous in (Z_t, t), it is also discontinuous in $(Z_{t+s}, t+s)$. Then,

$$\{\Gamma \leq t\} = \{u^-(Z_t, t) \neq u^+(Z_t, t)\} \cup [\{u^-(Z_t, t) = u^+(Z_t, t)\} \cap \{\Gamma = t\}] \,.$$

Since

$$\{u^-(Z_t, t) = u^+(Z_t, t)\} \cap \{\Gamma = t\} = \{u^-(Z_t, t) = u^+(Z_t, t)\} \cap \left[\bigcap_{n\geq 1} \{u^-(Z_{t+1/n}, t+1/n) \neq u^+(Z_{t+1/n}, t+1/n)\}\right],$$

the proof of the first assertion is done.
Remark that $Z_{t+1/n} = \phi_{t,1/n}(Z_t)$. So $\{\Gamma \leq t\} = Z_t^{-1}(A_t)$, with

$$A_t = \{u^-(\cdot, t) \neq u^+(\cdot, t)\} \cup \Bigg(\{u^-(\cdot, t) = u^+(\cdot, t)\} \cap \left[\bigcap_{n\geq 1} \{u^-(\phi_{t,1/n}, t+1/n) \neq u^+(\phi_{t,1/n}, t+1/n)\}\right]\Bigg)$$

Now we show that $\mathcal{F}^Z_0 = \mathcal{F}^Z_\Gamma$. First remark that if $\{b\} \neq \phi_{0,t}^{-1}(\phi_{0,t}(b))$, then $\tau(b) \leq t$. Thus for all Borel subset B and $t \geq 0$, we have $B \cap \{\tau > t\} = \phi_{0,t}^{-1}(\phi_{0,t}(B)) \cap \{\tau > t\}$ and

$$Z_0^{-1}(B) \cap \{\tau(Z_0) > t\} = Z_t^{-1}(\phi_{0,t}(B)) \cap \{\tau(Z_0) > t\} \,,$$

$$Z_0^{-1}(B) \cap \{\Gamma > t\} = Z_t^{-1}(\phi_{0,t}(B)) \cap \{\Gamma > t\} \in \mathcal{F}^Z_t \,.$$

This means that $Z_0^{-1}(B) \in \mathcal{F}^Z_\Gamma$.

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For the second assertion, since $Z_t \mathbb{1}_{\gamma\leq t} = Z'_t \mathbb{1}_{\Gamma\leq t}$, it is easy to see that $\mathrm{E}[F|Z'_t]\mathbb{1}_{\Gamma\leq t}$ is $\sigma(Z'_t)\cap\sigma(Z_t)$ measurable; for all bounded Borel function h,

$$\mathrm{E}\big(h(Z_t)\mathrm{E}[F|Z'_t]\mathbb{1}_{\Gamma\leq t}\big) = \mathrm{E}\big(h(Z'_t)\mathrm{E}[F|Z'_t]\mathbb{1}_{\Gamma\leq t}\big) = \mathrm{E}\big(h(Z'_t)F\mathbb{1}_{\Gamma\leq t}\big)$$

$$= \mathrm{E}\big(h(Z_t)F\mathbb{1}_{\Gamma\leq t}\big) = \mathrm{E}\big(h(Z_t)\mathrm{E}[F|Z_t]\mathbb{1}_{\Gamma\leq t}\big).$$

Hence, $\mathrm{E}[F|Z'_t]\mathbb{1}_{\Gamma\leq t} = \mathrm{E}[F|Z_t]\mathbb{1}_{\Gamma\leq t}$ a.s.

Proof of proposition 3.2

1) The restriction $[0,\gamma[\ni t\longmapsto u(Z_t,t)$ is monotone. Thus, the process $\mathbb{R}_+\ni t\longmapsto u(Z_t,t)\mathbb{1}_{t<\gamma}$ is a bounded variational process. It is adapted to $\mathcal{F}^X$ since γ is an optional time of this filtration.

2) We have $\mathcal{F}^X_0 = \mathcal{F}^X_\gamma$. So for all t, the r.v. $u_0(X_0)\mathbb{1}_{t<\gamma}$ is $\mathcal{F}^X_t$-measurable. Since $\mathcal{F}^X_t = \sigma(X_t)$, we get

$$\begin{aligned} u(Z_t,t)\mathbb{1}_{\gamma\leq t} &= u(X_t,t)\mathbb{1}_{\gamma\leq t} = \underbrace{E\left[u_0(X_0)|X_t\right]}_{M_t}\mathbb{1}_{\gamma\leq t} \\ &= M_t - E\left[u_0(X_0)\mathbb{1}_{t<\gamma}|X_t\right] = M_t - u_0(X_0)\mathbb{1}_{t<\gamma}. \end{aligned}$$

Then for all t, $u(Z_t,t) = \left[u(Z_t,t)-u_0(X_0)\right]\mathbb{1}_{t<\gamma} + M_t$.

3) Same proof as previous, using the fact that $\mathcal{F}^Z_0 = \mathcal{F}^Z_\gamma$ and $E\left[u_0(X_0)|X_t\right]\mathbb{1}_{\gamma\leq t} = E\left[u_0(X_0)|Z_t\right]\mathbb{1}_{\gamma\leq t}$ (lemma 3.4)

4) Simple application of lemma 3.3. For all t,

$$\begin{aligned} u(Z_t,t)\mathbb{1}_{\gamma\leq t} &= u(X_t,t)\mathbb{1}_{\gamma\leq t} = E\left[u_0(X_0)|X_t\right]\mathbb{1}_{\gamma\leq t} = \underbrace{E\left[u_0(X_0)|Z_t\right]}_{M_t}\mathbb{1}_{\gamma\leq t} \\ &= M_{\gamma\vee t} - \Delta_\gamma\mathbb{1}_{t<\gamma} - M^-_\gamma\mathbb{1}_{t<\gamma} \end{aligned}$$

with $\Delta_\gamma = M_\gamma - M^-_\gamma$.

Remark that assertion 3) is a consequence of 4). Indeed, if $\gamma = \tau(Z_0)$, then $\mathcal{F}^Z_0 = \mathcal{F}^Z_\gamma$ (lemma 3.4). So M^-_γ and M_γ are $\mathcal{F}^Z_\gamma$ measurable and the processes $t\longmapsto M_{\gamma\vee t}$, $M_\gamma\mathbb{1}_{t<\gamma}$, $\Delta_\gamma\mathbb{1}_{t<\gamma}$ are adapted to $\mathcal{F}^Z$. Thus, the process $M_{\gamma\vee t} - \Delta_\gamma\mathbb{1}_{t<\gamma}$ is a backward martingale of $\mathcal{F}^Z$. Hence the process $t\longmapsto u(Z_{t,},t)$ is a semi-martingale of $\mathcal{F}^Z$.

In fact, $, M_\gamma\mathbb{1}_{t<\gamma} =, M_0\mathbb{1}_{t<\gamma} =, M^-_\gamma\mathbb{1}_{t<\gamma}$. So the martingale part is M.

Now we precise, under more general assumptions, when the velocity of turbulence is a martingale.

c) *Martingales and soft turbulence*

In this part, we show that the martingality of the velocity turbulence implies that all mass of any turbulent interval is concentrated in at most one point

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(*single turbulent point*). Let $\mathcal{T}$ be the set of turbulent intervals which are not reduced to single points.

d) Corollary (Turbulence martingales and prime-delta-shocks)

1. *The process* $t \longmapsto u(Z_t, t)$ *is a martingale of* $\mathcal{F}^X$ *iff a.s.* $Z \equiv X$.
2. *Suppose that* γ *is an optional time of* $\mathcal{F}^Z$ *(which is effectively the case when* $\mathcal{S}$ *is an interval). The process* $t \longmapsto u(Z_t, t)$ *is a martingale of* $\mathcal{F}^X$ *iff a.s.* $Z_0 = E[X_0|Z_0]$. *Furthermore, if* $A_i = \Omega$, *then a.s.* $Z \equiv Z^i \equiv X$.

The following describes the turbulent intervals and clusters when the velocity of their borders are martingales.

e) Proposition

If $Z_0 = X_0$ *a.s., then* $\mathcal{T}$ *is at most countable and the interior of all turbulent interval is a vacuum.*

1. *Case* $Z \equiv Z^3$ $(A_3 = \Omega)$: *we have a.e.* $Z^3 \equiv Z^1 \equiv X$ *and* $Z^2 \equiv Z^4$.

$$\forall\, [\alpha,\, \beta] \in \mathcal{T},\ \mu_0(]\alpha,\, \beta]) = 0; \qquad P(Z_0^3 \neq (Z_0^4) = \sum_{[\alpha,\,\beta]\in\mathcal{T}} \mu_0(\{\alpha\})$$

2. *Case* $Z \equiv Z^4$ $(A_4 = \Omega)$: *we have a.e.* $Z^4 \equiv Z^2 \equiv X$ *and* $Z^1 \equiv Z^3$.

$$\forall\, [\alpha,\, \beta] \in \mathcal{T},\ \mu_0([\alpha,\, \beta[) = 0; \qquad P(Z_0^3 \neq (Z_0^4) = \sum_{[\alpha,\,\beta]\in\mathcal{T}} \mu_0(\{\beta\})$$

   any turbulent interval is also a cluster at turbulent time.

3. *Case* $A_1 = A_2 = \emptyset$: *we have a.e.* $Z^1 \mathbb{1}_{A_3} \equiv Z^2 \mathbb{1}_{A_3}$ *and* $Z^2 \mathbb{1}_{A_4} \equiv Z^4{}_{A_4}$.

$$\forall\, [\alpha,\, \beta] \in \mathcal{T},\ \mu_0([\alpha,\, \beta]) = \mu_0(\{\alpha\}) + \mu_0(\{\beta\});$$

$$P(Z_0^3 \neq (Z_0^4) = \sum_{[\alpha,\,\beta]\in\mathcal{T}} [\mu_0(\{\alpha\}) + \mu_0(\{\beta\})].$$

4. *Case* $Z \equiv Z^1$ $(A_1 = \Omega)$: *we have a.e.* $Z^3 \equiv Z^1 \equiv X$ *and* $Z^2 \equiv Z^4$.

$$\forall\, [\alpha,\, \beta] \in \mathcal{T},\ \mu_0(]\alpha,\, \beta]) = 0; \qquad P(Z_0^1 \neq (Z_0^2) = \sum_{[\alpha,\,\beta]\in\mathcal{T}} \mu_0(\{\alpha\})$$

5. *Case* $Z \equiv Z^2$ $(A_2 = \Omega)$: *we have a.e.* $Z^4 \equiv Z^2 \equiv X$ *and* $Z^1 \equiv Z^3$.

$$\forall\, [\alpha,\, \beta] \in \mathcal{T},\ \mu_0([\alpha,\, \beta[) = 0; \qquad P(Z_0^1 \neq (Z_0^2) = \sum_{[\alpha,\,\beta]\in\mathcal{T}} \mu_0(\{\beta\})$$

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Proof: Let us study each semi-martingale.

1. For $Z = Z^3$: A necessary condition is that $E\left[u_0(Z_0^3)\right] = E\left[u(Z_\gamma^3)\right] = E\left[u(X_\gamma, \gamma)\right] = E\left[u_0(X_0)\right]$. But $X_\gamma = X_0 + \gamma u_0(X_0) \leq Z_0 + \gamma u_0(Z_0)$ and $Z_0^3 \mathbb{1}_{\gamma>0} = X_0 \mathbb{1}_{\gamma>0}$. Then $\gamma^{-1}(X_0 - Z_0^3)\mathbb{1}_{\gamma>0} = u_0(Z_0^3) - u_0(X_0) \geq 0$ and $E\left[\gamma^{-1}(X_0 - Z_0^3)\mathbb{1}_{\gamma>0}\right] = 0$. So $X_0 = Z_0^3 = Z_0^1$ a.e. Thus $X \equiv Z^3 \equiv Z^1$ a.e.

   In the other hand, we have a.e. $u_0(Z_0^3) \geq u(X_\gamma, \gamma)$ and $E\left[u_0(Z_0^3) - u(X_\gamma, \gamma)\right] = 0$. So $u_0(Z_0^3) = u(X_\gamma, \gamma)$ a.e.
   Now we show that $Z_0^2 \equiv Z_0^4$. If $Z_0^3 = Z_0^4$, then $Z_0^2 = Z_0^4$. If $Z_0^3 \neq Z_0^4$, then $\exists\, \alpha < \beta$ s.t. $[Z_0^3, Z_0^4] = [\alpha, \beta]$. If moreover $\mu_0\left([\alpha, \beta]\right) = 0$, then $[\alpha, \beta] \in \mathcal{T}$ and $Z_0^2 = Z_0^4 = \beta$. If $\mu_0\left([\alpha, \beta]\right) > 0$, then

$$u_0(\alpha) = u_0(Z_0) = u(X_\gamma, \gamma) = \frac{\int_{[\alpha, \beta]} u_0(x)\mu_0(\mathrm{d}x)}{\mu_0\left([\alpha, \beta]\right)}.$$

   From corollary 2.3, we get $\mu_0\left([\alpha, \beta]\right) = 0$ and $[\alpha, \beta] \in \mathcal{T}$. And again $Z_0^2 = Z_0^4 = \beta$. Thus a.e. : $Z_0^2 = Z_0^4$ and $Z_0^2 \equiv Z_0^4$.

   Moreover $\{Z_0^3 \neq Z_0^4\} \subset \underset{[\alpha, \beta] \in \mathcal{T}}{\cup} \{\alpha \leq X_0 \leq \beta\}$. Conversely, if $[\alpha, \beta] \in \mathcal{T}$, $\exists\, [\alpha', \beta'] = [Z_0^3, Z_0^4] \supset [\alpha, \beta]$, with $\mu_0(]\alpha', \beta'[) = 0$. This implies $[\alpha, \beta] = [\alpha', \beta']$. So $\{Z_0^3 \neq Z_0^4\} = \underset{[\alpha, \beta] \in \mathcal{T}}{\cup} \{\alpha \leq X_0 \leq \beta\}$ and $\mu_0(]\alpha, \beta[) = 0$ for all $[\alpha, \beta] \in \mathcal{T}$. But the set of vacuums is at most countable. So is $\mathcal{T}$. We then get the result $P\left(\{Z_0^3 \neq Z_0^4\}\right)$.

2. For $Z = Z^4$: Analogous to previous case.

3. For $Z = Y$: The process $t \mapsto u(Y_t, t)$ is a martingale if only if its bounded variational part vanishes :

$$a.e\ \left[u(Y_t, t) - u(Y_\gamma, \gamma)\right] \mathbb{1}_{t<\gamma}, \ \forall\, t.$$

   This equivalent to $u_0(Y_0) = u(Y_t, t) = u(Y_\gamma, \gamma) = u(X_\gamma, \gamma)$ for all $t < \gamma$. Then a.e.

$$\tau(Y_0) = \gamma, \qquad u_0(Y_0) = u(X_\gamma, \gamma), \qquad Y_0 + t\gamma u_0(Y_0) = X_0 + t\gamma u_0(X_0).$$

   If $Z_0^3 \neq Z_0^4$, then $\exists \alpha < \beta$ s.t. $[Z_0^3, Z_0^4] = [\alpha, \beta]$ and $Y_0 = \alpha$ or $Y_0 = \beta$. If moreover $\mu_0([\alpha, \beta]) > 0$, then

$$u_0(\alpha) = u_0(Y_0) = u(X_\gamma, \gamma) = \frac{\int_{[\alpha, \beta]} u_0(x)\mu_0(\mathrm{d}x)}{\mu_0\left([\alpha, \beta]\right)}.$$

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In this case, If $Y_0 = \alpha$, then from the corollary 2.3, we get $\mu_0([\alpha, \beta]) = 0$ and $Y_0 = Z_0^1 = Z_0^3 = \alpha$. Then $P(Y_0 = \alpha \neq X_0) = \mu_0(]\alpha, \beta]) = 0$. In the same way, if $Y_0 = \beta$, we get $Y_0 = Z_0^2 = Z_0^4 = \beta$ and $P(Y_0 = \beta \neq X_0) = \mu_0([\alpha, \beta[) = 0$. In any case, $[\alpha, \beta] \in \mathcal{T}$ and $P(Y_0 \neq X_0,\, \alpha \leq X_0 \leq \beta) = 0$.

We conclude that $\mathcal{T}$ is at most countable and

$$\begin{array}{ccc} \forall\,[\alpha,\,\beta] \in \mathcal{T} & , & \mu_0([\alpha,\,\beta]) = \max\left(\mu_0(\{\alpha\}),\, \mu_0(\{\beta\})\right); \\ \left\{Z_0^3 \neq Z_0^4\right\} & = & \bigcup_{[\alpha,\,\beta]\in\mathcal{T}} \{\alpha \leq X_0 \leq \beta\} \end{array}$$

This give the results.

4. For $Z = Z^1$: $E[u_0(Z_0^1)] = E[u(Z_\gamma^1, \gamma)] = E[u_0(X_0)]$. But $u_0(Z_0^1) - u_0(X_0) = \gamma^{-1}(X_0 - Z_0^1)\mathbb{1}_{0<\gamma} \geq 0$. Then a.e. : $Z_0^1 = X_0$ and $Z_0^1 \equiv X_0$. Furthermore,

$$\left\{Z_0^2 \neq Z_0^1\right\} = \bigcup_{\substack{[\alpha,\,\beta]\in\mathcal{T} \\ \alpha<\beta}} \{\alpha \leq X_0 \leq \beta\}$$

and for all $[\alpha,\,\beta] \in \mathcal{T}$, we have

$$\mu_0(]\alpha,\,\beta]) = P(\alpha < X_0 \leq \beta) \leq P(Z_0^1 \neq X_0) = 0.$$

So $\mathcal{T}$ is at most countable and we get the result for $P(Z_0^2 \neq Z_0^1)$.

5. $Z = Z^4$: Analogous to previous case.

Now, we are interested in the conservation laws.

IV. Conservation Laws

In this section, we investigate if a process of type of Z can provide solutions to the scalar conservation law

$$\partial_t M + \partial_x\left(A(M)\right) = 0 \tag{12}$$

and to the pressure-less gas system

$$\begin{cases} \partial_t(\mu) + \partial_x(u\mu) = 0 \\ \partial_t(u\mu) + \partial_x(u^2\mu) = 0 \\ \mu_t \to \mu_0, \qquad u(\cdot,\,t)\mu_t \to u_0\mu_0 \; weakly\; as\; t \to 0 \end{cases} \tag{13}$$

It is well known, from the sticky particles model, that the mass distribution μ_t of the matter and their velocity functions $u(\cdot, t)$ provide a weak solution (in the sense of distributions) to the system (13). The first line of

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(13) is usually called *conservation law* of mass and the second is a conservation law of *momentum*. Moreover, the couple c.d.f and the momentum function provide an entropy solution to (12). Precisely, $\forall\,(x,\,t)\ \in \mathbb{R}\times\mathbb{R}_+$, $M(x,t)=\mu_t(]-\infty,x])$

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$$\forall\, m\in(0,1),\qquad A(m)=\int_0^m u_0(M_0^{-1}(z))dz, \tag{14}$$

where $M_0\ =\ M(\cdot,0)$. The equation (12) is *conservation law of mass and momentum*. Can we have the same thing for the function $N\ :\ (x,t)\ \mapsto P(Z_t\leq x)$ with the same flux (14) ?

a) Proposition

Consider the real function

$$v_0\ :\ a\mapsto\frac{\int_{Z_0=a}u_0(X_0)\mathrm{d}P}{P(Z_0=a)}=E\left[u_0(X_0)|Z_0=a\right]. \tag{15}$$

The couple $(N,\,A)$ *is a weak solution of the conservation law (12) if only if* Z *coincides with the sticky particles process defined from* $(N_0,\,v_0)$.

Before the proof, let us describe what happens in our investigation. From the point of view of the matter, our investigation consists in a change of distributions. We recall that the paths of Z are "extracted" from significant paths of X on which rise turbulences. The extraction procedure redistributes the mass. If τ is constant on $[a,\,b]$ and $[\alpha,\,\beta]$ is the cluster which contains $[a,\,b]$ at time $\tau(a)$, one of the four particles α, a, b or β are extracted. We call them "turbulent particles". All the mass of $[a,\,b]$ is initially re-affected to these particles. In order to expect the preservation of the conservation law, one can also re-affect the momentum as follows. First remark that each event A_i of section 3.1 is of type "$X_0\in\ E_i$".

- The mass $\mu_0\left([a,\,b]\cap E_3\right)$ and the momentum $\int_{[a,\,b]\cap E_3}u_0(x)d\mu_0(x)$ are affected to α.
- The mass $\mu_0\left([a,\,b]\cap E_1\right)$ and the momentum $\int_{[a,\,b]\cap E_1}u_0(x)d\mu_0(x)$ are affected to a.
- The mass $\mu_0\left([a,\,b]\cap E_2\right)$ and the momentum $\int_{[a,\,b]\cap E_2}u_0(x)d\mu_0(x)$ are affected to b.
- The mass $\mu_0\left([a,\,b]\cap E_4\right)$ and the momentum $\int_{[a,\,b]\cap E_4}u_0(x)d\mu_0(x)$ are affected to β.
- The total mass and total momentum of α and β are aggregations of the masses and momenta extracted from turbulent intervals inside $[\alpha,\,\beta]$.

Algorithm : Extraction along the time and aggregation of mass and momentum to α *(resp.* β*) until it is hinted from the left (resp. the right).*

The momentum transferred to turbulent particles can also be computed from the flux A (14) and c.d.f $N_0 := N(\cdot,\,0)$ of Z_0. Indeed, the turbulent particles in $[a,\,b]$ have the momentum

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$$
\begin{aligned}
\int_{\{a\leq Z_0\leq b\}} u_0(X_0)\mathrm{d}P &= \int_{\left\{a\leq N_0^{-1}\leq b\right\}} u_0(M_0^{-1}(z))\mathrm{d}z = \int_{N_0(a)}^{N_0(b)} u_0(M_0^{-1}(z))\mathrm{d}z \\
&= A(N_0(b)) - A(N_0(a^-))
\end{aligned}
$$

However, the velocity function induced by this momentum is not the correct one (u_0) for the real dynamics of Z. Indeed, each turbulent particle of initial position a' received the mass $P(Z_0 = a')$ and the momentum $\int_{Z_0=a'} u_0(X_0)\mathrm{d}P$. This induces the velocity

$$
\frac{\int_{Z_0=a'} u_0(X_0)\mathrm{d}P}{P(Z_0=a')} = E\left[u_0(X_0)|Z_0=a'\right] = v_0(a')
$$

So, $A(N_0(b)) - A(N_0(a^-)) = \int_{N_0(a)}^{N_0(b)} u_0(N_0^{-1}(z))\mathrm{d}z$ is the momentum of another sticky particles dynamics, the one from $(N_0,\, v_0)$.

Proof of proposition 4.1: Such a weak solution is an entropy solution which is unique once imposed the initial datum N_0. Let   be the flow constructed from $(N_0,\, v_0)$ and define $N_t = N(\cdot,\, t)$. One has $N_t^{-1} = \quad (N_0^{-1},\, t)$ for all t. Since $Z_t = \phi(Z_0,\, t)$, one has also $N_t^{-1} = \phi(N_0^{-1},\, t)$. So $\quad (\cdot,\, t) = \phi(\cdot,\, t)$ on the support of the law of Z_0, and one gets $Z_t = \quad (Z_0,\, t)$ for all t.

Now we consider the momentum which corresponds to the dynamics of Z, the function $B\, :\, (0,\, 1) \ni\mapsto \int_0^m v_0(N_0^{-1}(z))\mathrm{d}Z$. It is also the momentum function of the sticky particles dynamics defined from $(N_0,\, v_0)$.

b) Proposition

Suppose that $\gamma = \tau\,(Z_0)$ a.e. We have

$$
\partial_t N + \partial_x\left(A(N)\right) = -\partial_x\left(C(N,t)\right) \tag{16}
$$

$$
\partial_t N + \partial_x\left(B(N)\right) = \partial_x\left(D(N,t)\right) \tag{17}
$$

with, $\forall\,(m,t)\in(0,1)\times\mathbb{R}_+$,

$$
C(m,t) = \int_0^m \Delta_0\left(N_0^{-1}(z)\right)\mathbb{1}_{t<\tau\left(N_0^{-1}(z)\right)}dz
$$

$$
D(m,t) = \int_0^m \Delta_0\left(N_0^{-1}(z)\right)\mathbb{1}_{t\geq\tau\left(N_0^{-1}(z)\right)}dz
$$

and $\Delta_0 = u_0 - v_0$.

Surprisingly, as shown in the sequel, these results lead to the homogeneous conservation law of the momentum. For all t, let ν_t be the distribution of Z_t, i.e. $\nu_t(B) = P(Z_t\in\, B)$ for all Borel set B.

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c) Gas system with turbulence force

For all t, let ν_t be the distribution of Z_t, i.e. $\nu_t(B) = P(Z_t \in B)$ for all Borel set B.

d) Corollary

Let us define $\delta(x,t) = E[\Delta_0(Z_0)|Z_\gamma = x, \gamma = t]$ and consider the law $P_{Z_t,\gamma}$ of (Z_t, γ). If $\gamma = \tau(Z_0)$ a.e, then we have

$$\begin{cases} \partial_t(\nu) + \partial_x(u\nu) = 0 \\ \partial_t(u\nu) + \partial_x(u^2\nu) = -\delta P_{Z_\gamma,\gamma} \\ \nu_t \to \mu_0, \qquad u(\cdot,t)\nu_t \to u_0\nu_0 \text{ weakly as } t \to 0 \end{cases} \tag{18}$$

The couple (ν, u) is thus a weak solution of a pressure gas system of initial datum is (ν_0, u_0).

Proof of proposition 4.2: $u(Z_t,t) = E\left[v_0(Z_0) + \Delta_0(Z_0)\mathbb{1}_{t<\tau(Z_0)}|Z_t\right]$. Using $w(Z_0,t) := v_0(Z_0) + \Delta_0(Z_0)\mathbb{1}_{t<\tau(Z_0)}$, we have, for any test function f on $\mathbb{R}\times\mathbb{R}_+^*$:

$$\int\int f_t(x,t)N(x,t)\mathrm{d}t\mathrm{d}x = E\int\int f_t(x,t)H(x-Z_t)\mathrm{d}t\mathrm{d}x$$

$$= E\int\int f(x,t)u(Z_t,t)\delta_{Z_t}(\mathrm{d}x)\mathrm{d}t = E\int f(Z_t,t)u(Z_t,t)\mathrm{d}t$$

$$= E\int f(Z_t,t)w(Z_0,t)\mathrm{d}t = -E\int\int f_x(x,t)H(x-Z_t)w(Z_0,t)\mathrm{d}t\mathrm{d}x$$

$$= -\int\int f_x(x,t)E\left[H(x-Z_t)w(Z_0,t)\right]\mathrm{d}t\mathrm{d}x$$

and

$$E\left[H(x-Z_t)w(Z_0,t)\right] = \int_0^{N(x,t)} w(N_0^{-1}(z))\mathrm{d}z$$

$$= A(N(x,t)) + C(N(x,t),t) = B(N(x,t)) - D(N(x,t),t).$$

Proof of corollary 4.3

$$E\left[H(x-Z_t)w(Z_0,t)\right] = E\left[H(x-Z_t)u(Z_t,t)\right] = \int_{-\infty}^{x} u(y,t)\mathrm{d}\nu_t(y),$$

$$\partial_x\left[A(N(x,t)) + C(N(x,t),t)\right] = u(x,t)\mathrm{d}\nu_t(x).$$

From previous proposition, one gets $\partial_t M + u(x,t)\mathrm{d}\nu_t(x) = 0$. Then, in order to have the first equation of gas system, use the fact that

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$$\partial_t \partial_x M = \partial_x \partial_t M.$$

It remains the last equation. For any test function f on $\mathbb{R}_+^*$ and any test function g on $\mathbb{R}$,

$$\int\int f'(t)g(x)u(x,\, t)\mathrm{d}\nu_t(x)\mathrm{d}t = E\int f'(t)g(Z_t)u(Z_t,\, t)\mathrm{d}t$$

$$= E\int f'(t)g(Z_t)v_0(Z_0)\mathrm{d}t + E\int f'(t)g(Z_t)\Delta_0(Z_0)\mathbb{1}_{t<\tau(Z_0)}\mathrm{d}t$$

$$= -E\int f'(t)g(Z_t)u(Z_t,\, t)v_0(Z_0)\mathrm{d}t$$

$$+E\left[f(\gamma)g(Z_\gamma)\Delta_0(Z_0)\right] - E\int\int f'(t)g(Z_t)\Delta_0(Z_0)\mathbb{1}_{t<\tau(Z_0)}\mathrm{d}t$$

$$= -E\int f(t)g'(Z_t)u^2(Z_t,\, t)\mathrm{d}t + E\left[f(\gamma)g(Z_\gamma)\Delta_0(Z_0)\right]$$

This ends the proof.

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Acknowledgments

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The following is the official style and template developed for publication of a research paper. Authors are not required to follow this style during the submission of the paper. It is just for reference purposes.

Manuscript Style Instruction (Optional)

- Microsoft Word Document Setting Instructions.
- Font type of all text should be Swis721 Lt BT.
- Page size: 8.27" x 11'", left margin: 0.65, right margin: 0.65, bottom margin: 0.75.
- Paper title should be in one column of font size 24.
- Author name in font size of 11 in one column.
- Abstract: font size 9 with the word "Abstract" in bold italics.
- Main text: font size 10 with two justified columns.
- Two columns with equal column width of 3.38 and spacing of 0.2.
- First character must be three lines drop-capped.
- The paragraph before spacing of 1 pt and after of 0 pt.
- Line spacing of 1 pt.
- Large images must be in one column.
- The names of first main headings (Heading 1) must be in Roman font, capital letters, and font size of 10.
- The names of second main headings (Heading 2) must not include numbers and must be in italics with a font size of 10.

Structure and Format of Manuscript

The recommended size of an original research paper is under 15,000 words and review papers under 7,000 words. Research articles should be less than 10,000 words. Research papers are usually longer than review papers. Review papers are reports of significant research (typically less than 7,000 words, including tables, figures, and references)

A research paper must include:

a) A title which should be relevant to the theme of the paper.
b) A summary, known as an abstract (less than 150 words), containing the major results and conclusions.
c) Up to 10 keywords that precisely identify the paper's subject, purpose, and focus.
d) An introduction, giving fundamental background objectives.
e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition, sources of information must be given, and numerical methods must be specified by reference.
f) Results which should be presented concisely by well-designed tables and figures.
g) Suitable statistical data should also be given.
h) All data must have been gathered with attention to numerical detail in the planning stage.

Design has been recognized to be essential to experiments for a considerable time, and the editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned unrefereed.

i) Discussion should cover implications and consequences and not just recapitulate the results; conclusions should also be summarized.
j) There should be brief acknowledgments.
k) There ought to be references in the conventional format. Global Journals recommends APA format.

Authors should carefully consider the preparation of papers to ensure that they communicate effectively. Papers are much more likely to be accepted if they are carefully designed and laid out, contain few or no errors, are summarizing, and follow instructions. They will also be published with much fewer delays than those that require much technical and editorial correction.

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Format Structure

It is necessary that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

All manuscripts submitted to Global Journals should include:

Title

The title page must carry an informative title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) where the work was carried out.

Author details

The full postal address of any related author(s) must be specified.

Abstract

The abstract is the foundation of the research paper. It should be clear and concise and must contain the objective of the paper and inferences drawn. It is advised to not include big mathematical equations or complicated jargon.

Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Keywords

A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy: planning of a list of possible keywords and phrases to try.

Choice of the main keywords is the first tool of writing a research paper. Research paper writing is an art. Keyword search should be as strategic as possible.

One should start brainstorming lists of potential keywords before even beginning searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in a research paper?" Then consider synonyms for the important words.

It may take the discovery of only one important paper to steer in the right keyword direction because, in most databases, the keywords under which a research paper is abstracted are listed with the paper.

Numerical Methods

Numerical methods used should be transparent and, where appropriate, supported by references.

Abbreviations

Authors must list all the abbreviations used in the paper at the end of the paper or in a separate table before using them.

Formulas and equations

Authors are advised to submit any mathematical equation using either MathJax, KaTeX, or LaTeX, or in a very high-quality image.

Tables, Figures, and Figure Legends

Tables: Tables should be cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g., Table 4, a self-explanatory caption, and be on a separate sheet. Authors must submit tables in an editable format and not as images. References to these tables (if any) must be mentioned accurately.

Figures

Figures are supposed to be submitted as separate files. Always include a citation in the text for each figure using Arabic numbers, e.g., Fig. 4. Artwork must be submitted online in vector electronic form or by emailing it.

Preparation of Eletronic Figures for Publication

Although low-quality images are sufficient for review purposes, print publication requires high-quality images to prevent the final product being blurred or fuzzy. Submit (possibly by e-mail) EPS (line art) or TIFF (halftone/ photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Avoid using pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings). Please give the data for figures in black and white or submit a Color Work Agreement form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

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Tips for Writing a Good Quality Science Frontier Research Paper

Techniques for writing a good quality Science Frontier Research paper:

1. Choosing the topic: In most cases, the topic is selected by the interests of the author, but it can also be suggested by the guides. You can have several topics, and then judge which you are most comfortable with. This may be done by asking several questions of yourself, like "Will I be able to carry out a search in this area? Will I find all necessary resources to accomplish the search? Will I be able to find all information in this field area?" If the answer to this type of question is "yes," then you ought to choose that topic. In most cases, you may have to conduct surveys and visit several places. Also, you might have to do a lot of work to find all the rises and falls of the various data on that subject. Sometimes, detailed information plays a vital role, instead of short information. Evaluators are human: The first thing to remember is that evaluators are also human beings. They are not only meant for rejecting a paper. They are here to evaluate your paper. So present your best aspect.

2. Think like evaluators: If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

3. Ask your guides: If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

4. Use of computer is recommended: As you are doing research in the field of science frontier then this point is quite obvious. Use right software: Always use good quality software packages. If you are not capable of judging good software, then you can lose the quality of your paper unknowingly. There are various programs available to help you which you can get through the internet.

5. Use the internet for help: An excellent start for your paper is using Google. It is a wondrous search engine, where you can have your doubts resolved. You may also read some answers for the frequent question of how to write your research paper or find a model research paper. You can download books from the internet. If you have all the required books, place importance on reading, selecting, and analyzing the specified information. Then sketch out your research paper. Use big pictures: You may use encyclopedias like Wikipedia to get pictures with the best resolution. At Global Journals, you should strictly follow here.

6. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

7. Revise what you wrote: When you write anything, always read it, summarize it, and then finalize it.

8. Make every effort: Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

9. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several unnecessary diagrams will degrade the quality of your paper by creating a hodgepodge. So always try to include diagrams which were made by you to improve the readability of your paper. Use of direct quotes: When you do research relevant to literature, history, or current affairs, then use of quotes becomes essential, but if the study is relevant to science, use of quotes is not preferable.

10. Use proper verb tense: Use proper verb tenses in your paper. Use past tense to present those events that have happened. Use present tense to indicate events that are going on. Use future tense to indicate events that will happen in the future. Use of wrong tenses will confuse the evaluator. Avoid sentences that are incomplete.

11. Pick a good study spot: Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

12. Know what you know: Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

13. Use good grammar: Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice.

Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

14. Arrangement of information: Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. Multitasking in research is not good: Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. Never copy others' work: Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. Go to seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. Refresh your mind after intervals: Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

20. Think technically: Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

21. Adding unnecessary information: Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

22. Report concluded results: Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

The introduction: This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear: Adhere to recommended page limits.

Mistakes to avoid:

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

Title page:

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

- o Single section and succinct.
- o An outline of the job done is always written in past tense.
- o Concentrate on shortening results—limit background information to a verdict or two.
- o Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

The following approach can create a valuable beginning:

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

Materials may be reported in part of a section or else they may be recognized along with your measures.

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- o You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- o Give details of all of your remarks as much as possible, focusing on mechanisms.
- o Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- o One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- o Recommendations for detailed papers will offer supplementary suggestions.

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When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

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