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The Role of Noncommutative Quantum Gravity in Galactic Dynamics and Dark Matter Phenomena

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The Role of Noncommutative Quantum Gravity in Galactic Dynamics and Dark Matter Phenomena

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I. Introduction

In the section 2, we briefly reviewed the theory of noncommutative quantum gravity. In the paper [1] [2], we introduce the theory of noncommutative quantum gravity. Papers [3-5] discuss the macroscopic effects of noncommutative quantum gravity, including how gravity operates within a noncommutative framework and its potential observational consequences. In the section 3, we explained how quantum gravity can replace dark matter to explain the observed effects of dark matter, such as dark matter halos and the flatness of the rotation curves of galaxies.

II. Noncommutative Quantum Gravity

In the paper [1] and paper [2], we introduce the theory of noncommutative quantum gravity. In this theory, we give the field function of gravity. It is a semiclassical graviton, it can be be interpreted as a particle with zero point vibration. Its form is a Dirac-δ function as follows

$$\xi^i(x,r) = \begin{cases} \xi^r = r + C^r(x)\exp(-\dfrac{r}{l_P}) \\ \xi^\theta = \theta(x) \\ \xi^\phi = \phi(x) \\ \xi^t = t + C^t(x)\exp(-\dfrac{|t|}{t_P}) \end{cases} \tag{2.1}$$

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The Lagrangian density is

$$\mathscr{L} = -\frac{\eta^{\mu\nu}}{2}\frac{\partial\xi^i(x,r)}{\partial x^\mu}\frac{\partial\xi^j(x.r)}{\partial x^\nu}\eta_{ij} \tag{2.2}$$

The energy-momentum tensor is

$$\begin{aligned} T_{\mu\nu} &= \eta_{\mu\nu}\mathscr{L} - \frac{\partial\mathscr{L}}{\partial(\partial^\mu\xi^i)}\partial_\nu\xi^i \\ &= -\frac{\eta_{\mu\nu}}{2}\partial^\lambda\xi^i\partial_\lambda\xi^j\eta_{ij} + \partial_\mu\xi^i\partial_\nu\xi^j\eta_{ij} \end{aligned} \tag{2.3}$$

The free field equation is a wave equation

$$\partial^\mu\partial_\mu\xi^i = 0 \tag{2.4}$$

The Green's function can be written as

$$\tilde{G}^i(k) = \begin{cases} \tilde{G}^r(k) = -\dfrac{1}{(k^r)^2}\cdot\delta\Big(k^r - \dfrac{i}{l_P}\Big) \\ \tilde{G}^\theta(k) = -\dfrac{1}{(k^\theta)^2} \\ \tilde{G}^\phi(k) = -\dfrac{1}{(k^\phi)^2} \\ \tilde{G}^t(k) = -\dfrac{1}{\omega^2}\cdot\delta\Big(\omega - \dfrac{i}{t_P}\Big) \end{cases} \tag{2.5}$$

In the general theory of relativity, the energy-momentum tensor of gravitational field itself is:

$$\begin{aligned} t_{\mu\nu} &= \frac{1}{8\pi G}\left(\frac{1}{2}\eta_{\mu\nu}R^{(1)} - R^{(1)}_{\mu\nu}\right) \\ &= \frac{1}{8\pi G\cdot C}\left(\frac{1}{2}\eta_{\mu\nu}\frac{\partial\xi^i}{\partial x^\kappa}\frac{\partial\xi_i}{\partial x_\kappa} - \frac{\partial\xi^i}{\partial x^\mu}\frac{\partial\xi_i}{\partial x^\nu}\right) \end{aligned} \tag{2.6}$$

Up to a factor of a constant, it is equal to Eq.[2.3]. It is a strong evidence to prove that the quantum field theory constructed in the paper [1] and paper [2] is classically equivalent to the general theory of relativity.

In the paper [3] and [4], we discussed the self-interaction of gravitational fields. In momentum space, the metric of the gravitational field with self-interaction can be written as follows

$$
\begin{aligned}
g_{\mu\nu}[\lambda(\xi)] &= \frac{\partial\lambda(\xi^{\alpha})}{\partial x^{\mu}}\frac{\partial\lambda(\xi^{\beta})}{\partial x^{\nu}}\eta_{\alpha\beta} \\
&= \frac{\partial\Big(\xi^{\alpha}(x,X)\big|_{X=0}+\Delta\xi^{\alpha}\Big)}{\partial x^{\mu}}\frac{\partial\Big(\xi^{\beta}(x,X)\big|_{X=0}+\Delta\xi^{\beta}\Big)}{\partial x^{\nu}}\eta_{\alpha\beta} \\
&\equiv g_{\mu\nu}[\,\xi\,]+g^{(1)}_{\mu\nu}+g^{(2)}_{\mu\nu}
\end{aligned}
\tag{2.7}
$$

where

$$
\begin{aligned}
g_{\mu\nu}[\xi] = \Bigg[&\int d^4k\left(ik_{\mu}C^{\alpha}(k)\exp(ikx)-ik_{\mu}\left(C^{\alpha}(k)\right)^{*}\exp(-ikx)\right) \\
&\cdot\int d^4k\left(ik_{\nu}C^{\beta}(k)\exp(ikx)-ik_{\nu}\left(C^{\beta}(k)\right)^{*}\exp(-ikx)\right)\Bigg]\cdot\eta_{\alpha\beta}
\end{aligned}
\tag{2.8}
$$

$$
\begin{aligned}
g^{(1)}_{\mu\nu} = 2\cdot\Bigg[&\int d^4k\left(\frac{2|L_P(k)|}{1+ikL_P(k)}ik_{\mu}C^{\alpha}(k)\exp(ikx)-\frac{2|L_P(k)|}{1-ikL_P(k)}ik_{\mu}\left(C^{\alpha}(k)\right)^{*}\exp(-ikx)\right) \\
&\cdot\int d^4k\left(ik_{\nu}C^{\beta}(k)\exp(ikx)-ik_{\nu}\left(C^{\beta}(k)\right)^{*}\exp(-ikx)\right)\Bigg]\cdot\eta_{\alpha\beta}
\end{aligned}
\tag{2.9}
$$

$$
\begin{aligned}
g^{(2)}_{\mu\nu} = \Bigg[&\int d^4k\left(\frac{2|L_P(k)|}{1+ikL_P(k)}ik_{\mu}C^{\alpha}(k)\exp(ikx)-\frac{2|L_P(k)|}{1-ikL_P(k)}ik_{\mu}\left(C^{\alpha}(k)\right)^{*}\exp(-ikx)\right) \\
&\cdot\int d^4k\left(\frac{2|L_P(k)|}{1+ikL_P(k)}ik_{\nu}C^{\beta}(k)\exp(ikx)-\frac{2|L_P(k)|}{1-ikL_P(k)}ik_{\nu}\left(C^{\beta}(k)\right)^{*}\exp(-ikx)\right)\Bigg]\cdot\eta_{\alpha\beta}
\end{aligned}
\tag{2.10}
$$

Denote

$$f(k) \equiv \frac{2|L_P(k)|}{1 + ik_\mu L_P(k)}; \quad f^*(k) \equiv \frac{2|L_P(k)|}{1 - ik_\mu L_P(k)} \tag{2.11}$$

Using the mean value theorem of definite integrals, we have

$$g^{(1)}_{\mu\nu} = 2 \cdot \left[\int d^4k \left(f(k) \cdot ik_\mu C^\alpha(k) \exp(ikx) \right) - d^4k \left(f^*(k) \cdot ik_\mu \left(C^\alpha(k)\right)^* \exp(-ikx) \right) \right]$$

$$\cdot \left[\int d^4k \left(ik_\nu C^\beta(k) \exp(ikx) - ik_\nu \left(C^\beta(k)\right)^* \exp(-ikx) \right) \right] \cdot \eta_{\alpha\beta} \tag{2.12}$$

$$= 2 \cdot \left[f(\zeta_r) \cdot \int d^4k \left(ik_\mu C^\alpha(k) \exp(ikx) \right) - f^*(\zeta_r^*) \cdot \int d^4k \left(ik_\mu \left(C^\alpha(k)\right)^* \exp(-ikx) \right) \right)$$

$$\cdot \int d^4k \left(ik_\nu C^\beta(k) \exp(ikx) - ik_\nu \left(C^\beta(k)\right)^* \exp(-ikx) \right) \right] \cdot \eta_{\alpha\beta}$$

$$g^{(2)}_{\mu\nu} = \left[\int d^4k \left(f(k) \cdot ik_\mu C^\alpha(k) \exp(ikx) \right) - d^4k \left(f^*(k) \cdot ik_\mu \left(C^\alpha(k)\right)^* \exp(-ikx) \right) \right]$$

$$\cdot \left[\int d^4k \left(f(k) \cdot ik_\mu C^\beta(k) \exp(ikx) \right) - d^4k \left(f^*(k) \cdot ik_\mu \left(C^\beta(k)\right)^* \exp(-ikx) \right) \right] \cdot \eta_{\alpha\beta} \tag{2.13}$$

$$= \left[f(\zeta_r) \cdot \int d^4k \left(ik_\mu C^\alpha(k) \exp(ikx) \right) - f^*(\zeta_r^*) \cdot \int d^4k \left(ik_\mu \left(C^\alpha(k)\right)^* \exp(-ikx) \right) \right]$$

$$\cdot \left[f(\zeta_r) \cdot \int d^4k \left(ik_\mu C^\beta(k) \exp(ikx) \right) - f^*(\zeta_r^*) \cdot \int d^4k \left(ik_\mu \left(C^\beta(k)\right)^* \exp(-ikx) \right) \right] \cdot \eta_{\alpha\beta}$$

where $f(\zeta)$ is the mean value of $f(k)$, $f^*(\zeta^*)$ is the mean value of $f^*(k)$. The general static isotropic metric is:

$$ds^2 = g_{rr}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 - g_{tt}dt^2$$

$$g_{rr} = \left[1 - \frac{2MG}{r}\right]^{-1}, \quad g_{tt} = \left[1 - \frac{2MG}{r}\right] \tag{2.14}$$

Denote

$$\Delta_r \equiv \frac{f(\zeta_r) + f(\zeta_r^*)}{2}, \qquad \Delta_t \equiv \frac{f(\zeta_t) + f(\zeta_t^*)}{2} \tag{2.15}$$

Then the metric $g_{\mu\nu}$ of the general static isotropic gravitational field with self-interaction can be written as

$$ds^2 = (1+\Delta_r)^2 \cdot \left[1 - \frac{2MG}{r}\right]^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

$$- (1+\Delta_t)^2 \cdot \left[1 - \frac{2MG}{r}\right] dt^2 \tag{2.16}$$

It can also be expressed in the equivalent isotropic form. By introducing a new radius variable ρ

$$r = \rho(1+\Delta_r)\left(1 + \frac{MG}{2\rho}\right)^2 \tag{2.17}$$

Then the isotropic form can be written as

$$ds^2 = (1+\Delta_r)^2 \cdot \left(1 - \frac{MG}{2\rho}\right)^4 \left(d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2\theta d\phi^2\right)$$

$$- (1+\Delta_t)^2 \cdot \left(\frac{1 - MG/2\rho}{1 + MG/2\rho}\right)^2 dt^2 \tag{2.18}$$

We can find a scale transformation $r \to \left[(1+\Delta_r)\cdot r\right]$. It means that the gravity with self-interaction at a distance of $\left[(1+\Delta_r)\cdot r\right]$ from the gravitational source is equal to the gravity of the inverse square law at a distance of r from the gravitational source.

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Obviously, the strength of self-interaction depends on the function Δ_r. In the paper [5], we calculate the function Δ_r in coordinate space as follows:

$$\Delta_r \equiv F_r(r, M) \cdot \left[1 - \frac{2MG}{r}\right]^{1/2} \tag{2.19}$$

where

$$F_r(r, M)$$

$$= KMG \cdot \ln \frac{\left(\sqrt{r^2 + r} - \sqrt{(2MG)^2 + 2MG}\right) + \left(\sqrt{r(1 + 2MG)} - \sqrt{2MG(1 + r)}\right)}{\left(\sqrt{r^2 + r} - \sqrt{(2MG)^2 + 2MG}\right) - \left(\sqrt{r(1 + 2MG)} - \sqrt{2MG(1 + r)}\right)} \tag{2.20}$$

Notice that the maximum value of the function Δ_r is located at $r = 2 \cdot (2MG)$. Therefore, this function can explain the flatness of the rotation curves of galaxies.

III. Dark Matter and Macroscopic Effect of Non-Commutative Quantum Gravity

Given the challenges in detecting dark matter directly, alternative theories such as modifications to gravityparticularly those emerging from quantum gravi-tyhave been explored as potential explanations for observed galactic rotation curves. If dark matter is understood as the self-interaction of gravitational field, then the observational phenomena related to dark matter can be explained by the theory of noncommutative quantum gravity, such as dark matter halos and the flatness of the rotation curves of galaxies. Let's discuss as follows:

To explain the flatness of the rotation curves of galaxies, the galaxy serving as the gravitational source for a star located at the edge of the galaxy can be considered as an equivalent mass point M. The equivalent mass means that for the star located at the edge of the galaxy, the gravity from M is equivalent to the gravity from total mass of the galaxy. From the equivalent mass M we get the Schwarzschild radius $r_s = 2MG$. Note that the Schwarzschild radius r_s of the equivalent mass M does not imply the event horizon of a black hole, but only represents a scale of length.

Then Eq.(2.20) can be written as

$$F_r(r, M) = \frac{r_s}{2} K \cdot \ln \frac{\left(\sqrt{r^2 + r} - \sqrt{r_s^2 + r_s}\right) + \left(\sqrt{r(1 + r_s)} - \sqrt{r_s(1 + r)}\right)}{\left(\sqrt{r^2 + r} - \sqrt{r_s^2 + r_s}\right) - \left(\sqrt{r(1 + r_s)} - \sqrt{r_s(1 + r)}\right)} \tag{3.1}$$

For example, in the case of $r_s = 100$, the curve of the function δ_r is

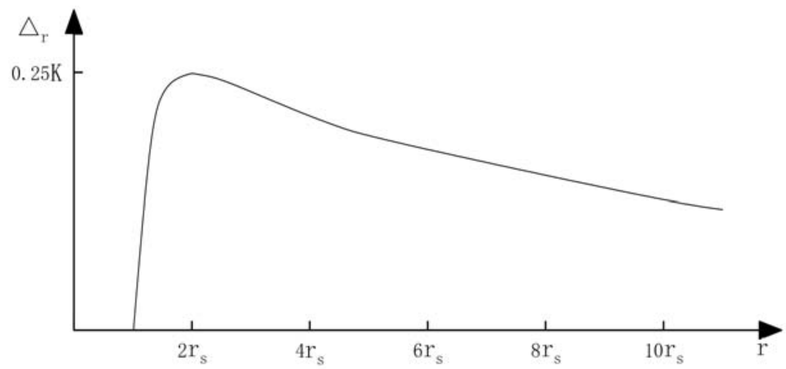

Figure 1: function Δ_r

The maximum of the function Δ_r is located at $r = 2r_s$. In the interval $r \in (r_s, 2r_s]$, the effect of self-interaction of gravitational field continues to increase and reaches its maximum at $r = 2r_s$. Therefore, in the interval $r \in (r_s, 2r_s]$, the rotation curves can remain flat with increasing distance. The flatness of the rotation curves is not due to the modified Newtonian dynamics in weak-field, nor is it due to the gravitational effect of dark matter, but rather due to the continuous enhancement of self-interaction in the gravitational field. If $r > 2r_s$, the self-interaction effect start to weaken, and due to the inverse square law, the velocity of stars located at the edge of the galaxy start to rapidly decrease with increasing distance. By superimposing the calculated rotation curve of the galaxy in the absence of dark matter with the curve of the function Δ_r in Figure 1, we can obtain the flatness of the rotation curves of galaxies without the need for dark matter halo, consistent with the observed rotation curve of the galaxy. This offers a reasonable explanation for the flatness of the rotation curves of galaxies without dark matter. In this way we can conclude that the dark matter we observe is distributed near $r = 2r_s$. The self-interaction gradually strengthen before $r = r_s$, then decrease with $r > 2r_s$, formed the dark matter halo.

In the case of galaxy cluster CL0024+17, the map of dark matter in the galaxy cluster is actually the macroscopic effect of self-interaction of the gravitational field of the galaxy cluster. Due to the disk like shape of galaxy clusters, dark matter is distributed in a circular ring pattern. According to the discussion in this paper, this distribution pattern of dark matter is universal. Another similar situation is the Milky Way. The rotation curve of the central part of the Milky Way follows the inverse square law, so there is no dark matter in the central region. Dark matter appears from the base of the spiral arms in the Milky Way galaxy, this is consistent with the self-interaction curve shown in Figure 1. Therefore, on the plane of galactic disk, the dark matter is also distributed in a circular ring pattern. In 3D space, the dark matter in the Milky Way is distributed ellipsoidal surface around the entire galaxy. The dark matter located in the central part of the galaxy cluster CL0024+17 is also due to the same reason. The gravitational fields of each galaxy in the central region of the galaxy cluster CL0024+17 have their own self-interactions, which superimpose to form a dark matter halo in the central region of the galaxy cluster.

Due to the gravitational effect of dark matter being the self-interaction effect of the gravitational field of baryonic matter, this indicates that the density distribution of dark matter in galaxy clusters is consistent with that of baryonic matter.

In the case of bullet cluster 1E0657-56, the dark matter images of galaxy cluster obtained from gravitational lensing effect and X-ray observations can be interpreted as the gravitational fields between two separating galaxy clusters cancel each other out, the effects of self-interaction also cancel each other out. Therefore, no dark matter was observed between galaxy clusters. Dark matter is distributed on the outer side of two separating galaxy clusters.

The self-interaction of the gravitational field can also explain the universal enhancement of the gravitational lensing effect observed of galaxy clusters, which is consistent with dark matter theory. According to the calculations in the paper [5], the gravitational effect of self-interaction has almost no gravitational redshift, which is a difference from dark matter theory.

From Figure 1, it can be seen that the attenuation of self-interaction of the gravitational field begins after $r > 2r_s$, but the attenuation rate is very small, therefore the effect of self-interaction can be understood as long-range interaction, although the effect of self-interaction becomes extremely weak at great distances. Maybe it also can be explain the Pioneer anomaly in the solar system.

IV. Conclusion

Our findings suggest that the self-interaction effects in noncommutative quantum gravity may provide an alternative explanation for dark matter-like gravitational phenomena, potentially offering a simpler theoretical framework. From the above analysis, we can see that the self-interaction effect of gravitational field may explain the gravitational effects caused by dark matter, such as dark matter halos and the flatness of the rotation curves of galaxies. The approach in this paper does not require modification of the inverse square law and can predict the location of dark matter. However, further work is required to compare this approach with observational data. Due to the difficulty in accurately determining the mass distribution of galaxies, the equivalent mass is also difficult to accurately determine, and thus the Schwarzschild radius r_s of the equivalent mass M is also difficult to accurately determine. At present, the Schwarzschild radius r_s of the equivalent mass M can only be roughly determined through observation data of dark matterThe location where dark matter is most concentrated is $r = 2r_s$the location where the rotation curve of the galaxy begins to descend is $r = 2r_s$. More detailed galaxy observation data will be available to verify the approach in this paper.

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