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Calculation of Macroscopic Effects of Quantum Gravity in General Static Isotropic Gravitational Fields

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Abstract- In this paper we calculated the self-interaction of the gravitational field in coordinate space by approximate method, and find the functional relationship between gravitational source and distance and self-interaction of quantum gravity. The calculation result shows that the self-interaction of quantum gravity can explain the gravitational effects of dark matter. In the solar system, the self-interaction is extremely weak, but it is enough to explain the the Pioneer anomaly.

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I. Introduction

In the paper [1] we discusses the effects of self-interaction of quantum gravity and given the expression of self-interaction in momentum space. In this paper we calculated the effects of self-interaction of quantum gravity in coordinate space. We find the functional relationship between self-interaction of quantum gravity and gravity source and distance. By substituting observational data directly into the calculation, we can determine whether it can be used to explain the gravitational effects of dark matter and the the Pioneer anomaly.

II. Self-Interaction of Noncommutative Quantum Gravity

This paper omits the introduction of the quantum gravity theory. For details of the quantum gravity theory, please refer to [1] [2] [3] [4].

Let's discuss the general static isotropic gravitational field. The spherical polar coordinate system of a general static isotropic gravitational field is $r^i = (r, \theta, \phi, t)$. The general static isotropic metric is

$$ds^2 = g_{rr}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 - g_{tt}dt^2$$

$$g_{rr} = \left[1 - \frac{2MG}{r}\right]^{-1}, \quad g_{tt} = \left[1 - \frac{2MG}{r}\right] \tag{2.1}$$

At the point r of the spherical polar coordinate system r^i, we establish a spherical polar coordinate system $l^m = (l, \Theta, \Phi, T)$. The direction of the polar axis l is consistent with the polar axis r of the spherical polar coordinate system r^i, the time axis T is the same as the time axis t. In the quantum gravitational

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field, there exists the self-interaction between gravitons, after considering the effects of all gravitons, the locally inertial coordinate system $\lambda(\xi^\alpha)$ at point r have to written as

$$\lambda(\xi^\alpha) = \xi^\alpha + \Delta\xi^\alpha$$

$$\Delta\xi^\alpha = \int d^4 l\, \xi^\alpha\big((r+l), |l|\big) \tag{2.2}$$

$$= \int d^4 l \left(C^\alpha(r+l) \cdot \exp(-\left|\frac{l}{L_P(x+l)}\right|)\right)$$

In a general static isotropic gravitational field, the equipotential surface are the spherical surface. On a small range and far away from the gravitational source, the equipotential surface can be regarded as a plane. Therefore the increment $\Delta\xi^m$ of the locally inertial coordinate system is

$$\Delta\xi^\alpha = \int dl\, d\Theta\, d\Phi\, dT \left(C^\alpha(r + l\cos\Theta) \cdot \exp(-\frac{l}{|L_P(r+l)|})\right) \tag{2.3}$$

At the point l of the coordinate system l^i, the metric of gravitational field can be written as

$$g_{ij}(r+l) = \frac{\partial C^\alpha(r+l\cos\Theta)}{\partial r^i} \frac{\partial C^\beta(r+l\cos\Theta)}{\partial r^j} \cdot \eta_{\alpha\beta}$$

$$ds^2 = \left[1 - \frac{2MG}{r+l\cos\Theta}\right]^{-1} dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2 - \left[1 - \frac{2MG}{r+l\cos\Theta}\right] dt^2 \tag{2.4}$$

Then we have

$$\begin{cases} \dfrac{\partial C^{\alpha}(r+l\cos\Theta)}{\partial r} = \left[1-\dfrac{2MG}{r+l\cos\Theta}\right]^{-1/2} \\ \dfrac{\partial C^{\alpha}(r+l\cos\Theta)}{\partial \theta} = 1 \\ \dfrac{\partial C^{\alpha}(r+l\cos\Theta)}{\partial \phi} = 1 \\ \dfrac{\partial C^{\alpha}(r+l\cos\Theta)}{\partial t} = \left[1-\dfrac{2MG}{r+l\cos\Theta}\right]^{1/2} \end{cases} \tag{2.5}$$

From Eq,[2.3] and Eq.[2.5], for the increment $\Delta\xi^m$ of locally inertial coordinate system, we have

$$\frac{\partial\Delta\xi^{\alpha}}{\partial r} = \left(\frac{\partial\Delta\xi^{l}}{\partial r}, \frac{\partial\Delta\xi^{\Theta}}{\partial r}, \frac{\partial\Delta\xi^{\Phi}}{\partial r}, \frac{\partial\Delta\xi^{T}}{\partial r}\right) \tag{2.6}$$

where

$$\begin{aligned} \frac{\partial\Delta\xi^{l}}{\partial r} &= \int dl\, d\Theta\, d\Phi \left(\frac{\partial C^{l}(r+l\cos\Theta)}{\partial r} \cdot \exp(-\frac{l}{|L_P(r+l)|})\right) \\ &= \int dl\, d\Theta\, d\Phi \left(\left[1-\frac{2MG}{r+l\cos\Theta}\right]^{-1/2} \cdot \exp(-\frac{l}{|L_P(r+l)|})\right) \\ &= \int dl\, d\Theta\, d\Phi \left(\sqrt{\frac{l\cos\Theta+r}{l\cos\Theta+r-2MG}} \cdot \exp(-\frac{l}{|L_P(r+l)|})\right) \end{aligned} \tag{2.7}$$

$$\frac{\partial\Delta\xi^{\Theta}}{\partial r} = 0$$

$$\frac{\partial\Delta\xi^{\Phi}}{\partial r} = 0$$

$$\frac{\partial\Delta\xi^{T}}{\partial r} = 0$$

We need to calculate the radial changes in the spatial part of the metric of gravitational field. Denote

$$F_r(r,M) \equiv \frac{\partial \Delta \xi^l}{\partial r} \tag{2.8}$$

From Eq.[2.7] we have

$$F_r(r,M) = \int dl\, d\Theta\, d\Phi \left(\sqrt{\frac{l\cos\Theta + r}{l\cos\Theta + r - 2MG}} \cdot \exp(-\frac{l}{\left|L_P(r+l)\right|}) \right) \tag{2.9}$$

First, integrate over l, it can be written as follows

$$\int_0^\infty dx \left(\sqrt{\frac{Ax+C}{Ax+D}} \cdot \exp(Bx) \right) \tag{2.10}$$

where

$$\text{variable}: \; x = l$$

$$\text{constant}: \; A = \cos\theta, \; B = -\frac{1}{\left|L_P(r+l)\right|}, \; C = r, \; D = r - 2MG \tag{2.11}$$

Using substitution rule for definite integrals, let

$$y = \sqrt{\frac{Ax+C}{Ax+D}} \tag{2.12}$$

Then

$$x = \frac{Dy^2 - C}{A(1-y^2)}$$

$$dx = \frac{2(D-C)y}{A(1-y^2)^2} dy \tag{2.13}$$

Using definite integration by parts, we have

$$\int dx \left(\sqrt{\frac{Ax+C}{Ax+D}} \cdot \exp(Bx)\right)$$

$$= \int dy \left(y \cdot \frac{2(D-C)y}{A(1-y^2)^2} \cdot \exp(B \cdot \frac{Dy^2-C}{A(1-y^2)})\right)$$

$$= \frac{1}{B} \cdot y \cdot \exp(B \cdot \frac{Dy^2-C}{A(1-y^2)}) - \frac{1}{B} \cdot \int dy \, \exp(B \cdot \frac{Dy^2-C}{A(1-y^2)}) \tag{2.14}$$

$$= -|L_P| \cdot \sqrt{\frac{Ax+C}{Ax+D}} \cdot \exp(-\frac{x}{|L_P|})\Bigg|_0^{\infty} - \frac{1}{B} \cdot \int dy \, \exp(B \cdot \frac{Dy^2-C}{A(1-y^2)})$$

$$= -|L_P| \cdot \sqrt{\frac{C}{D}} - \frac{1}{B} \cdot \int dy \, \exp(B \cdot \frac{Dy^2-C}{A(1-y^2)})$$

The integrand in the second term can be represented as the McLaughlin series

$$- \frac{1}{B} \cdot \int dy \, \exp(B \cdot \frac{Dy^2-C}{A(1-y^2)})$$

$$= |L_P| \cdot \int dy \left(\sum_{n=0}^{\infty} \frac{\left(-\frac{1}{|L_P|} \cdot \frac{Dy^2-C}{A(1-y^2)}\right)^n}{n!}\right) \tag{2.15}$$

$$= |L_P| \cdot \sum_{n=0}^{\infty} \left(\frac{1}{n!} \int_a^b dy \left(\frac{1}{|L_P|} \cdot \frac{Dy^2-C}{A(y^2-1)}\right)^n\right)$$

If $x \in (0, \infty)$, we have $y \in \left(\sqrt{\frac{C}{D}},\ 1\right)$. Notice that the McLaughlin series are undefined while $y = 1$. Since we only need to integrate within a small range

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at point r, we can make a infrared cutoff. To simplify the calculation, we set the integral interval as $y \in (a, b) = \left(\sqrt{\frac{C}{D}}, \sqrt{\frac{C+1}{D+1}}\right)$.

The McLaughlin series (2.15) rapidly decays if the variable x is extremely small. In weak-field approximation, we take the first two terms of the McLaughlin series for approximate calculation

$$
\begin{aligned}
&|L_P| \cdot \sum_{n=0}^{\infty} \left(\frac{1}{n!} \int_a^b dy \left(\frac{1}{|L_P|} \cdot \frac{Dy^2 - C}{A(y^2-1)} \right)^n \right) \\
&\approx |L_P| + |L_P| \cdot \int_a^b dy \left(\frac{1}{|L_P|} \cdot \frac{Dy^2 - C}{A(y^2-1)} \right) \\
&= |L_P| + \left(\int_a^b dy \left(\frac{Dy^2}{A(y^2-1)} \right) - \int_a^b dy \left(\frac{C}{A(y^2-1)} \right) \right) \\
&= |L_P| + \left(\frac{D}{A} y \right) \Big|_a^b - \frac{C-D}{A} \cdot \int_a^b dy \left(\frac{1}{y^2-1} \right) \\
&= |L_P| + \left(\frac{D}{A} y \right) \Big|_a^b - \left(\frac{C-D}{2A} \ln \frac{y-1}{y+1} \right) \Big|_a^b \\
&= |L_P| + \left(\frac{D}{A} \left(\sqrt{\frac{C+1}{D+1}}) - \sqrt{\frac{C}{D}} \right) \right) - \left(\frac{C-D}{2A} \ln \frac{y-1}{y+1} \right) \Big|_a^b
\end{aligned}
\tag{2.16}
$$

Then

$$
\begin{aligned}
&\int dx \left(\sqrt{\frac{Ax+C}{Ax+D}} \cdot \exp(Bx) \right) \\
&= -|L_P| \cdot \sqrt{\frac{C}{D}} + |L_P| + \left[\frac{D}{A} \left(\sqrt{\frac{C+1}{D+1}} - \sqrt{\frac{C}{D}} \right) \right] - \left(\frac{C-D}{2A} \ln \frac{y-1}{y+1} \right) \Big|_a^b
\end{aligned}
\tag{2.17}
$$

If $r \gg 2MG \gg 1$, we have

$$-|L_P| \cdot \sqrt{\frac{C}{D}} + |L_P| + \frac{D}{A}\left(\sqrt{\frac{C+1}{D+1}} - \sqrt{\frac{C}{D}}\right) \approx 0 \tag{2.18}$$

Then Eq.[2.17] can be written as

$$\int dx \left(\sqrt{\frac{Ax+C}{Ax+D}} \cdot \exp(Bx)\right) \approx -\left(\frac{C-D}{2A} \ln \frac{y-1}{y+1}\right)\Big|_a^b \tag{2.19}$$

Next, by calculate the multiple integrals over Θ and Φ, we can get a coefficient K. Then $F_r(r, M)$ can be written as

$$F_r(r,M) = \int d\Theta d\Phi \left[-\left(\frac{C-D}{2A} \cdot \ln \frac{y-1}{y+1}\right)\Big|_a^b\right]$$

$$= \int d\Theta d\Phi \left[-\frac{2MG}{2A}\left(\ln \frac{\sqrt{C+1}-\sqrt{D+1}}{\sqrt{C+1}+\sqrt{D+1}} - \ln \frac{\sqrt{C}-\sqrt{D}}{\sqrt{C}+\sqrt{D}}\right)\right]$$

$$= K \cdot MG \cdot \ln\left(\frac{\sqrt{C+1}+\sqrt{D+1}}{\sqrt{C+1}-\sqrt{D+1}} \cdot \frac{\sqrt{C}-\sqrt{D}}{\sqrt{C}+\sqrt{D}}\right) \tag{2.20}$$

Substituting Eq.(2.11) into Eq.(2.20) we have

$F_r(r, M)$

$$= KMG \cdot \ln \frac{\left(\sqrt{r^2+r} - \sqrt{(2MG)^2+2MG}\right) + \left(\sqrt{r(1+2MG)} - \sqrt{2MG(1+r)}\right)}{\left(\sqrt{r^2+r} - \sqrt{(2MG)^2+2MG}\right) - \left(\sqrt{r(1+2MG)} - \sqrt{2MG(1+r)}\right)} \tag{2.21}$$

Denote

$$\Delta_r \equiv F_r(r,M) \cdot \left[1 - \frac{2MG}{r}\right]^{1/2} \tag{2.22}$$

Then the metric g_{ij} of gravitational fields with self-interaction can be written as

$$ds^2 = (1+\Delta_r)^2 \cdot \left[1 - \frac{2MG}{r}\right]^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

$$- (1+\Delta_t)^2 \cdot \left[1 - \frac{2MG}{r}\right] dt^2 \qquad (2.23)$$

Similar as the Schwarzschild metric g_{ij}, we can also express the true metric g_{ij} in the equivalent isotropic form, by introducing a new radius variable ρ

$$\rho \equiv \frac{1}{2}\left[(1+\Delta_r)^{-1} \cdot r - MG + \left((1+\Delta_r)^{-2} \cdot r^2 - 2MG\,(1+F_r)^{-1} \cdot r \right)^{1/2} \right] \qquad (2.24)$$

or

$$r = \rho\,(1+\Delta_r)\left(1 + \frac{MG}{2\rho}\right)^2 \qquad (2.25)$$

Substituting it into Eq.(2.23) gives the isotropic form as follows

$$ds^2 = (1+\Delta_r)^2 \cdot \left(1 + \frac{MG}{2\rho}\right)^4 (d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\phi^2)$$

$$- (1+\Delta_t)^2 \cdot \left(\frac{1 - MG/2\rho}{1 + MG/2\rho}\right)^2 dt^2 \qquad (2.26)$$

From Eq.(2.25), we can find a radial scale change $r \to (1+\Delta_r)\cdot r$. Therefore due to the self-interaction, the spacetime described by the true metric g_{ij} has been expanded compared to the space described by the Schwarzschild metric g_{ij}. The radius has expanded to $(1+\Delta_r)$ times. Because the boundary condition is determined by the same gravitational field equation, from the view point of gravity, the extended spacetime described by the true metric g_{ij} is equivalent to the spacetime described by the Schwarzschild metric g_{ij}. The spacetime described by the metric g_{ij} follows the inverse square law, therefore the gravity of the extended spacetime is stronger than what is given by the inverse square law. In the spacetime described by the true metric g_{ij}, the gravity at a distance of $(1+\Delta_r)\cdot r$ from the gravitational source is equal to the gravity of the inverse square law at a distance of r from the gravitational source.

Because the general static isotropic metric doesn't implicit the time variable, the increment Δ_t is extremely small

$$\begin{aligned}
F_t(r,M) &= \frac{\partial \Delta\xi^T}{\partial t} \\
&= \left[1-\frac{2MG}{r}\right]^{1/2} \cdot \int_{-\infty}^{\infty} dT \exp(-\frac{|T|}{|L_P|}) \\
&= 2|L_P| \cdot \left[1-\frac{2MG}{r}\right]^{1/2} \\
\Delta_t &\equiv F_t(r,M)\left[1-\frac{2MG}{r}\right]^{-1/2} \\
&= 2|L_P|
\end{aligned} \tag{2.27}$$

From Eq.(2.21) and Eq.(2.22), we can see that the strength of the self-interaction is close to a linear relationship with the mass M of the gravitational source if $r \gg 2MG$. In the solar system, because the gravitational source M isn't strong, the self-interaction is extremely weak, but it is enough to explain the Pioneer anomaly. In galaxies containing supermassive black holes, the gravitational source M is very strong, then the self-interaction is strong enough to explain the gravitational effects of dark matter. Due to the mass of supermassive black holes is billions of times greater than that of the Sun, when the gravitational source is a supermassive black hole, the effect of the self-interaction is also billions of times stronger than the effect that caused the Pioneer anomaly, which is sufficient to explain the gravitational effect of dark matter. Because the increment Δ_t is extremely small, the gravitational redshift has no observable changes.

III. Conclusion

In this paper. we calculate the macroscopic effects of quantum gravity in the general static isotropic gravitational field. It can be seen that the strength of the self-interaction of quantum gravity depends on the strength of the gravitational source. In the solar system, due to the weak gravitational source, the self-interaction is extremely weak, but this is enough to explain the Pioneer anomaly.

When the gravitational source is a supermassive black hole, the effect of the self-interaction is very strong to explain dark matter. And there is no observable change in gravitational redshift due to the self-interaction in a static field.

This paper can lead to the conclusion that although the inverse square law is correct, the true gravitational field does not follow the inverse square law due to the self-interaction. The stronger the gravitational source, the greater the deviation from the inverse square law.

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