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Symbolic Collapse Intractability Hypothesis: P ≠ NP

By Jusn R Kornhaus

Abstract- The following proof focuses on the *Symbolic Collapse Intractability Hypothesis* and leverages symbolic entropy, recursive tractability, and structural complexity to argue that NP-complete problems with high entropy are intractable in polynomial time, implying P $\neq$ NP.

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Symbolic Collapse Intractability Hypothesis: P ≠ NP

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Abstract- The following proof focuses on the *Symbolic Collapse Intractability Hypothesis* and leverages symbolic entropy, recursive tractability, and structural complexity to argue that NP-complete problems with high entropy are intractable in polynomial time, implying P ≠ NP.

I. Formal Proof: P ≠ NP via Symbolic Entropy and Recursive Collapse

a) Definitions and Notations

Clause-Variable Incidence Graph:

- Let ϕ_n be a Boolean formula in conjunctive normal form (CNF) with ($v(n)$) variables $\{x_1,\ldots,x_{v(n)}\}$ and ($m(n)$) clauses $\{C_1,\ldots,C_{m(n)}\}$.
- Define the bipartite graph $G(\phi_n) = (V, C, E)$, where:
  - $V = \{x_1,\ldots, x_{v(n)}\}$ (variable nodes),
  - $C = \{C_1,\ldots, C_{m(n)}\}$ (clause nodes),
  - $E = \{(x_i, C_j) \mid x_i\ \neg x_i$ appears in $C_j\}$.
- Let $d_i = \deg(x_i)$ be the degree of variable x_i in $G(\phi_n)$, and $D = \sum_{i=1}^{v(n)} d_i$.

Symbolic Entropy:

- Define the normalized participation probability for variable x_i:

$$P(x_i) = \frac{d_i}{D}$$

- Define the symbolic entropy of ϕ_n:

$$\Sigma(\phi_n) = -\frac{1}{\log v(n)} \sum_{i-1}^{v(n)} P(x_i) \log P(x_i)\,,$$

Author: *Philosopher, B.A. Polical Science, Fairfield, Ohio. e-mail: justin.kornhaus@gmail.com*

where $\Sigma(\phi_n) \in [0, 1]$.

- $\Sigma(\phi_n) \to 1$: Maximal uniformity (high entanglement).
- $\Sigma(\phi_n) \to 0$: Skewed, localized structure.

Recursive Tractability Function:

- For constants $\alpha > 0, k \in N$, define:

$$R(n) = \alpha \cdot n^k \cdot (1 - \Sigma(\phi_n)).$$

- $R(n) \to 0$ when $\Sigma(\phi_n) \to 1$, indicating recursive collapse.

Structural Complexity Metric:

- Let $T_{\text{solve}}(n)$ be the time to decide satisfiability of ϕ_n.
- Define:

$$\text{SCM}(n) = \frac{T_{\text{solve}}(n)}{R(n)} = \frac{T_{\text{solve}}(n)}{\alpha \cdot n^k \cdot (1 - \Sigma(\phi_n))}$$

- When $R(n) \to 0$, $\text{SCM}(n) \to \infty$, indicating intractability.

Entropy-Preserving Reduction:

- For decision problems $L_1, L_2 \subseteq \{0,1\}^*$, a polynomial-time reduction $f: L_1 \to L_2$ is entropy-preserving if:
  - (f) is computable in time $(p(n))$ for some polynomial (p),
  - For any instance $x \in L_1, \Sigma(f(x)) \geq \Sigma(x)$.

b) Assumptions

- For any NP-complete language (L), there exists a polynomial-time reduction $f: L \to$ SAT such that high-entropy instances of (L) map to high-entropy instances of SAT (i.e., $\Sigma(f(x)) \to 1$ if $\Sigma(x) \to 1$.
- High symbolic entropy $(\Sigma(\phi_n) \to 1)$ correlates with exponential resolution proof length and super-polynomial circuit size or logarithmic depth, based on established results (Ben-Sasson & Wigderson, 2001; Håstad, 1987; Razborov-Smolensky, 1987).
- The class $\text{SRI} = \{L \subseteq \text{NP-complete} \mid \exists f: L \to \phi_n \in \text{SAT}, \Sigma(\phi_n) \to 1\}$ includes all NP-complete problems.

II. Theorem 1: Symbolic Entropy Implies Resolution Width Growth

For a family of random (k)-CNF formulas $\{\phi_n\}$ with $\Sigma(\phi_n) \to 1$:

- The resolution width $w(\phi_n) = \Omega(n)$,
- The resolution proof length $L(\phi_n) \geq 2^{\Omega(n)}$.

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a) Proof

- By Ben-Sasson & Wigderson (2001), for unsatisfiable (k)-CNF formulas, high clause-variable uniformity (implied by $\Sigma(\phi_n) \to 1$) forces large resolution width $w(\phi_n) = \Omega(n)$.
- The resolution length is bounded by $L(\phi_n) \geq 2^{\Omega(w(\phi_n))}$, so $w(\phi_n) = \Omega(n) \Rightarrow L(\phi_n) \geq 2^{\Omega(n)}$.
- High $\Sigma(\phi_n)$ ensures low compressibility, as variable participation is nearly uniform, preventing short resolution proofs.

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III. Theorem 2: Symbolic Entropy Implies Circuit Depth Growth

For a family of CNF formulas $\{\phi_n\}$ with $\Sigma(\phi_n) \to 1$, any Boolean circuit family $\{C_n\}$ deciding satisfiability of ϕ_n satisfies:

- Either $\text{Depth}(C_n) = \Omega\,(\log n)$,
- Or $\text{Size}(C_n) = 2^{\Omega(n^{\epsilon})}$ for some $\varepsilon > 0$.

a) Proof

- High $\Sigma(\phi_n) \to 1$ implies full variable-clause interaction, resembling random-like functions.
- By Håstad's switching lemma and Razborov-Smolensky results, functions with high uniformity resist bounded-depth computation (e.g.,AC^0).
- If $\text{Depth}(C_n) = \Omega \log n$, then $\text{Size}(C_n) = 2^{\Omega(n^{\epsilon})}$ for some $\varepsilon > 0$.
- Alternatively, deciding ϕ_n requires $\text{Depth}(C_n) = \Omega(\log n)$ to avoid exponential size.

IV. Lemma 1: Entropy Preservation in Reductions

For NP-complete languages L_1, L_2, and a standard polynomial-time reduction $f: L_1 \to L_2$, (f) is entropy-preserving: $\Sigma(f(x)) \geq \Sigma(x)$.

a) Proof

- Consider standard reductions (e.g., 3-SAT to Clique, SAT to Subset Sum). These reductions typically map instances to structures with equal or greater clause-variable or node-edge interactions.
- For example, in the 3-SAT to Clique reduction, each clause becomes a node in a graph, and edges reflect variable consistency. The resulting graph's entropy (based on node-edge incidence) is at least as high as the original clause-variable graph, as the reduction preserves or increases structural complexity.

- Formally, let $x \in L_1$ have incidence graph $(G(x))$. The reduction (f) constructs $f(x) \in L_2$ with incidence graph $(G(f(x)))$. Since (f) is polynomial-time, it does not collapse the structural complexity (otherwise, it would imply $L_1 \in P$). Thus, : $\Sigma(f(x)) \geq \Sigma(x)$.
- This holds for a large class of Karp reductions between NP-complete problems, as they map constraints to constraints without reducing variable interdependence.

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V. Theorem 3: Universality of Symbolic Collapse

For any NP-complete language (L), if there exists an entropy-preserving reduction $f: L \rightarrow \mathrm{SAT}$ such that $\Sigma(f(x)) \rightarrow 1$ implies $\mathrm{R}(n) \rightarrow 0$ and $\mathrm{SCM}(n) \rightarrow \infty$, then $L \notin P$.

a) *Proof*

- Let $x \in L$, and $f(x) = \phi_n \in \mathrm{SAT}$, where (f) is polynomial-time and entropy-preserving.
- If $\Sigma(x) \rightarrow 1$then $\Sigma(\phi_n) \rightarrow 1$ (by lemma 1).
- By Theorem 1, $\Sigma(\phi_n) \rightarrow 1 \Rightarrow w(\phi_n) = \Omega(n) \Rightarrow L(\phi_n) \geq 2^{\Omega(n)}$.
- By Theorem 2, $\Sigma(\phi_n) \rightarrow 1 \implies \mathrm{Size}(C_n) = 2^{\Omega(n^{\epsilon})}$ or $\mathrm{Depth}(C_n) = \Omega \log n$.
- Thus, $T_{\mathrm{solve}}(n)$ for ϕ_n is super-polynomial, and $R(n) \rightarrow 0 \implies \mathrm{SCM}(n) \rightarrow \infty$.
- Since (f) is polynomial-time, the intractability of ϕ_n implies (x) is intractable, so $L \notin P$.

VI. Theorem 4: Symbolic Collapse Intractability Hypothesis

If all NP-complete problems belong to the class $\mathrm{SRI} = \{ L \subseteq \text{NP-complete} \mid \exists f: L \rightarrow \phi_n \in \mathrm{SAT}, \Sigma(\phi_n) \rightarrow 1\}$, then $P \neq NP$.

a) *Proof*

- Let $L \in$ NP-complete. By assumption, there exists an entropy-preserving reduction $f: L \rightarrow \mathrm{SAT}$ such that for hard instances $x \in L, \phi_n = f(x)$ has $\Sigma(\phi_n) \rightarrow 1$.
- By Theorem 3, $\Sigma(\phi_n) \rightarrow 1 \Rightarrow L \notin P$.
- Since (L) is NP-complete, if $L \in P$, then $\mathrm{NP} \subseteq P$, implying $P = \mathrm{NP}$.
- However, $L \notin P$ due to the exponential proof length and circuit size/depth requirements (Theorems 1 and 2).
- Thus, $P \neq \mathrm{NP}$.

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VII. Contrapositive Argument

- If $P = \mathrm{NP}$, then there exists a polynomial-time algorithm for SAT, implying polynomial-size circuits and sub-exponential resolution proofs for all ϕ_n.
- For high-entropy ϕ_n $(\Sigma(\phi_n) \to 1)$:
    - Resolution proofs require length $2^{\Omega(n)}$ (Theorem 1),
    - Circuits require size $2^{\Omega(n^{\epsilon})}$ or depth $\Omega(\log n)$ (Theorem 2).
- This contradicts the existence of polynomial-time algorithms, as established lower bounds (Ben-Sasson & Wigderson, Håstad, Razborov-Smolensky) cannot be bypassed.
- Thus, $P = \mathrm{NP}$ is false, so $P \neq \mathrm{NP}$.

VIII. Conclusion

Assuming all NP-complete problems admit reductions to high-entropy SAT instances (NP-complete ⊆ SRI), and high symbolic entropy induces recursive collapse ($R(n) \to 0$, $(\mathrm{SCM}(n) \to \infty)$, no polynomial-time algorithm can exist for any NP-complete problem. Therefore:

$$\boxed{P \neq \mathrm{NP}}.$$

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