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Modelling the Plant Silicon Hydraulic Capacitance and Passive Uptake under Drought and Saline Conditions

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Strictly as per the compliance and regulations of:

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I. Introduction

The soil water hydraulic capacitance, β, a tank like the plant water reservoir, is being controlled by signals, valves like the switches. There are three types of signaling devices control the mass flow of water and nutrients, Geo, Bio, and weather controlled. The Darcian flow of water carries with it nutrients toward root in accordance with their concentration. The property of soil hydraulic capacitance determines the type of root water and nutrient uptake under drought and saline conditions in accordance with stress, strain, and weather-controlled forces. It was first discovered by the author when modeling the wheat root water uptake under drought and saline stress conditions [1]. The latter signaling devices not only control soil water hydraulic capacitance and uptake but also silicon passive uptake in such a way to make it equal the product of multiplying soil stress index with a modified value of soil water hydraulic capacitance, silicon like (β_Si).

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Treating plants with silica products was the common managerial practice used for enhancing uptake and plantation under the extreme drought and saline conditions. Soil moisture reservoir represents the plant available water affects the recharge of aquifers capacitors. It is also a vital source of upward atmospheric humidity and accordingly, the downward precipitation. The soil moisture reservoir represents the plant available water affects the recharge of aquifers capacitors. It is also a vital source of upward atmospheric humidity and accordingly, the downward precipitation. Soil moisture contributes to regulating the global energy balance of terrestrial ecosystems, controlling soil temperature, air humidity, and surface albedo [2]. Moisture deficit causes drought attack. By the year 2030, the global climate change will be changing the half of the world's population to be lived in areas of water scarcity. In some arid and semi-arid regions, it will displace up to 24: 700 million people due to water stress problem [3].United Nations (2010) chose the decade 2010–2020 for the fight against desertification. This is because the following reasons: 41.3% of the earth surfaces are drylands, approximately 44% of all the world's cultivated area located in the dry lands. The desertification, land degradation may cause severe dust storms, which can lead to respiratory diseases and other health problems [4]. Drought threats more than 1 billion people from the 2.1 billion people who live in the dry regions and responsible for an income loss of US$ 42 billion per year. In the most dried year, the cropping season 2009/2010, most of the world cropland faced severe drought waves which affected the crop productivity.

Soil salinization can be enhanced by the climatic changes, rising temperatures, increase soil evaporation, and crop water requirements. The coastal regions are vulnerable to see water intrusion, thereby deteriorating the coastal aquifers. The extreme storms and tsunamis waves can flow overland resulting in saltwater infiltration into soils contaminating soil and water resources [5].The problems of arid zones and their accompying results, salinization and desertification, are highlighten end in the countries where no renewable water resources exist such as Saudia Arabia [6]and Kwait [7].

Robinson, D.et al. [2] defined soil hydraulic function as the ability of soils to store water to serve the moisture pool that sustains the agro ecosystem system. They said that soil hydraulic functions are always estimated as static property, such as relating to hydraulic conductivity and soil water retention. They neglect the interaction between the agro-ecosysrem components which do their best either alone or in combination reducing the moisture deficit signal. The soil living organisms could change soil structural and accordingly the moisture deficit. As soil available water decreases due to drought and salinity stress conditions, the agro-ecosystem components, plant and soil, start to do on their relative extreme response functions. The relative extremes may be achieved by root through choosing the easiest bath to navigate soil system, categorizing the energy states of soil water, preferring the easiest available water to water, and following model of minimizing the consumed energy for combating stress conditions. The interaction between ecosystem components, plant and soil; due to drought cycles, the soil is shrinking its energy now to the half of its value at the wetting cycles to save plant's life and prevent the hydrological release. By limiting crown roots number per plant, plant causes a degree of response towards the drought stress, a compensatory growth in existing roots, which subsequently reaches deeper soil layers, categorizing the energy states of soil water seeking the easiest available water and nutrient for compensation. In tillering crops such as wheat, a drying soil was found to limit root growth at the top 30 cm using the hydraulic signal for increasing cell wall elastic modulus and maintaining turgidity while promoting root extension and growth into depth.

Estimating the root water uptake when using the stress form of modified Richard's equation, S= SSI*B, involves the term soil water hydraulic capacitance. SWHC represents the response of the soil moisture reservoir toward a certain plant water demand under a certain atmospheric condition. Soil profile consists of layered capacitors. Each of which is recharged during the precipitation, irrigation, capillarity, or water table rise forming the constant shaped soil moisture profile after moisture redistribution. The easy available water is being stressed by discharging the soil capacitors during the root water uptake creating the variably saturated zone. In between the sink/source terms of Richard's equation, the discharge/ recharge terms of soil capacitors or the root uptake, hydrological release terms, root navigates soil capacitors categorizing their energy states to select what has the easiest available water and bath to pass and water by exerting a minimum consumed energy. SHC depends on soil genesis, plant species, varieties, ages, adaptability, atmospheric conditions, and the high tensile strength of liquid phase. Under deficit irrigation and or the harsh extreme conditions of aridsols, SHC determines the types of water uptake in accordance with weathering induced stress- strain signaling devices [8]. The latter devices are being regulated for the sake of preventing the hydrological release and complete the life cycle.

It had been the big bang separated planet earth from star sun. The separated mass had cooled and the basic element had cocked in a hydrothermal environment to form minerals [9] (Figure 1)..Silicon and aluminum bonded with oxygen forming silica tetrahedron and alumina octahedron, the basic units of clay minerals with their distinguishable isomorphs substitution [10].

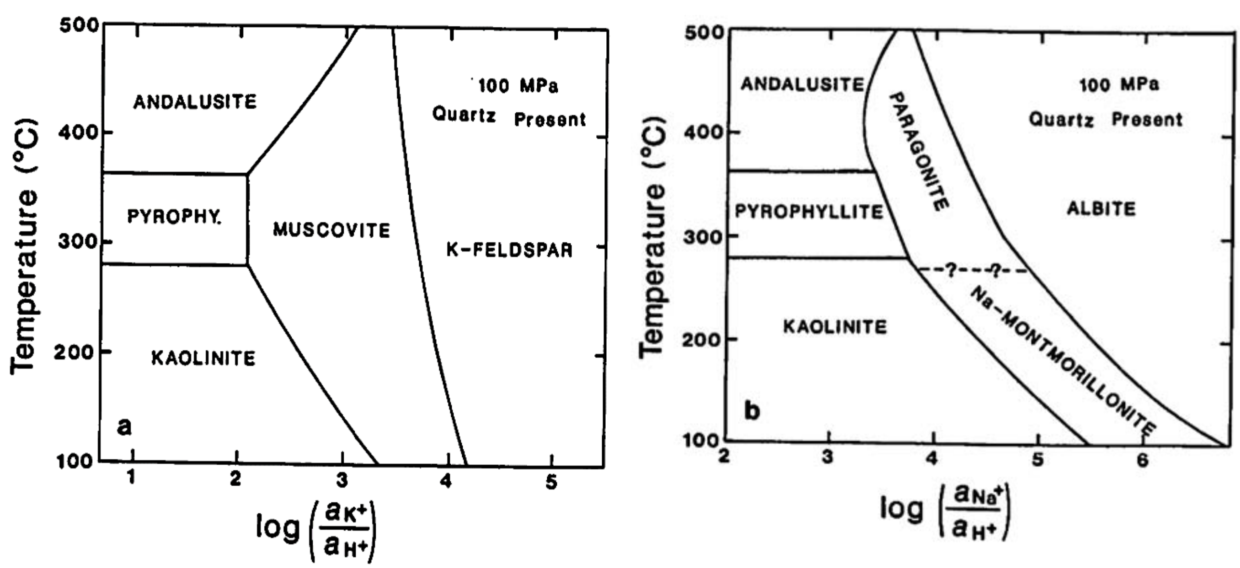

Figure (1): A: Temperature- Logarithmic Cation Activity Ratio Diagram in the System K_2O- AL_2O_3- SiO_2- H_2O at a Total Pressure of 100 MPa and Quartz Saturation. B:Temperature- Logarithmic Cation Activity Ratio Diagram in the System Na_2O- AL_2O_3- SiO_2- H_2O at a total Pressure of 100 MPa and Quartz Saturation [11]

The merciful behavior between silicon as a basic soil skeleton constituent and mankind as a clayey apped creature, under harsh extreme stress conditions, may be summarized as dust support dust in order to let the life continue [1]. Silicon the second abundant element in earth's crust, the beneficial functional nutrient element for human, animal, and plant is from the types of environmental managerial practices should have been taken into consideration when dealing with the natural abiotic stress of drought and salinity due to global climatic changes or artificial one due to irrigation deficit scenario because of the three incoming reasons:

1. Since the geo-signal is being induced by more compacted electric double layer, hysteric behavior and the Albert Ainkhtien's relativity of clayey lattice [8].
2. As the bio signal is being induced by osmoregulation strain and turgidity strain[1].
3. As the weather controlled signaling device is being induced by canopy evaporative demand [1].

Silicon enhances the overall signaling devices to be in favorable states under the abiotic stress conditions. It deposites in tricombs enhancing its function in cooling leaves acting as antenna reflects the solar short incoming radiation.. Treating the sulfuric acid aerosols in the stratosphere with silica aerosols reduces significantly the short incoming radiation. Accordingly, increases the Mie scattering in the stratosphere. Accordingly, reducing the impact of global temperature rise by cooling the at mosphere in general and plant canopy in particular. However, the silicon are rosols is an effective managerial practices for albedo modification, particular technology should be adopted control the silicon radioactivity.

Silicon deposits in root enhancing the plant roots to be in a higher negative potential state for overcoming the total potential of soil water. The siliceous nutrition of plants is not only scientifically intriguing but also important in a world where more food will have to be wrung from a finite area of land especially for the deficit irrigation and partial root zone drying scenarios which will put crops under stress. [12and 14].

In the agro-lands of MENA countries with marine and lacustrine origins, the overexploitation of groundwater under the conditions of water scarcity and global climatic changes causes the intrusion of seawater [5]. As the saline water intrudes the variably saturated zone under a harsh extreme condition, the potentially guided Darian flow charges the capacitors from the bottom upwardly. Charging the capacitors in the electrical model [15] is different than that of the AMUN_SHC electrical model [1] as the former did not consider that water quality affects its uptake. For instance, the water salinity decreases the availability of irrigation water. AMUN_SHC model takes the concept of total energy states into consideration. Assessing the impact of specific ion toxicity has a certain value of salt index under abiotic stress conditions may be achieved by making SSI speciation. The latter speciation may be done in order to know the combined stress of drought, salinity and specific ion effects such as in the case of using potassium chloride instead of potassium sulfate fertilizers. SSI speciation, SSIS, is still under investigation. Moreover, SSIS needs to be validated when assessing the impact of saline contaminated groundwater with iron and manganese on plant growth under deficit irrigation scenarios.

The plant response under environmental abiotic stressed conditions could be discussed from the side of dynamics in plant roots and shoots used to minimize stress, reduce consumed energy, and maintain water and nutrient uptake [16 and17]. Firstly, plant enhances the uptake of nutrient element responsible for combating abiotic stress, silicon. Secondly, the beneficial functional element, silicon, interact positively with macro and micronutrient and stimulate the biophysical functions inside plant's tissues [12]. Thirdly, Silicon also appears to be a part of the osmoregulation within cells subjected to drought stress which enables the plant to uptake and transpirate more water for combating the stressed conditions [18]. Fourthly, the wheat's hydraulic signal reduces water loss via transpiration by decreasing leaf area index and increasing leaves rolling [19]. Fifthly, the adaptive root growth [20], the compensated root water uptake [21], and root hydraulic redistribution to cope with the heterogeneity in soil moisture regime [22]. Under such conditions,, It makes sense to butsilicon uptake in one side andother nutrients and water uptake in the other in order to complete the plant life and prevent the hydrological release.

The object of this research is to quantify the Darcian silicon uptake by changing the water uptake response function, S= (β, SSI), to be the silicon Darcian uptake response function, Si_Up= (β_Si, SSI).Silicon, the master element under such stress conditions, may be correlated with other nutrients and water uptake.

II. Material and Method

Whey is the foliar application of silicate fertilizers or sprinkler fertigation of silicate ions recommended under stress conditions?

The following points will clarify the reasons and limit the debates:

1. The fast supplying reservoir of the soluble form of silica ready for plant root uptake and translocation to do its best for combating the adverse impacts of global climatic changes provided 40.79, 55.9, and 54.58 ppm in the experimental plot 1, 2, and 3 respectively (Table 1).

2. Silicon which forms inner and outer sphere complexation may be desorped to support plant growth under stress conditions.
3. Ground application silicate fertilizers may refresh the soluble quick equilibrium between the up taken form and the soluble available form of silicates.
4. As silicon is the second abundant element in the earth's crust, the slowly dissociation of silicate minerals replenishes and compensates the depletion of the absorbed silicic acid in the rizosphere by silicon accumulator crops such as sugarcane and wheat.
5. As silicon is mainly absorbed by mass flow of water, the factors affecting soil water hydraulic capacitance and uptake may affect the silicon hydraulic capacitance and therefore, the passive silicon uptake. Accordingly, the highly energetic soil water due to drought and salinity, reduces water uptake and soil silicon passive uptake.
6. The foliar application of silicate fertilizers is recommended than ground one in the case of the abiotic stresses of drought and salinity due to its abundance in soil skeleton, slow release from silicate minerals, but less available due to the high negative total soil potential may reduce the water and soil silicon uptake or even the hydrological releases under the harsh extreme abiotic stress conditions.

$$AL_2Si_2O_5 + 2H^+ + 3H_20 \longrightarrow 2AL^{3+} + 2H_4SIO_4 \text{Log } K^0 = 15.12$$

$$AL_2Si_2O_5\,(OH)_4 + 6H^+ \longrightarrow 2AL_3^+ + 2H_4SIO_4 + H_20 \text{Log } K^0 = 5.45$$

As drought and salinity reduce the silicon uptake from soil solution, they were managed by foliar spraying of silica products as potassium silicates and sodium silicates in three concentrations 0.0, 30.6, and 40.8 ppm. All of them were foliar sprayed at ages of 45, 60, and 75 days from seed emergence at the early morning. The 6 treatment combinations were distributed in 3 salinity levels, 3.12, 4.82, and 5.12 dS/m (Table 1), in a split-split plot design with four replicates. Soil hydro physical properties and water flow parameters were estimated by HYDRUS- 1D (Vr. 4.17) at depth z dimension (Table 2). ET_C was calculated from meteological data [23]. The irrigation interval each 20: 25 day. The relation between soil stress index and plant stress index was developed [8]. The AMUN_SiHC was built and run to calculate silicon hydraulic capacitance, β_Si (Figure 2).All the read input parameter were taken, processed to output PSI, SSI, β, β_Si, and Si_Up, and by run the program [24].

AMUN_SHC Processing Code

- $\mathrm{SSI} = \frac{(1+(\alpha\Psi)^n)^m}{(1+(\alpha\psi_*)^n)^m} \qquad (m=1/n)$
- $\beta = \left(\frac{1}{\mathrm{SSI}(z,t)}\right)\left(\frac{1}{Z2-Z1}\left(k\frac{\Psi2-\Psi1}{z2-z1} - k\right) - c(h)\,\frac{\Psi2-\Psi1}{t2-t1}\right)$
- $\mathrm{Bsi} = c\left(\frac{1}{\mathrm{SSI}(z,t)}\right)\left(\frac{1}{Z2-Z1}\left(k\frac{\Psi2-\Psi1}{z2-z1} - k\right) - c(h)\,\frac{\Psi2-\Psi1}{t2-t1}\right)$
- $S(h, z, t) = \beta \,.\, SSI\,(h, z, t) \qquad \beta = Tp.\,\frac{PSI}{SSI}$
- $S_{si}(h, z, t) = \beta si \,.\, SSI\,(h, z, t) \qquad \beta_{si} = (Tp).\, c.\,\frac{PSI}{SSI}$
- $(PSI)^i_{j+1} = \left(\left(\begin{matrix}(SSI)^i_{j+1}\\ -(SSI)^i_j\end{matrix}\right)\Big|(SSI)^i_{j+1}\right)\left(KC\left(\frac{\sum_{j=1}^{j=n}(PSI)}{n}\right)\right)+\left(\frac{\sum_{j=1}^{j=n}(PSI)}{n}\right)$
- **Compensated Silicon Uptake =**

$$(c.\,\theta_V.\,\Delta z^2.\,h^*/Tp)\left[\left(\begin{matrix}(PSI)_n^{SSI+\Delta SSI}\\ -(PSI)_s^{SSI+\Delta SSI}\end{matrix}\right)+\left(\begin{matrix}-(PSI)_n^{SSI}\\ (PSI)_s^{SSI}\end{matrix}\right)\right]\left[\left(\begin{matrix}(Tc)_n^{h+\Delta h}\\ -(Tc)_s^{h+\Delta h}\end{matrix}\right)+\left(\begin{matrix}-(Tc)_n^{h}\\ (Tc)_s^{h}\end{matrix}\right)\right]$$

Figure 2: AMUN_SiHC Processing Code

Table 1: The Major Physical and Chemical Characteristics of the Soil and Irrigation Water of the Three Plots of the Experiment at Oraby Village, Maryout area Alexandria (2009/2010)

Characters	Plots and Irrigation Water			
	Plot 1	Plot 2	Plot 3	Irrigation Wt.
Sand (%)	17	23	15	
Silt(%)	37	38	38	
Clay (%)	46	39	47	
Texture	Clayey	Clayey L.	Clayey	
pH	8.35	8.4	8.31	7.6
ECe (mS/cm)	9.6	6.3	10.4	0.26
Sol. Si(ppm)	40.79	55.9	54.58	1.5
Sol. Na	6.8 (meq/l)	9.0 (meq/l)	6.37 (meq/l)	38.3 ppm
Sol. K	0.72 (meq/l)	0.89 (meq/l)	0.71	0.72 ppm

Table 2: Soil Hydro-Physical Properties According to HYDRUS 1D [25]

	θ_s (v/v)	θ_r (v/v)	Ks(cm/day)	α (1/cm)	n	m
Clayey Texture	0.38	0.068	4.8	0.008	1.09	0.083
Clayey Loam Texture	0.41	0.095	6.24	0.019	1.31	0.24

The earth nutritional alkaloids were determined as follow: K^{+}, Na^{+}, Mg^{+2}, and Ca^{+2} by the Atomic Absorption Spectrophotometer (Perkin Elmer 1100). Concentrations of K^{+}, Na^{+}, Mg^{+2}, and Ca^{+2} were calculated as percentage. Determination of silicon was performed based on the colorimetric determination of blue silicomolybdic acid procedure and measured using Perkin-Elmer Spectrophotometer at wavelength 820 nanometer and was calculated as percentage [26]. Nutrient uptake were calculated as kg per feddan. Regression analysis of nutrients uptake (Kg/Fed.) was performed according to Microsoft Excel 2010 (Table 3).SSI and available water content were calculated with depth and time respectively according to [8].

III. Results and Discussion

Figure 3 indicated that silicon hydraulic capacitance (SiHC) and therefore passive silicon uptake (Si_Up) decreases. Whereas the slightly saline soils have SiHC much more than saline ones and so as in the case of irrigation water. Accordingly, seawater intrusion diminishes the SiHC with the exception for the case of special soil, water, and plant managerial practices such as leaching and drainage for soil, magnetism for water, and silicon foliar application and gene transfer for the plant which use to raise SHC, Si_HC, and Si_Up in some degree of extend. In halophytes, PSI is always greater than SSI. Accordingly, SiHC and all the related parameter will be exceeded, according to the Tp and the opposite is true in sensitive plants. As silicon concentrates straw sap, it makes the root sap more strained. Therefore silicon allowed plants to withstand abiotic stress by shaking down the limits of actual stress plant roots may bear from the log phase until disappear. SiHC in control treatment is higher than in silicon treatments. The latter means that SSI is greater than PSI. Moreover, a little pulse of environmental abiotic stress will cause a greater negative response on crop yield without silica fertilization. Potassium silicate is better than sodium silicate in the latter manner because the soil under investigation is rich in clay mineral fixes potassium between its layers, At tapulgite (Figure3). As SiHC reveals the overall interaction between the ecosystem components the managerial practices affect the plant, soil and atmosphere and their interaction each to other may affect it. Treating the plant and soil with silica affects SiHC (Figure3).It will be possible to correlate the other nutrients to silicon because: The skeletal element, silicon is mainly up taken passively by root system. Silicon is a key nutrient element under the studied abiotic stress conditions.

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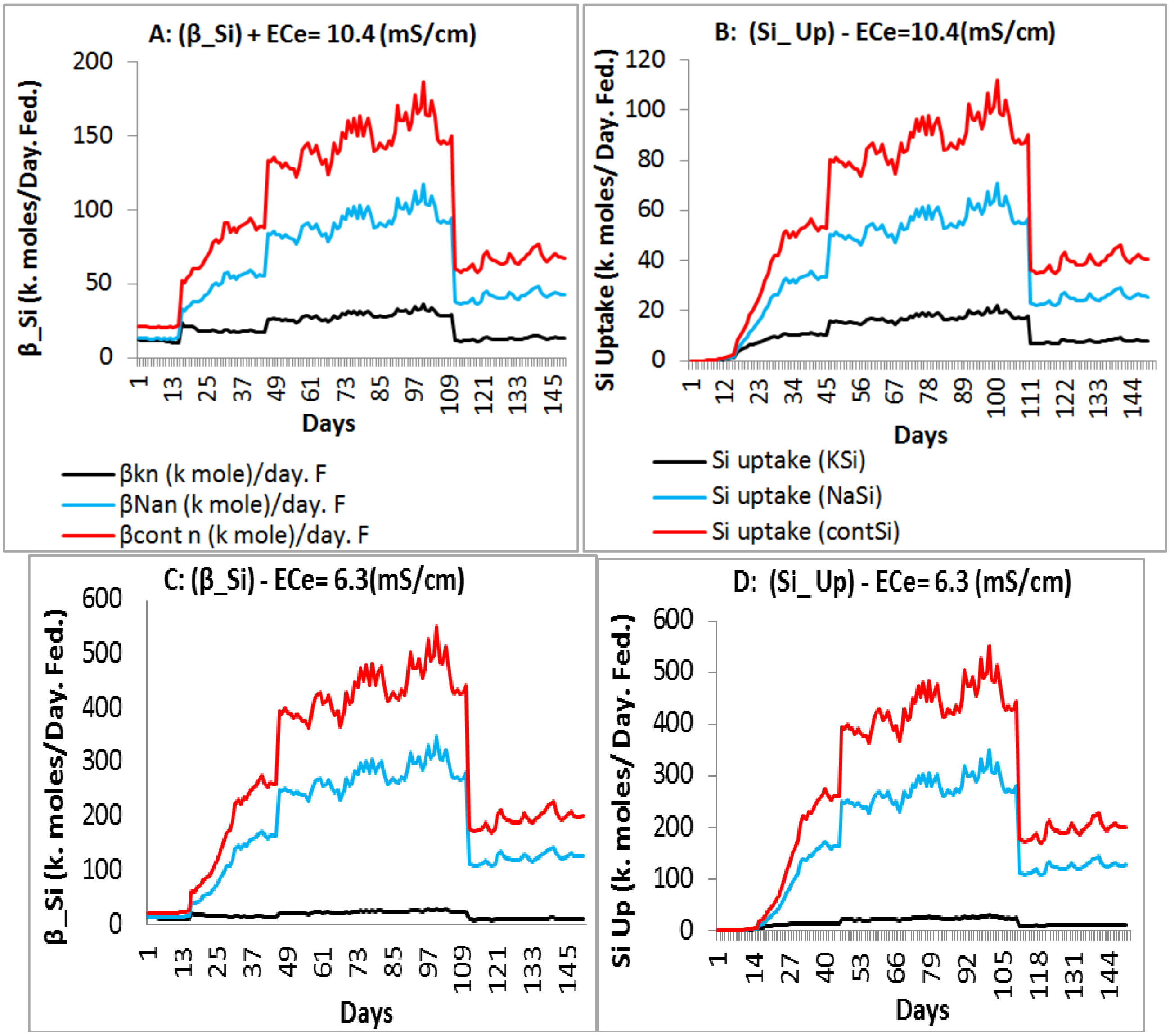

Figure 3: (A): Silicon Hydraulic Capacitance, (β_Si), for Saline Soil ECe= 10.4. (B): Silicon Passive Uptake, Si_ Up, for Saline Soil ECe= 10.4. (C): Silicon Hydraulic Capacitance, (β_Si), for Non-Saline Soil, ECe= 6.3.(D): Silicon Passive Uptake, Si_ Up, for Non-Saline Soil ECe= 6.3. The Unit of both β_Si and Si_Up is K. moles/ Day. Fed

Table 3: Regression Analysis of Some Nutrients Uptake Response Fuctions of Wheat Plant to Silicon Application Under Drought and Salinity Stress Conditions. Feddan= 1.05 acre

Dependent variable(y)	Potassium silicate(x)	Sodium silicate(x)
Magnesium uptake (kg/feddan)	$y = 7.3903\ln(x) + 6.4128$ $R^2 = 0.9999$	$y = 4.1105\ln(x) + 6.535$ $R^2 = 0.9807$
Calcium uptake (kg/feddan)	$y = 13.09\ln(x) + 9.6154$ $R^2 = 0.9947$	$y = -2.645x^2 + 13.425x - 0.94$ $R^2 = 1$
Sodium uptake(kg/feddan)	$y = -16.16\ln(x) + 37.352$ $R^2 = 0.9901$	$y = 10.1x^2 - 37.4x + 65.03$ $R^2 = 1$
Potassium uptake(kg/feddan)	$y = 12.428\ln(x) + 8.7508$ $R^2 = 0.9894$	$y = 5.2199\ln(x) + 8.9357$ $R^2 = 0.8635$
Silicon uptake(kg/feddan)	$y = 174.76\ln(x) + 140.03$ $R^2 = 1$	$y = 92.159\ln(x) + 139.22$ $R^2 = 0.9978$
K/Na uptake ratio	$y = 0.794\ln(x) + 0.2091$ $R^2 = 0.9966$	$y = -0.18x^2 + 0.77x - 0.37$ $R^2 = 1$

Three types of models are used for predicting the dynamics of geo-alkaloids towards the root system under stress conditions. They are as follows:

1. A multiple regression between soil salinity, drought, and available silicon in soil solution may control the water flow and geo-alkaloids dynamics towards plant root under abiotic stress conditions (Table 3).
2. The geo-alkaloids dynamics through variably saturated porous media are usually described using convection–dispersion equations [21].
3. AMUN_SiHC: Silicon, the skeletal second abundant element in earth's crust, absorbs passively with water flow and interacts +Ca, +Mg, +K, and –Na. Daily water uptake, daily soil water hydraulic capacitance, daily silicon uptake, daily silicon like hydraulic capacitance, daily potassium uptake, daily magnesium uptake, daily calcium uptake, and daily sodium uptake are the outputs

A further incoming research may compare between the experimental solution obtained from the latter multiple regression and the analytical ones.

AMUN_SiHC supports the applied research and development aiding manager to assess the impact of climatic changes to take the right decision

Some points of general significance in modelling plant water and nutrient uptake under drought and saline conditions using AMUN_SiHC They are as follow:

1. AMUN_SiHC supports the applied research and development aiding manager to assess the impact of climatic changes to take the right decision.
2. Allows methods for interpolation, extrapolation, and prediction.
3. Contains new valuable tools for environmental impact assessment which should be combined to show a unified valuable macrosopic picture for the soil system under stress conditions, SHC, SiHC, W_up, and Si_up. Accordingly, it provides new ideas and experimental approaches and allows the best use of data.
4. AMUN_SiHC provides a mathematical (Figure 2) and FORTRAN computer code for hypotheses to quantify the plant response to silicon foliar application under abiotic stress conditions.
5. Daily water uptake, daily soil water hydraulic capacitance, daily silicon uptake, daily silicon like hydraulic capacitance, daily potassium uptake, daily magnesium uptake, daily calcium uptake, and daily sodium uptake are the outputs of AMUN_SiHC. Accordingly, environmental modelling using AMUN_SiHC is in formative where precise fertigation is used. It allows pedologest to minimize the residual remaining fertilizers in soil after process fertigation. Therefore, it enables pedologest to phytocontain the fertilizers' nutritional elements in root zone using the concepts of soil water hydraulic capacitance and silicon like capacitance. The latters are the newborns of soil stress index model, SSIMOD (Figures 4 and 5).

$$S = \beta * SSI$$

$$\int_{(SSI)}^{(SSI+\Delta SSI)} S \;\; dSSI = \int_{(SSI)}^{(SSI+\Delta SSI)} \beta * SSI \;\; dSSI$$

$$\int_{(SSI)}^{(SSI+\Delta SSI)} S \;\; dSSI = 1/2 \int_{(SSI)}^{(SSI+\Delta SSI)} \beta \;\; d(SSI)^2$$

$$= 1/2[\; \beta * (SSI)^2 - \int_{(SSI)}^{(SSI+\Delta SSI)} (SSI)^2 \;\; d\beta\;]_{(SSI)}^{(SSI+\Delta SSI)}$$

$$= 1/2[\; \beta * (SSI)^2 + \alpha * Tp \int_{(SSI)}^{(SSI+\Delta SSI)} (SSI)^2 * (SSI)^{-2} \;\; dssi\;]_{(SSI)}^{(SSI+\Delta SSI)}$$

$$= 1/2[\; \beta * (SSI)^2 + \alpha * Tp * SSI]_{(SSI)}^{(SSI+\Delta SSI)}$$

$$\therefore S \begin{pmatrix} SSI^{(SSI+\Delta SSI)} \\ SSI^{(SSI)} \end{pmatrix} = 1/2 \begin{pmatrix} \beta^{SSI+\Delta SSI} (SSI)^{2(SSI+\Delta SSI)} + \alpha 2Tp2 * (SSI)^{SSI+\Delta SSI} \\ -\beta^{SSI} (SSI)^{2\,(SSI)} - \alpha 1Tp1 * (SSI)^{(SSI)} \end{pmatrix}$$

$$\therefore S = \frac{\begin{pmatrix} \beta^{SSI+\Delta SSI} (SSI)^{2(SSI+\Delta SSI)} + \alpha 2Tp2 * (SSI)^{SSI+\Delta SSI} \\ -\beta^{SSI} (SSI)^{2\,(SSI)} - \alpha 1Tp1 * (SSI)^{(SSI)} \end{pmatrix}}{\begin{pmatrix} SSI^{(SSI+\Delta SSI)} \\ SSI^{(SSI)} \end{pmatrix}} \qquad \text{(Eq. 1)}$$

$$\therefore S\_Si = \frac{\begin{pmatrix} \beta si^{SSI+\Delta SSI} (SSI)^{2(SSI+\Delta SSI)} + \alpha 2Tp2 * (SSI)^{SSI+\Delta SSI} \\ -\beta si^{SSI} (SSI)^{2\,(SSI)} - \alpha 1Tp1 * (SSI)^{(SSI)} \end{pmatrix}}{\begin{pmatrix} SSI^{(SSI+\Delta SSI)} \\ SSI^{(SSI)} \end{pmatrix}} \qquad \text{(Eq.2)}$$

Where:

- SSI: Soil stress index
- β : Soil water hydraulic capacitance
- βsi: Silicon hydraulic capacitance
- $\Delta SSI = SSI2 - SSI1 = SSI^{(SSI+\Delta SSI)} - SSI^{(SSI)}$
- α_1 , α_2 , Tp_1, and Tp_2 areplant stress indices and potential transpiration constants for corresponding values of soil stress indices SSI_1, SSI_2
- S : water uptake due to ΔSSI
- S_Si: silicon uptake due to ΔSSI

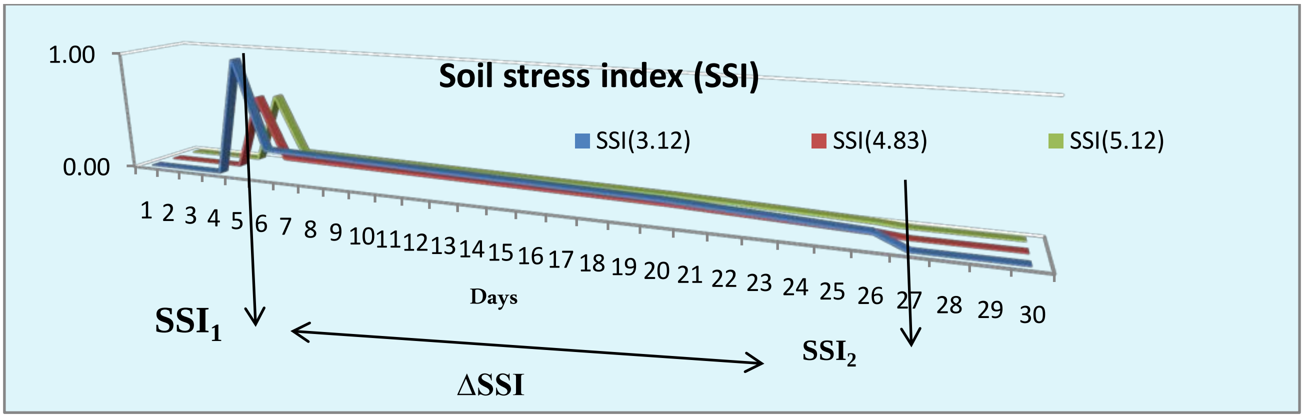

Figure 4.1: As Time is a Moisture and Accordingly a Soil Water Potential, ΔSSI Cauases Change in Water and Silicon Uptake (Eqs. 1 and 2)

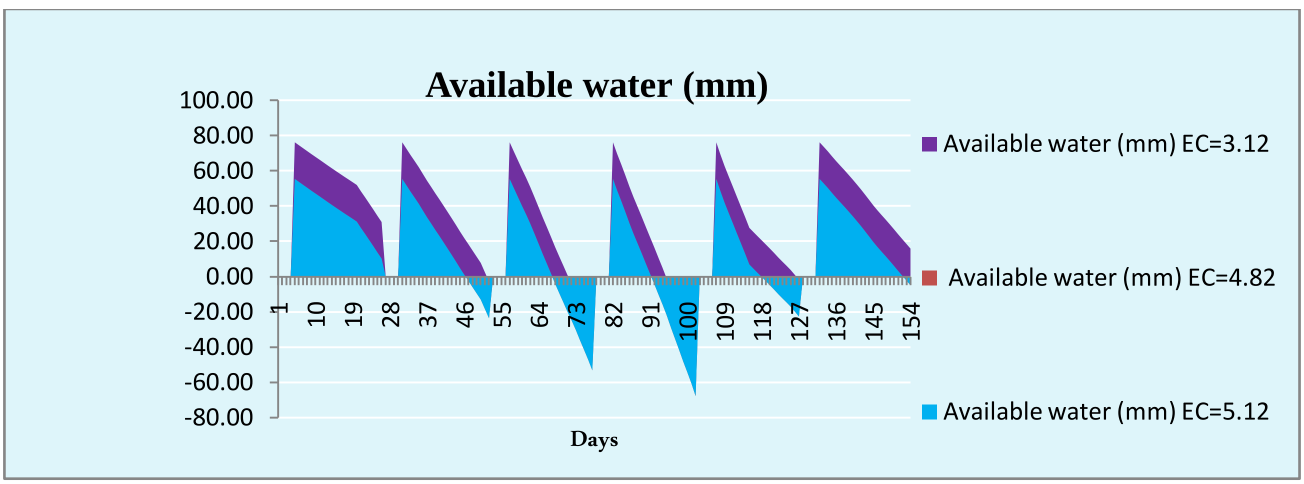

Figure 4.2: Time is a Moisture and Accordingly a Stress Index

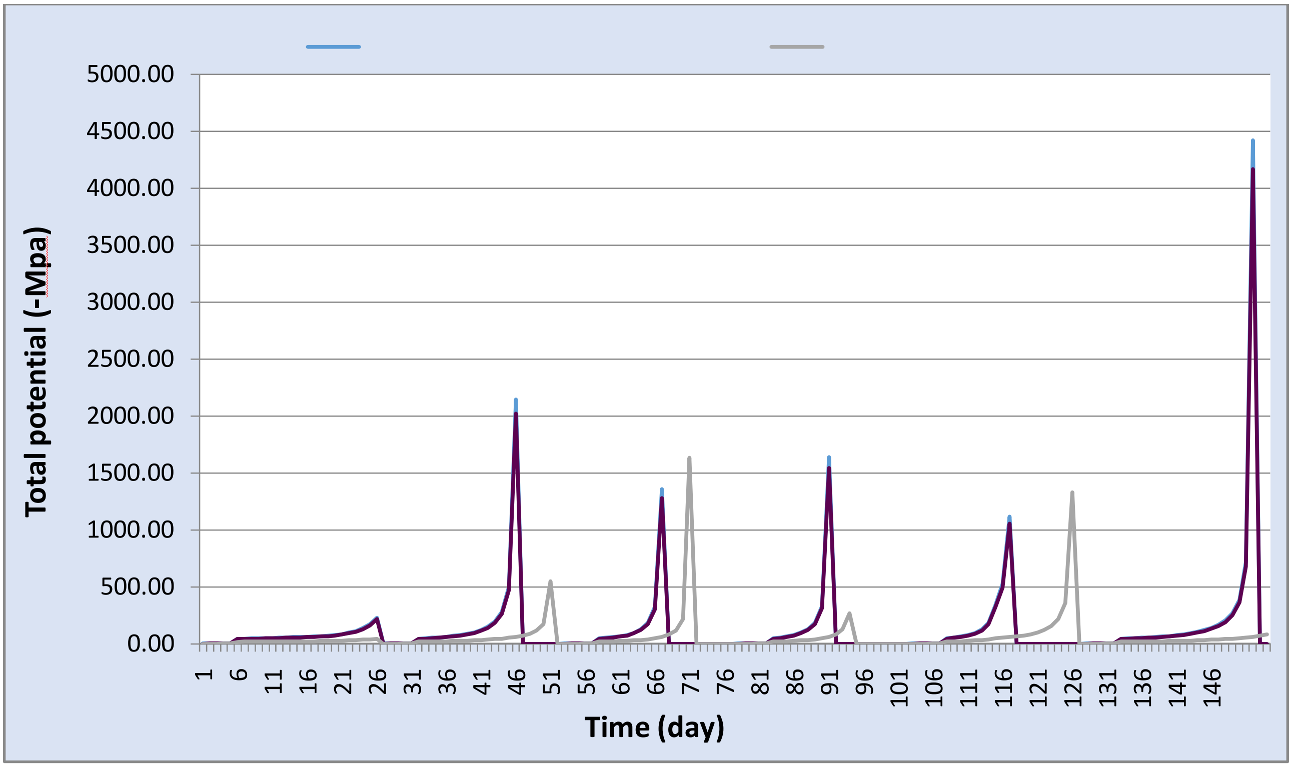

Figure 4.3: Time is a Soil Water Potential and Accordingly a Stress Index

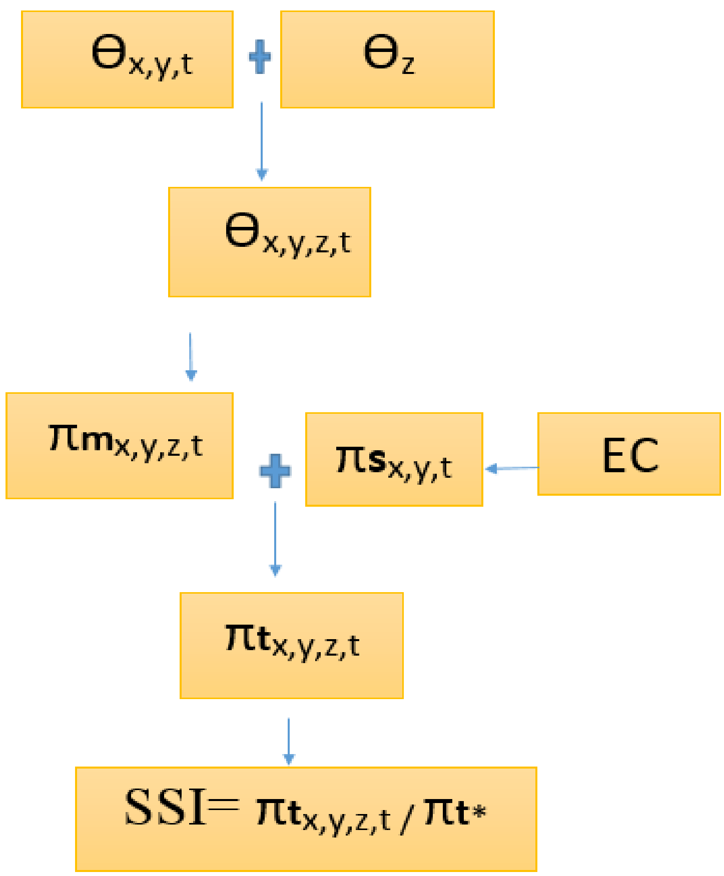

Figure 4.4: Schematic Presentation of (SSIMOD)

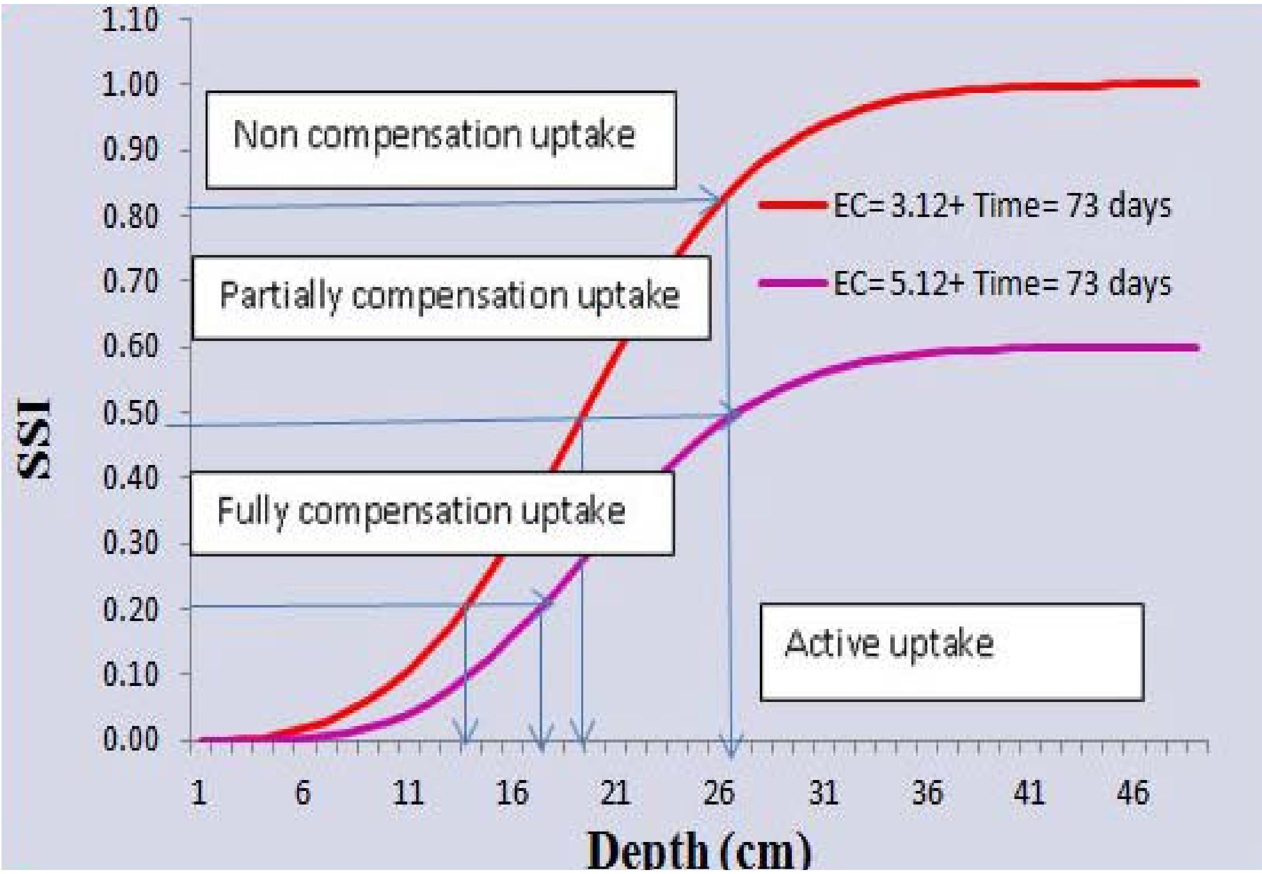

Figure 4.5: Logical Model of SSI

IV. Conclusion

AMUN_SHC was modified to calculate the silicon hydraulic capacitance and mass flow passive silicon uptake, AMUN_SiHC. The latter was achieved by replacing β with β_Si. Silicon, is the nutrien element responsible for aiding plants to complete theirs under stress conditions. The concept silicon like hydraulic capacitance indicates that it like to move passively with water path in accordance with soil water hydraulic capacitance. Β and β_Si may distinguish the compensated and uncompensated water and nutrient uptake under saline and drought conditions. The data clearly showed the positive interaction between silicon uptake and soil hydraulic capacitance under abiotic stress conditions. Silicon plays vital roles in reducing the side effects of climatic changes on the agro ecosystem continuum.

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Appendix (1): AMUN_SiHC FORTRAN Code

Symbols:

Mo: soil moisure(v/v), S: soil salinity(ds/cm), PSI: plant astress index, dSSI: change in SSI, AVG: average PSI, KS: saturated hydraulic conductivity(cm/day), hfc: optimum conditiond of total soil potential due to plant species, growth stage, and soil type, Ta: actual transpiration (mm/day). Ta_cum: cumulative tanspiration, Pr_Ta: predicted actual transpiration, Tp: potential transpieration (mm/day).b, βn: soil and silicon hydraulic capacitance, respectively, hs: soil water potential (mpa),hr = root water ptential(mpa).dh: total potential (mpa). Mp, op: mattric and osmotic potential (mpa) respectively. Con: soil silicon concentration (ppm), C: water holding capacity. PCWU: partially compenstaed water uptake, FCWU: fully compenstaed water uptake, PCSIU: partially compenstaed, silicon uptake, FCSIU: fully compenstaed silicon uptake, UCWU: uncompensted water uptake, UCSIU: un compenstaed silicon uptake, Si_pas: silicon passive uptake, Si_pas_cum: cumulative silicon passive uptake, Ob_ PSI: observed plant stress index, AMUN_Si_acc: AMUN_SiHC_accuracy for predicting Si uptake, AMUN_PSI_acc: AMUN_SiHC_accuracy for predicting PSI, AMUN_Ta_acc: AMUN_SiHC_accuracy for predicting actual transpiration, TSi: observed cumulative Si uptake.

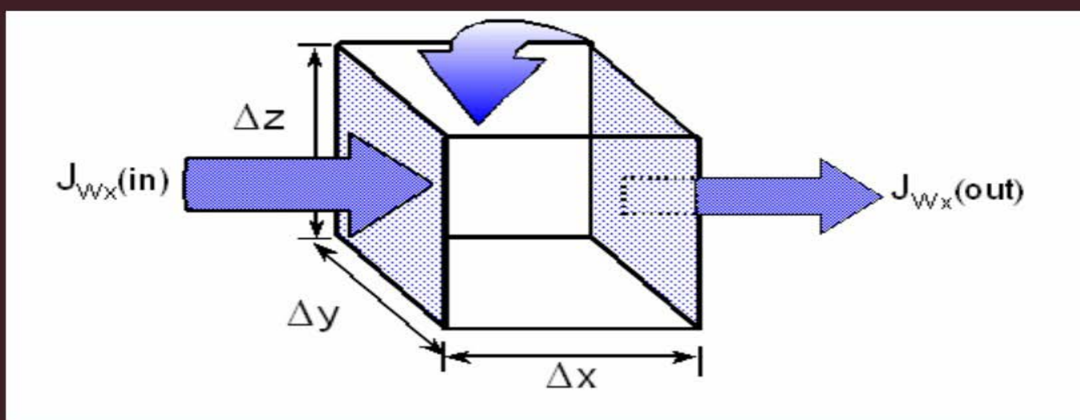
```
PROGRAM AMUN_SiHC
IMPLICIT NONE
REAL, DIMENSION(:,:), ALLOCATABLE :: MO, S, SSI,PSI
INTEGER z,I,J,dJ,t,Min,sec,dSec,IERR
REAL dssi, AVG, KC, hfc ,B ,B1 , B2 , hs, ht, dh,Mp,Op,Ex,Wy,Zs,Zp, c,con, Bn, TP
REAL PCWU, PCSIU, FCWU, FCSIU, UCWU, UCSIU, PrSi_pascum,PrSi_pas, pr_Tacum
REAL AMUN_Si_up_acc, TSi, Pr_Ta,
```

```
AMUN_Tacum_acc,Ob_Tacum
Ex=0
WY=0
Zs=0
zp=0
I=1
dssi=0
avg=0
kc=0
Tp=0
ht=0
dh=0
B1=0
B2=0
con=0
c=0
PCWU=0
PCSIU=0
FCWU=0
FCSIU=0
UCWU=0
UCSIU=0
Ob_Tacum=0
PR_Tacum=0
WRITE (*,*) 'inter hfc, dssi, avg, kc, c,con,I,J,dJ,Min,sec,dSec '
READ (*,*) hfc, dssi, avg, kc, c,con,I,J,dJ,Min,sec,dSec
ALLOCATE (Mo(Z,T),S(Z,T))
OPEN(UNIT=7,FILE='Amun_in.txt',status= 'unknown',IOSTAT=IERR)
OPEN(UNIT=77,FILE='Amun_out.txt')
READ (7,*) ((Mo(Z,T),S(Z,T),Z=I,J,dJ),T= min,sec,dsec)
DO Z=I,J,dJ
DO T= min,sec,dsec
DO WHILE (IERR.GE.0)
MO(I,J)= Ex
Mp=(Ex)*0.0174
S(I,J)=Wy
op=(Wy)*0.036
hs=Op+Mp
dh=hs- ht
ht=hs
SSI(Z,T)= hs/hfc
SSI(Z,T)=Zs
dssi= SSI(Z+1,T)-SSI(Z+1,T)
PSI(Z,T)=(Avg * kc*dSSI/zs)+(Avg)
PSI(Z,T)= zp
B1= (zp*Tp)/ Zs
B2= (1/Zs)*(((kc*dh)-kc)-(c*dh))
B= (B1+B2)/2
Bn= B * con
IF (B .LT.tp)THEN
PCWU=B*Zs
PR_Ta=PCWU
PR_Tacum=PR_Tacum+ PR_Ta
PCSIU=Bn* Zs
PCSIU=PrSi_pas
PrSI_pascum= PrSI_pascum+ PrSi_pas
WRITE(77,*)"PCWU,PCSIU,SSI,PSI,B,Bn,PrSi_pas, PrSi_pas_cum =", PCWU,PCSIU,SSI,PSI,B,Bn,PrSi_pas, PrSi_pascum
ELSEIF (B.GT.Tp)THEN
 FCWU=B*Zs
PR_Ta=FCWU
PR_Tacum=PR_Tacum+ PR_Ta
 FCSIU=Bn* Zs
PrSi_pas= FCSIU
PrSI_pascum= PrSI_pascum+ PrSi_pas
 WRITE(77,*)"FCWU,SSI,PSI, B,Bn, PrSi_pas,PrSi_pas_cum =",FCWU,SSI,PSI, B,Bn, PrSi_pas,PrSi_pascum
ELSE
 UCWU=B* Zs
PR_Ta=UCWU
PR_Tacum=PR_Tacum+ PR_Ta
 UCSIU=Bn* Zs
PrSi_pas= UCSIU
PrSI_pascum= PrSI_pascum+ PrSi_pas
WRITE(77,*)" UCWU,SSI,PSI, B,Bn,PrSi_pas, PrSi_pas_cum= ",UCWU,SSI,PSI, B,Bn,PrSi_pas, PrSi_pascum
11 format(1x,f7.4,1x)
END IF
END DO
END DO
END DO
WRITE(77,*)" Predicted Si Uptake= ", PrSi_pascum
WRITE(77,*)" Observed Si Uptake= ", TSi
WRITE(77,11) (MO(I,J)),(S(I,J)),(SSI(I,J)),(PSI(I,J))
CalLsaln(S,TSi)
AMUN_Si_up_acc = (( PrSi_pascum)/ TSi)*100
PR_Tacum= PR_Tacum+ PR_Ta
AMUN_Tacum_acc=(PR_Tacum/ Ob_Tacum)*100
Write (77,*) " AMUN_SHC Accuracy Tacum, SI uptake=", AMUN_Tacum_Acc,AMUN_Si_up_acc
DEALLOCATE (MO),(S),(SSI),(PSI)
END AMUN_SiHC
SUBROUTINE SALN(S,TSi)
REAL S,TSi
WRITE (*,*) "Inter Salinity Level"
READ S
DO WHILE (3.0 .GE.S .AND. S.LE. 6.0)
TSi=-59.7*S**2 + 206*S + 85.84
END DO
RETURN
END
REAL FUNCTION Ob_Ta(Ahr,Ahs,Ar)
!Ahr: ROOT TOTAL POTENTIAL(MPa)
!Ahs: TOTAL SOIL POTENTIAL(MPa)
!AR: HYDRAULIC RESISTANCE
IMPLICIT NONE
REAL Ar,ks, Ahs,Ahr,Ob_Ta, Ob_Tacum
```

```
DIMENSION Ahs(:,:),Ahr(:,:)
INTEGER  z,t, i, j, dj, min, sec, dsec
WRITE(*,*) "INTER ks, i, j, dj, min, sec, dsec"
READ  ks, i, j, dj, min, sec, dsec
OPEN(UNIT=777,FILE='AMUN_SiHC_in.txt')
!OPEN(UNIT=7777,FILE='AMUN_SiHC_OUT.txt')
ks=0
DO z = i,j,dj
DO t = min,sec,dsec
READ (777,*) Ahs(z,t),Ahr(z,t)
AR= dj/ks
DO WHILE (Ahs(z,t) <Ahr(z,t))
Ob_Ta= (Ahr(z,t)-Ahs(z,t))*10197.44/AR
Ob_Tacum= Ob_Tacum+Ob_Ta
END DO
 END DO
END DO
RETURN
END
```

Appendix (1): Boxes

BOX (1):



Mass Balance Method for Solving the Equation of Water Flow.

Mass Balance Method

A. ($Q_{in}= Q_{out}$)= Accumulation

$Q_{in}= q_{in}.\ A = \ q_{i-1.}\ A$

$Q_{out=}\ q_{out}.\ A. = \ q_{i.}\ A$

$Q_{in}= Q_{I-1}= -A.\ K.\ [\frac{hi - hi-1}{dx}]$

$Q_{out}=Q_{I} = -A.\ K.\ [\frac{hi+1 - hi}{dx}]$

$Accum. = -A.\ K.\ [\frac{hi - hi-1}{dx} - \frac{hi+1 - hi}{dx}]$

$(\theta_i^{j+1} - \theta_i^{j})/\Delta t = -A.\ K.\ [\frac{h(i-1)-2hi+h(i+1)}{\Delta x2}]$

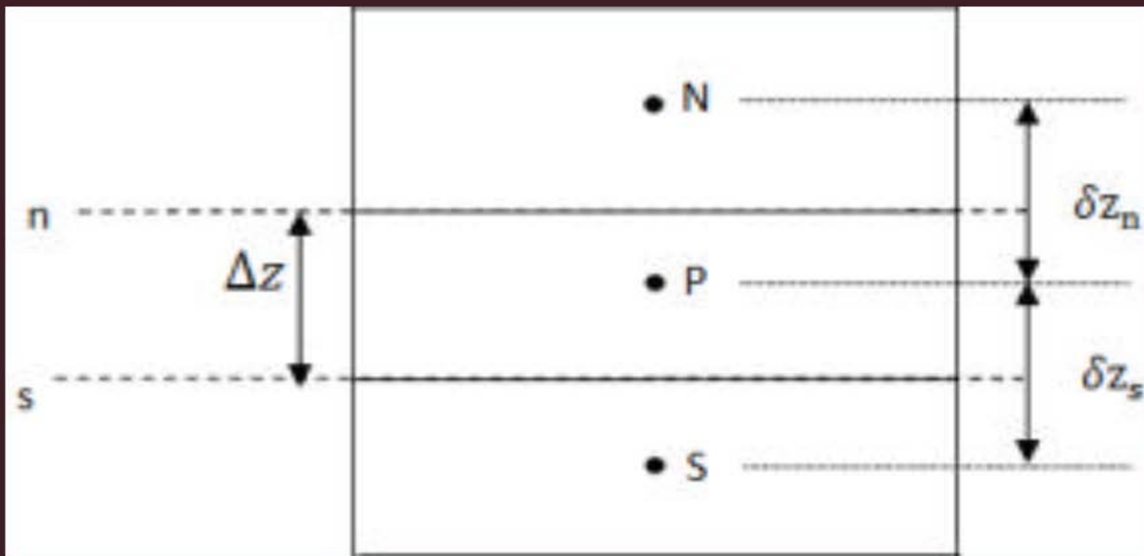
BOX (2):

Finite Volume Method

$$C(\psi)\frac{\partial \psi}{\partial t} - \nabla.K(\psi)\nabla\psi + \frac{\partial K(\psi)}{\partial z} = 0 \quad \text{Pressure-based} \qquad (1)$$

$$\frac{\partial \theta}{\partial t} - \nabla.D(\theta)\nabla\psi + \frac{\partial K(\theta)}{\partial z} = 0 \quad \text{Moisture-based} \qquad (2)$$

$$\frac{\partial \theta}{\partial t} - \nabla.K(\theta)\nabla\psi + \frac{\partial K(\psi)}{\partial z} = 0 \quad \text{Mixed-form} \qquad (3)$$



A Portion of 1-D Grid

BOX (3):

$$\frac{\partial \theta}{\partial t} - \frac{\partial}{\partial z}\left(K\frac{\partial \psi}{\partial z} + k\right) = 0 \qquad (4)$$

The control volume integration, which forms the key step of the finite volume method that distinguishes it from finite difference technique, for the control volume defined above, yields the following form:

$$\int_n^s \int_t^{t+\Delta t} \frac{\partial \theta}{\partial t} dtdz - \int_t^{t+\Delta t} \int_n^s \frac{\partial}{\partial z}\left(K\frac{\partial \psi}{\partial z} + K\right) dzdt = 0 \qquad (5)$$

Integrating from first term of this equation respect to time

$$\int_n^s \int_t^{t+\Delta t} \frac{\partial \theta}{\partial t} dtdz = \int_n^s (\theta^{t+\Delta t} - \theta^t) dz \qquad (6)$$

In finite volume method, the whole specification of fluid and media is constant in a control volume. Therefore, equation 6 converts to follow equation:

$$\int_n^s \int_t^{t+\Delta t} \frac{\partial \theta}{\partial t} dtdz = (\theta_p^{t+\Delta t} - \theta_p^t)\Delta z \qquad (7)$$

After integrating from second term of equation 5, the result would be:

$$\int_t^{t+\Delta t} \int_n^s \frac{\partial}{\partial z}\left(K\frac{\partial \psi}{\partial z} + K\right) dzdt = \int_t^{t+\Delta t} \left[\begin{array}{l} \left(K\frac{\partial \psi}{\partial z} + K\right)_s \\ -\left(K\frac{\partial \psi}{\partial z} + K\right)_n \end{array} \right] dt \qquad (8)$$

BOX (4):

For implicit solution, the discretized form of (4)

$$(\theta_p^{t+\Delta t} - \theta_p^t)\Delta z = \left[\begin{array}{l} \left(K^t\frac{\partial \psi^{t+\Delta t}}{\partial z} + K^t\right)_s \\ -\left(K^t\frac{\partial \psi^{t+\Delta t}}{\partial z} + K^t\right)_n \end{array} \right] \Delta t \qquad (9)$$

where θ^t and K^t denotes the value of θ and K at time t, respectively, Dt is the time step, Y^{t+Dt} denotes the Y at time t+Dt and the solution is assumed to be known at time level t and unknown at time level t+Dt. Because the velocity of water flow in soil is very small, it is logical to consider hydraulic conductivity is constant during a time step.

$$\left(K^t \frac{\partial \psi^{t+\Delta t}}{\partial z} + K^t\right)_s = K_s^t \frac{\psi_s^{t+\Delta t} - \psi_p^{t+\Delta t}}{\delta z_s} + K_s^t \qquad (10)$$

$$\left(K^t \frac{\partial \psi^{t+\Delta t}}{\partial z} + K^t\right)_n = K_n^t \frac{\psi_p^{t+\Delta t} - \psi_n^{t+\Delta t}}{\delta z_n} + K_n^t \qquad (11)$$

Substitution of equations (10) and (11) into (9) gives:

$$(\theta_p^{t+\Delta t} - \theta_p^t)\Delta z = \begin{bmatrix} \left(K_s^t \frac{\psi_s^{t+\Delta t} - \psi_p^{t+\Delta t}}{\delta z_s} + K_s^t\right) \\ -\left(K_n^t \frac{\psi_p^{t+\Delta t} - \psi_n^{t+\Delta t}}{\delta z_n} + K_n^t\right) \end{bmatrix} \Delta t \qquad (12)$$

For solution (10) an iterative method must be used. Considering 'm' as iteration level and using Picard iterative method, gives:

BOX (5):

$$(\theta_p^{t+\Delta t, m+1} - \theta_p^t)\Delta z = \begin{bmatrix} \left(K_s^t \frac{\psi_s^{t+\Delta t, m+1} - \psi_p^{t+\Delta t, m+1}}{\delta z_s} + K_s^t\right) \\ -\left(K_n^t \frac{\psi_p^{t+\Delta t, m+1} - \psi_n^{t+\Delta t, m+1}}{\delta z_n} + K_n^t\right) \end{bmatrix} \Delta t \qquad (11)$$

There are two unknowns vector θ and Y in 'm+1'th iteration level. As proposed by researchers, for predicting $\theta_p^{t+Dt,m+1}$, a Taylor series is expanded as follow [12]:

$$\theta_p^{t+\Delta t, m+1} = \theta_p^{t+\Delta t, m} + \left(\frac{d\theta}{d\psi}\right)_p^t (\psi_p^{t+\Delta t, m+1} - \psi_p^{t+\Delta t, m}) \qquad (12)$$

In a uniform grid $Dz = dz_s = dz_n$ and (10) can be re-arranged as:

$$\alpha \psi_n^{t+\Delta t} + \beta \psi_p^{t+\Delta t} + \gamma \psi_s^{t+\Delta t} = \lambda \tag{13}$$

where

$$\alpha = K_n^t \left(\frac{\Delta t}{\Delta z^2} \right)$$

$$\beta = -\left[(K_n^t + K_s^t)\left(\frac{\Delta t}{\Delta z^2} \right) + C_p^t \right]$$

$$\gamma = K_s^t \left(\frac{\Delta t}{\Delta z^2} \right)$$

and

$$\lambda = \left[(K_n^t - K_s^t)\left(\frac{\Delta t}{\Delta z} \right) \right] - C_p^t \psi_p^{t+\Delta t,m} + (\theta_p^{t+\Delta t,m} - \theta_p^t)$$