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Life Energy By Symmetric Quartic Power Curves

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Life Energy
By Symmetric Quartic Power Curves

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I. Introduction to our New Model of life based on Quartics, Rather than on Lognormals

Life may be modelled as a Power Curve in time ranging between birth and death, with a peak somewhere in between.

This author's 2020 book "Evo-SETI" (https://link.springer.com/book/10.1007/978-3-030-51931-5) gave the equations of three such Power Curves:

1) b-lognormals, namely a lognormal between birth and descending inflection followed by a tangent straight line down to death;
2) logpars, namely a lognormal between birth and peak, followed by a parabola down to death;
3) logells, namely a lognormal between birth and peak, followed by an ellipse down to death.

II. Mathematical Introduction to Quartics

By "quartics" we mean algebraic equations of the fourth degree having the time t as their independent variable. Thus, a quartic in the time has five coefficients (A, B, C, D, E) and reads

$$\text{general\_quartic} = A\,t^4 + B\,t^3 + C\,t^2 + D\,t + E \qquad (1)$$

This quartic, however, is too general for our purposes, for, in general, it has a graph like this one

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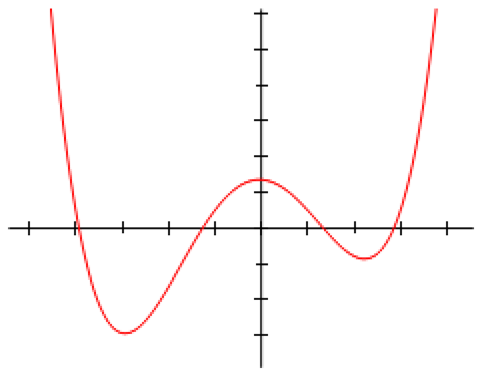

In order for the quartic to represent the lifetime of a living species, we rather need a quartic starting at instant *b* (birth), reaching its maximum at time *p* (peak time) and finally intercepting the time axis at death time *d*. For instance, supposing the lifetime of a human being is 80 years, we need a quartic looking like this one:

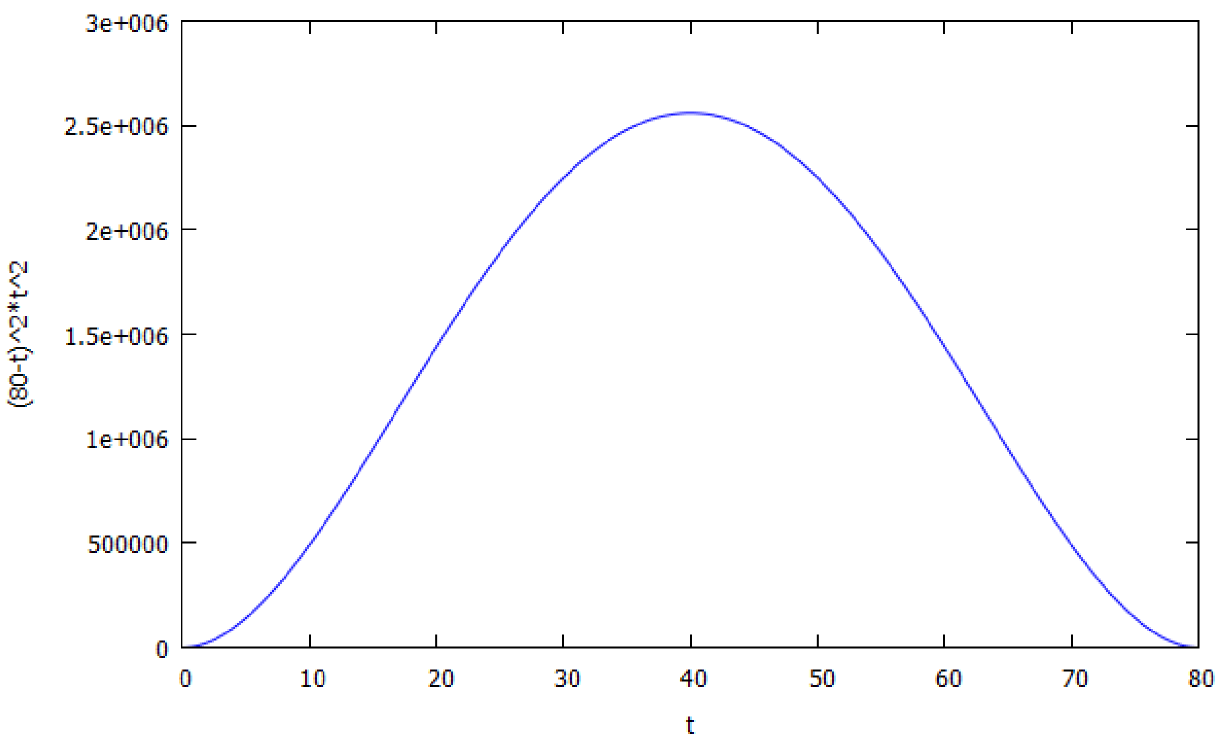

Figure 1: The symmetric quartic of an 80-years-old human being, clearly symmetric with respect to age 40. Notice that the tangent to this curve at both the birth and death is horizontal, that is the derivative of the symmetric quartic at both birth and death is zero. Pay no notice to the numbers on the vertical axis, please, since this quartic is not normalized at all. Later we will normalize it so that the area under the quartic equals one by convention. This normalized-to-one quartic may thus be regarded as a probability density function (pdf) in the time. We will assume this quartic to be a *power* curve, measured in Watts. Since the power is the time derivative of the energy, and so the energy is the time integral of the power, the area under the above quartic is the total *energy* of that human, that is the energy this human needs to live up to age 80.

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In his mathematical book "Evo-SETI" (standing for Evolution and SETI) (website https://link.springer.com/book/10.1007/978-3-030-51931-5), this author proved (see the pages 693-713) that the equation of every symmetric quartic between birth b and death d reads

$$\text{symmetric quartic between birth } b \text{ and death } d \qquad quartic(t) = (t-b)^2 (d-t)^2 \qquad (2)$$

Mathematically speaking, (2) is much easier to handle than the general quartic (1). In addition, the sign of the content inside each parenthesis of the symmetric quartic (2) is unimportant since both parentheses are raised to the square. This paper uses symmetric quartics to mathematically describe a variety of phenomena in Astrobiology and related disciplines, like Human History.

This author presented his first paper about Quartics during the SETI 2 Session of the International Astronautical Congress (IAC 2019) held in Washington D.C. in 2019 https://www.iafastro.org/events/iac/iac-2019/, paper # IAC-19-A4,2,x49155. In the Maxima Code of that paper the incorrect wording "quadric" is sometimes used instead of "quartic". Apologies.

III. Our Three Lognormal-Based Power Curves

In his 2020 mathematical book "Evo-SETI" (standing for "Evolution and SETI") (website: https://link.springer.com/book/10.1007/978-3-030-51931-5) this author represented the lifetime of every living being as either of the following three different Power Curves (namely we have time on the horizontal axis and Watt-measured Power on the vertical axis):

1) b-lognormal (a lognormal curve between birth b and the descending inflection s called senility, followed by a straight line between senility s and death d, intercept with the time axis). Its equation is (only for $t > b$):

$$\text{b\_lognormal}(t;\mu,\sigma,b) = \frac{e^{-\frac{(\log(t-b)-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\,\sigma\,(t-b)}.$$

However, we will not use this equation in this paper, which is why we won't number it.

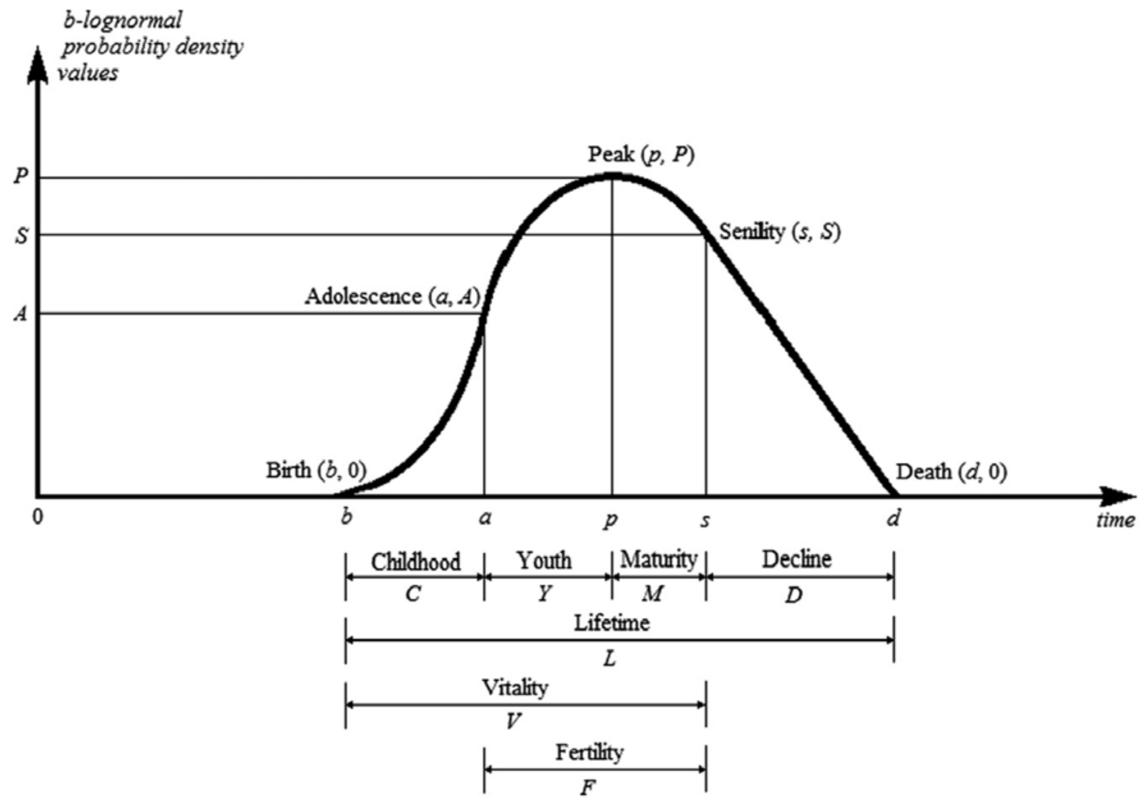

Figure 2: This curve we called b-lognormal (since around 2010) and it ranges from birth b to senility s (the descending inflection) as a lognormal, and from s to death d as a straight line. It has two independent parameters: $\sigma > 0$ and $-\infty < \mu < \infty$. It basically is exp (Gaussian pdf).

It became useful to us since we discovered (again since after 2010) its History Formulae:

$$\text{b-lognormal History Formulae} = \begin{cases} \sigma = \dfrac{d-s}{\sqrt{d-b}\cdot\sqrt{s-b}} \\ \mu = \ln(s-b) + \dfrac{(d-s)(d+b-2s)}{(d-b)(s-b)}. \end{cases}$$

2) Logpar (= LOG normal plus PAR abola). This is a lognormal curve between birth b and the peak time p, followed by a parabola starting at the peak time p and having the same horizontal tangent at p, then getting down as a parabola to intersect with the time axis at death time d.

It became useful to us only after November 22nd, 2015, when we discovered the relevant History Formulae appearing hereafter (their proof is not obvious and it implies the minimum energy principle, meaning that every living being uses only the minimum energy it necessitates to survive during its own lifetime):

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$$\text{Logpar History Formulae} = \begin{cases} \sigma = \dfrac{\sqrt{2}\sqrt{d-p}}{\sqrt{2d-(b+d)}} \\ \mu = \ln(p-b) + \dfrac{2(d-p)}{2d-(b+d)}. \end{cases}$$

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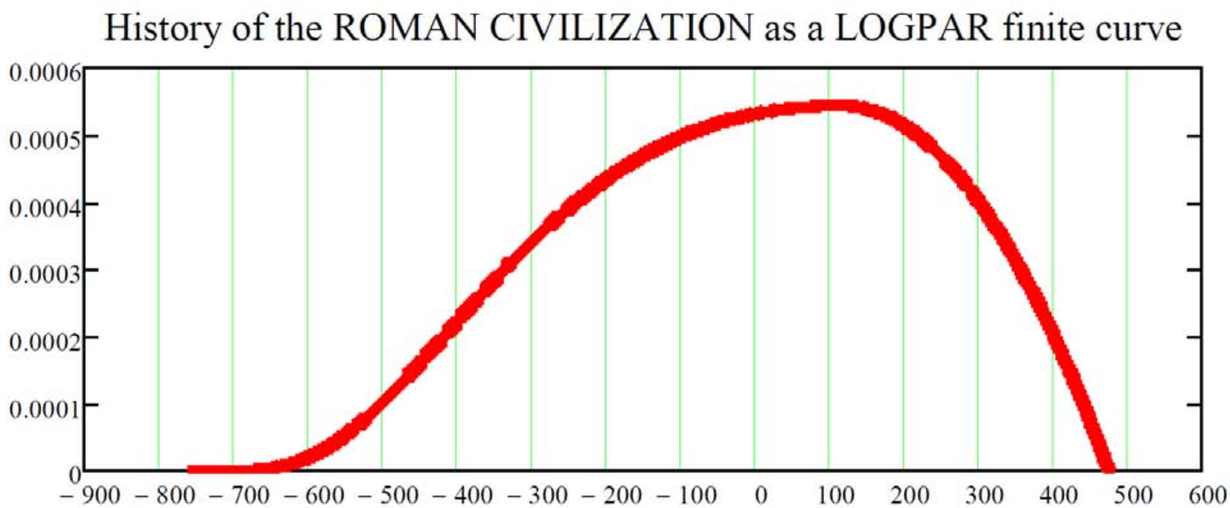
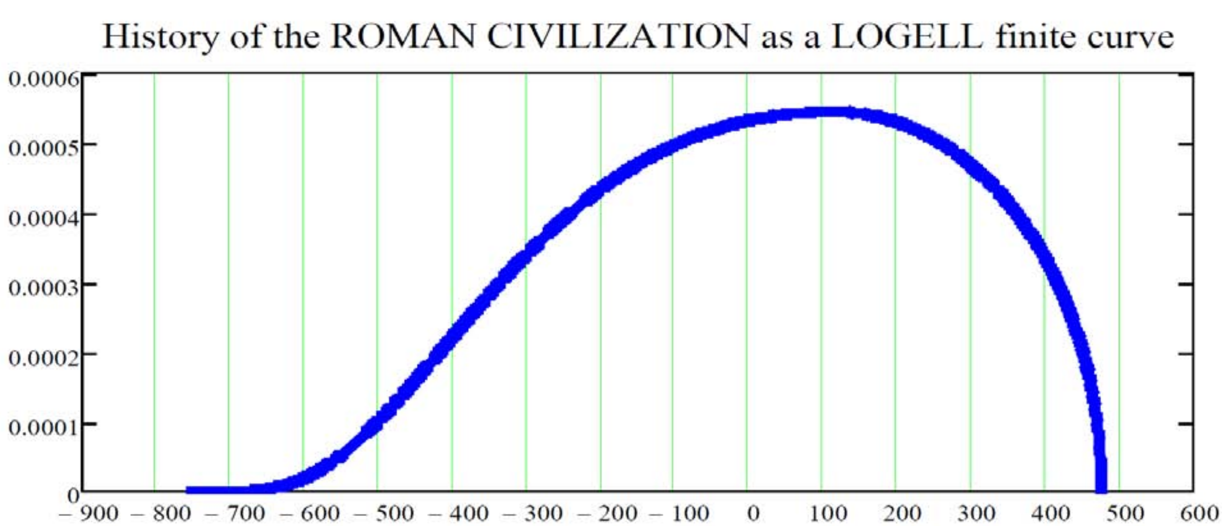
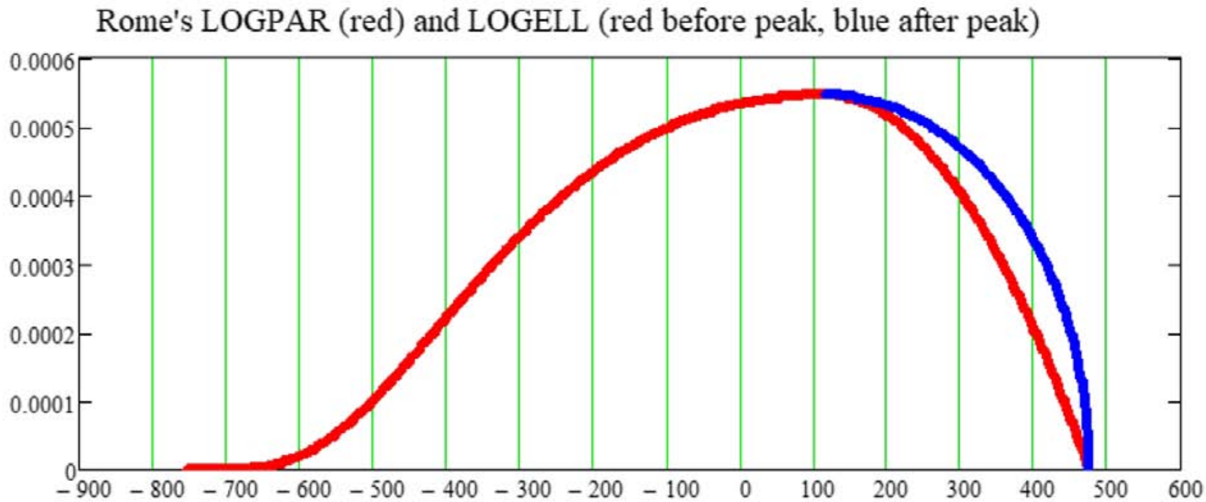
As a good Logpar example, just look at the following History of the Roman Civilization (753 B.C. through 476 A.D.) as a Logpar power curve (the peak is at A.D. 117 when emperor Trajan died and the Roman Empire reached it largest extent ever):



3) Logell(= LOGnormal plus ELLipse). This power curve is a lognormal curve between birth b and the peak time p, followed by the quarter of an ellipse in between the peak time p and the intercept time d (death) between the ellipse and the time axis. In the example of the Roman Civilization used already to describe the Logpar, the Logell looks like this:





The Logell History Equations again follow from the Minimum Energy Principle and read:

$$\begin{cases} \sigma = \dfrac{\sqrt{\pi}\sqrt{d-b}}{\sqrt{\pi(d-b)+(\pi-4)(p-b)}} \\ \mu = \ln(p-b) + \dfrac{\pi(d-p)}{\pi(d-b)+(\pi-4)(p-b)}. \end{cases}$$

IV. Quartic Power Curves do Not Need any History Equation

Let us now go back to our Quartic power curves, the general equation of which is(2), that is

$$(t-b)^2 (d-t)^2 \tag{3}$$

This equation contains two independent parameters, b and d, with $d > b$.

In the practice, however, it is traditional to count the age (in years) of a person starting from his/her year of birth. If we so decide and call $L = d - b$ the number of years a person lives on, that is

$$\begin{cases} b = 0 \\ L = d - b \quad with \quad L > 0. \end{cases} \tag{4}$$

then (3) takes the new and easier form

$$\text{quartic} = t^2 (L-t)^2. \tag{5}$$

The advantage of (5) over (3) is that (5) contains one parameter only: the lifetime duration L.

Notes

#1 Theorem: our quartic is perfectly symmetrical with respect to the peak vertical axis

$$\text{peak time} = \frac{d-b}{2} = \frac{L}{2}. \tag{6}$$

Notes

Proof. Just compute the first derivative of (3) with respect to t and set it equal to zero. Then you get three roots: b=0, d=0 and (6), as expected and intuitively obvious.

#2 Theorem: The two inflection times of the quartic (3) occur at the two instants

$$\begin{cases} \text{left inflection time} = \frac{(d-b)}{2}\cdot\left(1-\frac{1}{\sqrt{3}}\right) = \frac{L}{2}\cdot\left(1-\frac{1}{\sqrt{3}}\right) \\ \text{right inflection time} = \frac{(d-b)}{2}\cdot\left(1+\frac{1}{\sqrt{3}}\right) = \frac{L}{2}\cdot\left(1+\frac{1}{\sqrt{3}}\right). \end{cases} \tag{7}$$

Proof. Just compute the second derivative of (3) with respect to t and set it equal to zero. Then solve the resulting quadratic equation with respect to the time t, and (7) follows after a few reductions.

#3 Theorem: The "chord length between the two inflection times", which is the amount of time in between the left- and right-inflection times (7), simply is given by

$$\frac{d-b}{\sqrt{3}} = \frac{L}{\sqrt{3}} = 0.577\ldots\cdot L \approx 57.7\,\%\cdot L. \tag{8}$$

In other words, the chord in between the two inflection times is about the 57.7 % of the whole lifetime $(d-b)=L$ of our living being. We shall we give it the special name of Vitality, and Vitality described in the next Section 5.

Please also look at the plot in *Figure 2*, which is the same as Figure 1 with the added horizontal line intercepting the symmetric quartic of an 80-years-old human being drawn through the two inflection points given by equations (7).

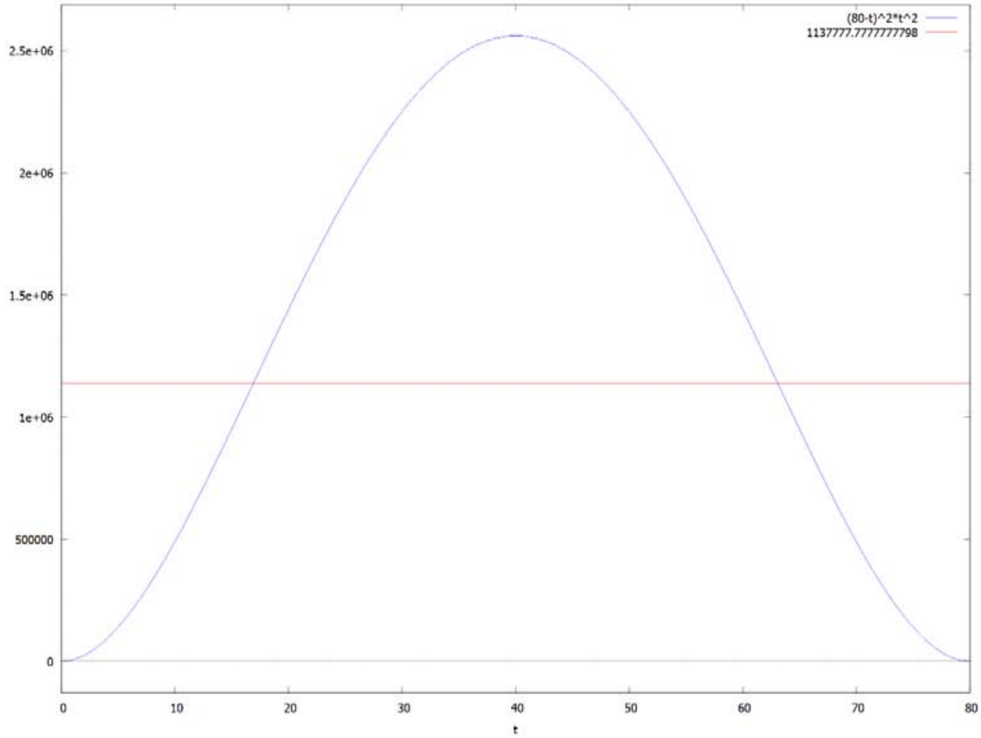


Figure 2: The symmetric quartic of an 80-years-old human being, just as the plot in Figure 1. Here, however, we have added the horizontal line in red intercepting the symmetric quartic at its two inflection points that have a height of 1137777.7777777798 above the time axis. This plot neatly shows the three parts of a 80-years-old human as Youth (left to the left intercept), Vitality (in between the left intercept and the right intercept) and Senility (right to the right intercept and down up to death).

V. Vitality is What we Call the Amount of a Lifetime in between the Two Quartic Inflections

Indeed, VITALITY is the most vital part of one's lifetime. For a Human aged 80 years, this vitality starts about the age of 16.9 years:

$$\left(1-\frac{1}{\sqrt{3}}\right)\cdot 80 \text{ years} \approx 16.905 \text{ years} \approx 17 \text{ years} . \qquad (9)$$

This is the age of majority, also known as legal age, namely the threshold of legal adulthood as recognized or declared in law. It is the moment when a person ceases to be considered a minor and assumes legal control over their person, actions, and decisions, thus terminating the control and legal responsibilities of their parents or guardian over them. Please see the Wikipedia website https://en.wikipedia.org/wiki/Age_of_majority.

On the contrary, the vitality's end turns out to be around the age of

$$\left(1+\frac{1}{\sqrt{3}}\right)\cdot 80 \text{ years} \approx 63.094 \text{ years} \qquad (10)$$

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Please see the Wikipedia website https://en.wikipedia.org/wiki/Retirement_age. But, then, the definition of YOUTH now follows…

VI. Youth is the Amount of a Lifetime in between Birth and the First Quartic Inflection

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Indeed, youth is the part of our quartic in between birth and the left-quartic-inflection, please see the Wikipedia site https://en.wikipedia.org/wiki/Youth
This is also related to the voting age https://en.wikipedia.org/wiki/Voting_age
And finally…

VII. Senility is the Amount of a Lifetime in between the Last Quartic Inflection and Death

Please see the related websites: https://en.wikipedia.org/wiki/Alois_Alzheimer and https://en.wikipedia.org/wiki/Gaetano_Perusini. Thanks.

VIII. Two Important Equations for Chord and Height of any Symmetric Quartic

On December 1st, 2023, this author discovered the Chord equation and the Height equation that we describe in the present section. Clearly, both these equations might have been known to mathematicians very long ago already, perhaps known even in the 1540s to Lodovico Ferrari, credited to have discovered the solution to the quartic algebraic equation prior to 1545, when Gerolamo Cardano published all that in his Latin book "Ars Magna" (please enjoy the website: https://en.wikipedia.org/wiki/Quartic_equation).

In order to discover these two equations, we start by noting that, at height h above the time axis, the two intercepts between our quartic and the horizontal h-height straight line are given by the new equation with the time denoted by z

$$h = z^2 \left(L - z\right)^2. \tag{11}$$

Since one has $h > 0$, $L > 0$, $z > 0$ and $z < L$, we may take the square root of (11) and get

$$\sqrt{h} = z \cdot \left(L - z\right). \tag{12}$$

This is just a quadratic equation in z, that we may solve for z getting the two roots

$$\begin{cases} z = -\left(\dfrac{\sqrt{L^2 - 4\sqrt{h}} - L}{2}\right) = \text{left (time) root} \\ z = \left(\dfrac{\sqrt{L^2 - 4\sqrt{h}} + L}{2}\right) = \text{right (time) root.} \end{cases} \tag{13}$$

Now, the chord length in between these two instants is the time segment parallel to the time axis and positioned at height h above the time axis: we shall call it *chord* and its length is given by the difference between the right (time) root minus the left (time) root, that is

$$chord = \left(\frac{\sqrt{L^2 - 4\sqrt{h}} + L}{2}\right) - \left[-\left(\frac{\sqrt{L^2 - 4\sqrt{h}} - L}{2}\right)\right] = \sqrt{L^2 - 4\sqrt{h}}. \tag{14}$$

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In other words, we have discovered the *chord equation* that will be important for practical applications:

$$chord = \sqrt{L^2 - 4\sqrt{h}}. \tag{15}$$

In addition, the above chord equation may immediately be solved for h yielding, after a few manipulations, the *height equation*

$$h = \frac{\left(L^2 - chord^2\right)^2}{16}. \tag{16}$$

This *height equation* also will be of paramount importance for the applications of symmetric quartics to most diverse fields of learning, like Mathematical History, Astrophysics, Medicine, and more. Let us just check the results that (16) yields in some cases that we considered already:

1) If the chord equals $\frac{L}{\sqrt{3}}$ then we get the height above the time axis of the straight line passing through the two inflection points of the quartic, that is

$$h = \frac{\left(L^2 - \frac{L^2}{3}\right)^2}{16} = \frac{L^4}{16} \cdot \left(\frac{2}{3}\right)^2 = \frac{L^4}{36}. \tag{17}$$

One might wish to check that the same result is found upon replacing either of equations (7) into the quartic of equation (3). We leave that to the reader as an exercise.

2) If the chord has zero length, then we are at the quartic PEAK, and then the peak height on the time axis is given by (16) with *chord*= 0 and reads

$$\text{peak height} = \frac{L^4}{16}. \tag{18}$$

3) If the chord equals the whole lifetime L, then (16) shows that $h = 0$, as intuitively obvious.

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IX. Energy of the Power Curves given by our Symmetric Quartics

Nowadays even schoolchildren know that $E = mc^2$, and freshmen know that "power is the time derivative of energy", so that, conversely, energy is the integral of a power curve with respect to the time. But in this section we face a more subtle Astrobiology question: "Suppose that all Humans have the same POTENTIAL ENERGY at birth, depending only on the Species they belong to. Then, are we going to NORMALIZE all our symmetric quartics by conventionally setting their potential energy to ONE ?

The answer depends on which field of science we wish to apply our symmetric quartics to.

X. Quintics Yielding the Energy of our Symmetric Quartic Power Curves

As mentioned, we assume that all Lifetimes considered in this paper are symmetric quartics. Thus, the energy requested for a living organism to live the whole of his/her lifetime is the integral of the symmetric quartic with respect to the time extended to that whole lifetime. That is, upon inserting (5) under the integral between 0 and L

$$\text{Whole Lifetime ENERGY} = \int_0^L t^2 \left(L-t\right)^2 dt. \tag{19}$$

It is actually more convenient define and compute just the energy requested by the living organism to live in between two assigned instants $Tstart$ and $Tend$ with $Tstart < Tend$, like this:

$$\begin{aligned} &ENERGY\_necessary\_to\_live\_from\_Tstart\_to\_Tend = \int_{Tstart}^{Tend} t^2 \left(L-t\right)^2 dt = \\ &= -((6Tstart^5 - 15L\,Tstart^4 + 10L^2 Tstart^3 - 6Tend^5 + 15L Tend^4 - 10L^2 Tend^3)/30). \end{aligned} \tag{20}$$

Let us now compute the tree most important cases of the last formula:

1) Youth Energy,
2) Vitality Energy,
3) Senility Energy.

The Youth Energy is obtained by replacing $Tstart = 0$ and $Tend = \frac{L}{2}\left(1 - \frac{1}{\sqrt{3}}\right)$ into (20). After a few reductions, one gets:

$$Youth\_Energy = \frac{\left(2-\sqrt{3}\right)L^5}{120}. \tag{21}$$

The Vitality Energy is obtained by replacing $Tstart = \frac{L}{2}\left(1-\frac{1}{\sqrt{3}}\right)$ and $Tend = \frac{L}{2}\left(1+\frac{1}{\sqrt{3}}\right)$ into (20). Again a few reductions yield the desired Vitality Energy:

$$Vitality\_Energy = \frac{L^5}{20\sqrt{3}}. \tag{22}$$

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Finally, the Senility Energy is clearly equal to the Youth Energy because of the perfect symmetry of the quartic around the vertical axis $\frac{L}{2}$ and so one gets:

$$Senility\_Energy = \frac{\left(2-\sqrt{3}\right)L^5}{120}. \tag{23}$$

It is now easy to *sum* the three energies (21), (22) and (23) to get the total energy that a living organism needs in order to live his/her whole life in between birth and death. Some reduction yield:

$$Total\ Energy\ to\ live\ from\ birth\ to\ death = \frac{L^5}{30}. \tag{24}$$

XI. Conclusion

We prefer to stop our description of the Energy of Symmetric Power Curves at this point.

More paper(s) will follow as soon as we discover both more detailed mathematics and applications to real cases of interest in Biology, Astrobiology and Mathematical History.

References Références Referencias

1. C. Maccone, *Evo-SETI – Life Evolution Statistics on Earth and Exoplanets*, a 878-pages mathematical book published by Springer Nature in 2020, but available only since March 2021, website https://link.springer.com/book/10.1007/978-3-030-51931-5.