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Macroscopic Effect of Quantum Gravity in General Static Isotropic Gravitational Field

By Gang Lee

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Macroscopic Effect of Quantum Gravity in General Static Isotropic Gravitational Field

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I. Introduction

This paper discusses the effects of self-interaction of gravitational field and finds that it can be used to explain dark matter. In section 2, we calculated the self-interaction of the gravitational field. We analyzed the effect of the self-interaction of a general static isotropic gravitational field using a semi classical approach. In section 3, we found a symmetry of the Klein-Gordon equation in the noncommutative quantum gravitational field.

II. Noncommutative Quantum Gravity and Self Interaction

Let's briefly review the theory of noncommutative quantum gravity while referring to [1] for more details.

Since the introduction of the uncertainty principle into the general theory of relativity, we get a semiclassical graviton approximate to the Dirac δ-function as follows

$$\xi^{\alpha}(x,X) = X + C^{\alpha}(x)\cdot\exp(-\left|\frac{X}{L_P(x)}\right|) \tag{2.1}$$

The free field equation is

$$\partial^{\mu}\partial_{\mu}\xi^{\alpha}(x) = 0 \tag{2.2}$$

At the point x, the local inertial coordinate is $\xi^{\alpha}(x,X)\big|_{X=0}$. Then the metric associated with ξ^{α} is

$$\begin{aligned} g_{\mu\nu}(x) &= \frac{\left(\partial\xi^{\alpha}\big|_{X=0}\right)}{\partial x^{\mu}}\frac{\partial\left(\xi^{\beta}\big|_{X=0}\right)}{\partial x^{\nu}}\eta_{\alpha\beta} \\ &= \frac{\partial C^{\alpha}(x)}{\partial x^{\mu}}\frac{\partial C^{\beta}(x)}{\partial x^{\nu}}\eta_{\alpha\beta} \end{aligned} \tag{2.3}$$

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In any point x of gravitational field, due to the ductility of gravitons, gravitons elsewhere will act on the point x together. all another gravitons excited at a distance of $l \equiv l^{\mu}$ from point x can be written as

$$\Delta\xi^{\alpha} = X + \int d^4 l\, \xi^{\alpha}\big((x+l), |l|\big)$$

$$= X + \int d^4 l \left(C^{\alpha}(x+l) \cdot \exp(-\left|\frac{l}{L_P(x+l)}\right|)\right) \tag{2.4}$$

Then after considering the effects of all gravitons, the locally inertial coordinate system ξ^{α} at point x have to written as

$$\lambda(\xi^{\alpha}) = \xi^{\alpha}(x, X)\big|_{X=0} + \Delta\xi^{\alpha} \tag{2.5}$$

The field $C^{\alpha}(x+l)$ also satisfy the free field equation, it can be written as

$$C^{\alpha}(x+l) = \int d^4 k \left(C^{\alpha}(k) \exp\big(ik(x+l)\big) + (C^{\alpha}(k))^{*} \exp\big(-ik(x+l)\big)\right)$$

$$= \int d^4 k \left(C^{\alpha}(k) \exp(ikx) \exp(ikl) + (C^{\alpha}(k))^{*} \exp(-ikx) \exp(-ikl)\right) \tag{2.6}$$

Then we have

$$\frac{\partial \Delta(\xi^{\alpha})}{\partial x^{\mu}} = \frac{\partial \int d^4 l \left(C^{\alpha}(x+l) \cdot \exp(-\left|\frac{l}{L_P(x+l)}\right|)\right)}{\partial x^{\mu}}$$

$$= \int d^4 k d^4 l \Bigg[\Big(ik_{\mu} C^{\alpha}(k) \exp(ikx) \exp(ikl)$$

$$- ik_{\mu} (C^{\alpha}(k))^{*} \exp(-ikx) \exp(-ikl)\Big) \cdot \exp(-\left|\frac{l}{L_P(k)}\right|)\Bigg] \tag{2.7}$$

$$= \int d^4 k \left(\frac{2|L_P|}{1 - ikL_P} ik_{\mu} C^{\alpha}(k) \exp(ikx) - \frac{2|L_P|}{1 + ikL_P} ik_{\mu} (C^{\alpha}(k))^{*} \exp(-ikx)\right)$$

In momentum space, the metric with the self-interaction can be written as follows

$$g_{\mu\nu}[\lambda(\xi)] = \frac{\partial\lambda(\xi^\alpha)}{\partial x^\mu}\frac{\partial\lambda(\xi^\beta)}{\partial x^\nu}\eta_{\alpha\beta}$$

$$= \frac{\partial\Big(\xi^\alpha(x,X)\big|_{X=0} + \Delta\xi^\alpha\Big)}{\partial x^\mu}\frac{\partial\Big(\xi^\beta(x,X)\big|_{X=0} + \Delta\xi^\beta\Big)}{\partial x^\nu}\eta_{\alpha\beta} \tag{2.8}$$

$$\equiv g_{\mu\nu}[\,\xi\,] + g^{(1)}_{\mu\nu} + g^{(2)}_{\mu\nu}$$

By Eq.(2.7) we get

$$g_{\mu\nu}[\xi] = \Bigg[\int d^4k\left(ik_\mu C^\alpha(k)\exp(ikx) - ik_\mu\left(C^\alpha(k)\right)^*\exp(-ikx)\right)$$
$$\cdot\int d^4k\left(ik_\nu C^\beta(k)\exp(ikx) - ik_\nu\left(C^\beta(k)\right)^*\exp(-ikx)\right)\Bigg]\cdot\eta_{\alpha\beta} \tag{2.9}$$

$$g^{(1)}_{\mu\nu} = 2\cdot\Bigg[\int d^4k\left(\frac{2|L_P(k)|}{1+ikL_P(k)}ik_\mu C^\alpha(k)\exp(ikx) - \frac{2|L_P(k)|}{1-ikL_P(k)}ik_\mu\left(C^\alpha(k)\right)^*\exp(-ikx)\right)$$
$$\cdot\int d^4k\left(ik_\nu C^\beta(k)\exp(ikx) - ik_\nu\left(C^\beta(k)\right)^*\exp(-ikx)\right)\Bigg]\cdot\eta_{\alpha\beta} \tag{2.10}$$

$$g^{(2)}_{\mu\nu} = \Bigg[\int d^4k\left(\frac{2|L_P(k)|}{1+ikL_P(k)}ik_\mu C^\alpha(k)\exp(ikx) - \frac{2|L_P(k)|}{1-ikL_P(k)}ik_\mu\left(C^\alpha(k)\right)^*\exp(-ikx)\right)$$
$$\cdot\int d^4k\left(\frac{2|L_P(k)|}{1+ikL_P(k)}ik_\nu C^\beta(k)\exp(ikx) - \frac{2|L_P(k)|}{1-ikL_P(k)}ik_\nu\left(C^\beta(k)\right)^*\exp(-ikx)\right)\Bigg]\cdot\eta_{\alpha\beta} \tag{2.11}$$

Denote

$$f(k) \equiv \frac{2|L_P(k)|}{1+ik_\mu L_P(k)}, \quad f^*(k) \equiv \frac{2|L_P(k)|}{1-ik_\mu L_P(k)} \tag{2.12}$$

Using the mean value theorem of definite integrals, we have

$$g^{(1)}_{\mu\nu} = 2\cdot\Bigg[\int d^4k\left(f(k)\cdot ik_\mu C^\alpha(k)\exp(ikx)\right) - d^4k\left(f^*(k)\cdot ik_\mu\left(C^\alpha(k)\right)^*\exp(-ikx)\right)\Bigg]$$
$$\cdot\Bigg[\int d^4k\left(ik_\nu C^\beta(k)\exp(ikx) - ik_\nu\left(C^\beta(k)\right)^*\exp(-ikx)\right)\Bigg]\cdot\eta_{\alpha\beta}$$

$$= 2 \cdot \left[f(\zeta_r) \cdot \int d^4k \left(ik_\mu C^\alpha(k) \exp(ikx) \right) - f^*(\zeta_r^*) \cdot \int d^4k \left(ik_\mu \left(C^\alpha(k) \right)^* \exp(-ikx) \right) \right)$$

$$\cdot \int d^4k \left(ik_\nu C^\beta(k) \exp(ikx) - ik_\nu \left(C^\beta(k) \right)^* \exp(-ikx) \right) \Bigg] \cdot \eta_{\alpha\beta} \tag{2.13}$$

$$g_{\mu\nu}^{(2)} = \left[\int d^4k \left(f(k) \cdot ik_\mu C^\alpha(k) \exp(ikx) \right) - d^4k \left(f^*(k) \cdot ik_\mu \left(C^\alpha(k) \right)^* \exp(-ikx) \right) \right]$$

$$\cdot \left[\int d^4k \left(f(k) \cdot ik_\mu C^\beta(k) \exp(ikx) \right) - d^4k \left(f^*(k) \cdot ik_\mu \left(C^\beta(k) \right)^* \exp(-ikx) \right) \right] \cdot \eta_{\alpha\beta} \tag{2.14}$$

$$= \left[f^*(\zeta_r) \cdot \int d^4k \left(ik_\mu C^\alpha(k) \exp(ikx) \right) - f^*(\zeta_r^*) \cdot \int d^4k \left(ik_\mu \left(C^\alpha(k) \right)^* \exp(-ikx) \right) \right]$$

$$\cdot \left[f(\zeta_r) \cdot \int d^4k \left(ik_\mu C^\beta(k) \exp(ikx) \right) - f^*(\zeta_r^*) \cdot \int d^4k \left(ik_\mu \left(C^\beta(k) \right)^* \exp(-ikx) \right) \right] \cdot \eta_{\alpha\beta}$$

where $f(\zeta)$ is the mean value of $f(k)$, $f^*(\zeta^*)$ is the mean value of $f^*(k)$.

Now we found the self-interaction $g_{\mu\nu}^{(1)} + g_{\mu\nu}^{(2)}$ of the noncommutative quantum gravity.

Let's discuss a macroscopic system by a semi classical approach, the case of a general static isotropic gravitational field. The general static isotropic metric is:

$$ds^2 = g_{rr} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 - g_{tt} dt^2$$

$$g_{rr} = \left[1 - \frac{2MG}{r} \right]^{-1}, \quad g_{tt} = \left[1 - \frac{2MG}{r} \right] \tag{2.15}$$

We can also express it in the equivalent isotropic form, by introducing a new radius variable ρ

$$\rho \equiv \frac{1}{2} \left[r - MG + (r^2 - 2mGr)^{1/2} \right] \tag{2.16}$$

or

$$r = \rho \left(1 + \frac{MG}{2\rho} \right)^2 \tag{2.17}$$

Substituting it into Eq.(2.15) gives the isotropic form as follows

$$ds^2 = \left(1 - \frac{MG}{2\rho}\right)^4 \left(d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\phi^2\right) - \left(\frac{(1 - MG/2\rho)^2}{(1 + MG/2\rho)^2}\right) dt^2 \quad (2.18)$$

Compare Eq.(2.9) and Eq.(2.15), we can obtain that the element g_{rr} is

$$g_{rr}[\xi] = \left[\int d^4k \left(ik_r C^\alpha(k) \exp(ikx) - ik_r \left(C^\alpha(k)\right)^* \exp(-ikx)\right)\right.$$

$$\left. \cdot \int d^4k \left(ik_r C^\beta(k) \exp(ikx) - ik_r \left(C^\beta(k)\right)^* \exp(-ikx)\right)\right] \cdot \eta_{\alpha\beta} \quad (2.19)$$

$$= \left[1 - \frac{2MG}{r}\right]^{-1}$$

Then for the element $g_{rr}[\xi]$ of the general static isotropic metric, we have

$$\int d^4k \left(ik_r C^\alpha(k) \exp(ikx)\right) = \int d^4k \left(-ik_r \left(C^\alpha(k)\right)^* \exp(-ikx)\right)$$

$$= \frac{1}{2}\left[1 - \frac{2MG}{r}\right]^{-1/2} \quad (2.20)$$

Denote

$$f(k) \equiv \frac{2|L_P(k)|}{1 + ikL_P(k)}, \qquad f^*(k) \equiv \frac{2|L_P(k)|}{1 - ikL_P(k)} \quad (2.21)$$

Using the mean value theorem of definite integrals, we have

$$\int d^4k \left(f(k) \cdot ik_r C^\alpha(k) \exp(ikx)\right)$$

$$= f(\zeta_r) \int d^4k \left(ik_r C^\alpha(k) \exp(ikx)\right) \quad (2.22)$$

$$= f(\zeta_r) \cdot \frac{1}{2}\left[1 - \frac{2MG}{r}\right]^{-1/2}$$

$$\int d^4k \left(f^*(k) \cdot \left(-ik_r \left(C^\alpha(k)\right)^* \exp(-ikx) \right) \right)$$

$$= f(\zeta_r^*) \int d^4k \left(-ik_r \left(C^\alpha(k)\right)^* \exp(-ikx) \right) \tag{2.23}$$

$$= f^*(\zeta_r^*) \cdot \frac{1}{2} \left[1 - \frac{2MG}{r} \right]^{-1/2}$$

Then $g_{rr}^{(1)}$ is

$$g_{rr}^{(1)} = 2 \cdot \left[\int d^4k \left(\frac{2|L_P(k)|}{1 + ik_r L_P(k)} ik_r C^\alpha(k) \exp(ikx) - \frac{2|L_P(k)|}{1 - ik_r L_P(k)} ik_r \left(C^\alpha(k)\right)^* \exp(-ikx) \right) \right.$$

$$\left. \cdot \int d^4k \left(ik_r C^\beta(k) \exp(ikx) - ik_r \left(C^\beta(k)\right)^* \exp(-ikx) \right) \right] \cdot \eta_{\alpha\beta}$$

$$= 2 \cdot \left[f(\zeta_r) \cdot \frac{1}{2} \left[1 - \frac{2MG}{r} \right]^{-1/2} + f^*(\zeta_r^*) \cdot \frac{1}{2} \left[1 - \frac{2MG}{r} \right]^{-1/2} \right] \cdot \left[1 - \frac{2MG}{r} \right]^{-1/2} \tag{2.24}$$

$$= \left(f(\zeta_r) + f^*(\zeta_r^*) \right) \cdot \left[1 - \frac{2MG}{r} \right]^{-1}$$

And $g_{rr}^{(2)}$ is

$$g_{rr}^{(2)} = \left[\int d^4k \left(\frac{2|L_P(k)|}{1 + ikL_P(k)} ik_r C^\alpha(k) \exp(ikx) - \frac{2|L_P(k)|}{1 - ikL_P(k)} ik_r \left(C^\alpha(k)\right)^* \exp(-ikx) \right) \right.$$

$$\left. \cdot \int d^4k \left(\frac{2|L_P(k)|}{1 + ikL_P(k)} ik_r C^\beta(k) \exp(ikx) - \frac{2|L_P(k)|}{1 - ikL_P(k)} ik_r \left(C^\beta(k)\right)^* \exp(-ikx) \right) \right] \cdot \eta_{\alpha\beta} \tag{2.25}$$

$$= \left[f(\zeta_r) \cdot \frac{1}{2} \left[1 - \frac{2MG}{r} \right]^{-1/2} + f^*(\zeta_r^*) \cdot \frac{1}{2} \left[1 - \frac{2MG}{r} \right]^{-1/2} \right]^2$$

$$= \left(\frac{f(\zeta_r) + f^*(\zeta_r^*)}{2} \right)^2 \cdot \left[1 - \frac{2MG}{r} \right]^{-1}$$

Denote the true metric $\mathrm{g}_{\mu\nu} \equiv g_{\mu\nu} + g^{(1)}_{\mu\nu} + g^{(2)}_{\mu\nu}$ for short. The element g_{rr} of the true general static isotropic metric can be written as follows

$$\mathrm{g}_{rr} = \left(1 + \frac{f(\zeta_r) + f(\zeta_r^*)}{2}\right)^2 \cdot \left[1 - \frac{2MG}{r}\right]^{-1} \tag{2.26}$$

And g_{tt} is

$$\mathrm{g}_{tt} = \left(1 + \frac{f(\zeta_t) + f(\zeta_t^*)}{2}\right)^2 \cdot \left[1 - \frac{2MG}{r}\right] \tag{2.27}$$

Due to the general static isotropic gravitational field, we have

$$\mathrm{g}_{\theta\theta} = r^2, \quad \mathrm{g}_{\phi\phi} = r^2 \sin\theta \tag{2.28}$$

Denote

$$\Delta_r \equiv \frac{f(\zeta_r) + f(\zeta_r^*)}{2}, \qquad \Delta_t \equiv \frac{f(\zeta_t) + f(\zeta_t^*)}{2} \tag{2.29}$$

Then the true metric $\mathrm{g}_{\mu\nu}$ can be written as

$$\begin{aligned} ds^2 = (1+\Delta_r)^2 \cdot \left[1 - \frac{2MG}{r}\right]^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \\ - (1+\Delta_t)^2 \cdot \left[1 - \frac{2MG}{r}\right] dt^2 \end{aligned} \tag{2.30}$$

We can also express the true metric $\mathrm{g}_{\mu\nu}$ in the equivalent isotropic form, by introducing a new radius variable ρ

$$\rho \equiv \frac{1}{2}\left[(1+\Delta_r)^{-1} \cdot r - MG + \left((1+\Delta_r)^{-2} \cdot r^2 - 2MG\,(1+\Delta_r)^{-1} \cdot r \right)^{1/2} \right] \tag{2.31}$$

or

$$r = \rho\,(1+\Delta_r)\left(1 + \frac{MG}{2\rho}\right)^2 \tag{2.32}$$

Substituting it into Eq.(2.15) gives the isotropic form as follows

$$\begin{aligned} ds^2 = (1+\Delta_r)^2 \cdot \left(1 - \frac{MG}{2\rho}\right)^4 \left(d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\phi^2\right) \\ - (1+\Delta_t)^2 \cdot \left(\frac{1 - MG/2\rho}{1 + MG/2\rho}\right)^2 dt^2 \end{aligned} \tag{2.33}$$

Compare with the metric $g_{\mu\nu}$ (2.15) or (2.18), due to the self-interaction, the spacetime described by the true metric $\mathrm{g}_{\mu\nu}$ has been expanded compared to the space described by metric $g_{\mu\nu}[\xi]$. The radius has expanded to $1+\Delta_r$ times. Because the boundary condition is determined by the same gravitational field equation, from the view point of gravity, the extended spacetime described by the true metric $\mathrm{g}_{\mu\nu}$ is equivalent to the spacetime described by the metric $g_{\mu\nu}[\xi]$. The spacetime described by the metric $g_{\mu\nu}[\xi]$ follows the inverse square law, therefore the gravity of the extended spacetime is stronger than what is given by the inverse square law. In the spacetime described by the true metric $\mathrm{g}_{\mu\nu}$, the gravity at a distance of $\left(1+\Delta_r\right)\cdot r$ from the gravitational source is equal to the gravity of the inverse square law at a distance of r from the gravitational source. It is not modified on the inverse square law, because the boundary condition still determined by the Einstein's field equation.

The mean value Δ_r is related to the boundary condition determined by the Einstein's field equation. In the general static isotropic gravitational field, the only parameter is the mass M of the gravitational source. We can expect that the stronger the gravitational source, the stronger the energy-momentum k of the excited gravitons, and the larger the median value Δ_r. So that if the galaxy with strong enough gravitational source is large enough, the distance from the gravitational source is far enough, the deviation from the inverse square law can be observed.

The self-interaction of the gravitational field also changes the energy-momentum tensor of the gravitational field itself. Let's briefly explain. From the canonical field theory, the energy-momentum tensor of the gravitational field itself is

$$\begin{aligned} t_{\mu\nu}(x) &= -\frac{\eta_{\mu\nu}}{2}\partial^\kappa\lambda(\xi^\alpha)\partial_\kappa\lambda(\xi^\beta)\eta_{\alpha\beta}+\partial_\mu\lambda(\xi^\alpha)\partial_\nu\lambda(\xi^\beta)\eta_{\alpha\beta} \\ &= -\frac{\eta_{\mu\nu}}{2}\partial^\kappa\left(\xi^\alpha\big|_{X=0}+\Delta\xi^\alpha\right)\partial_\kappa\left(\xi^\beta\big|_{X=0}+\Delta\xi^\beta\right)\eta_{\alpha\beta} \\ &\quad +\partial_\mu\left(\xi^\alpha\big|_{X=0}+\Delta\xi^\alpha\right)\partial_\nu\left(\xi^\beta\big|_{X=0}+\Delta\xi^\beta\right)\eta_{\alpha\beta} \end{aligned} \tag{2.34}$$

(We drop the subscript $X=0$ from now on.) It can be written as the classical part $t_{\mu\nu}(\xi)$ and the quantum part $t_{\mu\nu}(\Delta\xi)$

$$t_{\mu\nu}(x)\equiv t_{\mu\nu}(\xi)+t_{\mu\nu}(\Delta\xi) \tag{2.35}$$

The classical part $t_{\mu\nu}(\xi)$ is

$$t_{\mu\nu}(\xi)=-\frac{\eta_{\mu\nu}}{2}\partial^\kappa\xi^\alpha\partial_\kappa\xi^\beta\eta_{\alpha\beta}+\partial_\mu\xi^\alpha\partial_\nu\xi^\beta\eta_{\alpha\beta} \tag{2.36}$$

The energy-momentum tensor of gravitational field itself in the general theory of relativity is

$$t_{\mu\nu}=\frac{1}{8\pi G}\left(\frac{1}{2}\eta_{\mu\nu}R^{(1)}-R^{(1)}_{\mu\nu}\right) \tag{2.37}$$

In the paper[1], we have proven that it is equivalent to the classical part of energy-momentum tensor (2.36).

The quantum part $t_{\mu\nu}(\Delta\xi)$ is

$$t_{\mu\nu}(\Delta\xi) = -\frac{\eta_{\mu\nu}}{2}\Big[\partial^\kappa\xi^\alpha\partial_\kappa(\Delta\xi^\beta) + \partial^\kappa(\Delta\xi^\alpha)\partial_\kappa\xi^\beta + \partial^\kappa(\Delta\xi^\alpha)\partial_\kappa(\Delta\xi^\beta)\Big]\eta_{\alpha\beta}$$
$$+\Big[\partial_\mu\xi^\alpha\partial_\nu(\Delta\xi^\beta) + \partial_\mu(\Delta\xi^\alpha)\partial_\nu\xi^\beta + \partial_\mu(\Delta\xi^\alpha)\partial_\nu(\Delta\xi^\beta)\Big]\eta_{\alpha\beta} \tag{2.38}$$

It can be written as

$$t_{\mu\nu}(\Delta\xi) \equiv t^{(1)}_{\mu\nu} + t^{(2)}_{\mu\nu} \tag{2.39}$$

where

$$t^{(1)}_{\mu\nu} = -\frac{\eta_{\mu\nu}}{2}\Big[\partial^\kappa\xi^\alpha\partial_\kappa(\Delta\xi^\beta) + \partial^\kappa(\Delta\xi^\alpha)\partial_\kappa\xi^\beta\Big]\eta_{\alpha\beta}$$
$$+\Big[\partial_\mu\xi^\alpha\partial_\nu(\Delta\xi^\beta) + \partial_\mu(\Delta\xi^\alpha)\partial_\nu\xi^\beta\Big]\eta_{\alpha\beta} \tag{2.40}$$
$$t^{(2)}_{\mu\nu} = -\frac{\eta_{\mu\nu}}{2}\partial^\kappa(\Delta\xi^\alpha)\partial_\kappa(\Delta\xi^\beta)\eta_{\alpha\beta} + \partial_\mu(\Delta\xi^\alpha)\partial_\nu(\Delta\xi^\beta)\eta_{\alpha\beta}$$

The quantum part is the change in energy-momentum tensor caused by the self-interaction of the gravitational field.

III. Symmetry of Klein-Gordon Equation

Due to the fact that the graviton ξ^α satisfies the free field equation, and the free field equation is a wave equation, then the field $C^\alpha(x)$ can be expanded as a linear superposition of the form:

$$C^\alpha(x) = C^\alpha(k)\exp\big(ikx\big) + (C^\alpha(k))^*\exp\big(-ikx\big) \tag{3.1}$$

From this property, we can find a symmetry of the Klein-Gordon equation.

In the gravitational field with metric $g_{\mu\nu}$, the Lagrangian density of real scalar particle with spin 0 is

$$\mathscr{L} = g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi + m^2\phi^2 \tag{3.2}$$

We can obtain the Klein-Gordon equation in the gravitational field as follows

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial\Phi}{\partial x^\nu}\right) - m^2\Phi = 0 \tag{3.3}$$

It can be written as

$$g^{\mu\nu}\frac{\partial^2\Phi}{\partial x^\mu\partial x^\nu} + \frac{1}{2g}\frac{\partial g}{\partial x^\mu}g^{-1}_{\mu\nu}\frac{\partial\Phi}{\partial x^\nu} - g^{-1}_{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial x^\mu}g^{-1}_{\mu\nu}\frac{\partial\Phi}{\partial x^\nu} - m^2\Phi = 0 \tag{3.4}$$

The inverse of the metric $g_{\mu\nu}$ is

$$g^{-1}_{\mu\nu} = \frac{1}{g}\left[g^*\right]^{\mu\nu} \tag{3.5}$$

where $\left[g^*\right]^{\mu\nu}$ is the adjoint matrix of the metric $g_{\mu\nu}$.
Then Eq.(3.5) can be expressed as follows

$$g^{\mu\nu}\frac{\partial^2\Phi}{\partial x^\mu\partial x^\nu}+\left(\frac{1}{2g}\frac{\partial g}{\partial x^\mu}-\frac{1}{g}\left[g^*\right]^{\lambda\kappa}\frac{\partial g_{\lambda\kappa}}{\partial x^\mu}\right)g^{-1}_{\mu\nu}\frac{\partial\Phi}{\partial x^\nu}-m^2\Phi=0 \tag{3.6}$$

For the metric (2.9), using the mean value theorem of definite integrals, it can be written as

$$\begin{aligned} g_{\mu\nu}=-\Bigg[&\left(\varsigma_\mu\cdot\int d^4k\,C^\alpha(k)\exp(ikx)-\varsigma^*_\mu\cdot\int d^4k\,\left(C^\alpha(k)\right)^*\exp(-ikx)\right)\\ &\cdot\left(\varsigma_\nu\cdot\int d^4k\,C^\alpha(k)\exp(ikx)-\varsigma^*_\nu\cdot\int d^4k\,\left(C^\alpha(k)\right)^*\exp(-ikx)\right)\Bigg] \end{aligned} \tag{3.7}$$

where k_μ is the function of mean value, ς_μ and ς^*_μ are the mean value of k_μ.
For the form of metric (3.7), we have

$$\frac{1}{2g}\frac{\partial g}{\partial x^\mu}=\frac{1}{g}\left[g^*\right]^{\lambda\kappa}\frac{\partial g_{\lambda\kappa}}{\partial x^\mu} \tag{3.8}$$

For the true metric $\mathrm{g}_{\mu\nu}$, Eq.(3.8) still holds true.
So that Eq.(3.6) can be written as

$$g^{\mu\nu}\frac{\partial^2\Phi}{\partial x^\mu\partial x^\nu}-m^2\Phi=0 \tag{3.9}$$

Therefore the Klein-Gordon equation in the gravitational field be the usual form as follows

$$\left(\Box^2-m^2\right)\Phi=0 \tag{3.10}$$

where $\Box^2$ is the usual D'Alembertian operator in curved spacetime

$$\Box^2=g^{\mu\nu}\frac{\partial^2}{\partial x^\mu\partial x^\nu} \tag{3.11}$$

Due to the local inertial coordinate, i.e. the graviton, satisfies the wave equation, there is the symmetry of the Klein Gordon equation. It is a symmetry that only holds true in quantum gravity theory.

IV. Conclusion

From the calculation in this paper, it can be seen that although the inverse square law is correct, the true gravitational field does not follow the inverse square law due to the self-interaction. It can be used to explain dark matter.

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