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Highlights

- Finite Element Modeling
- Exploring Torus Black-Holes
- Collision Analysis of a Ship
- Symmetric Quartic Power Curves

Discovering Thoughts, Inventing Future

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Finite Element Modeling and Simulation of Collision Analysis of a Ship

By Dr. Md Shahidul Islam, Md Kaykuj Taky & Nusrat Jahan Omaia
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Abstract- Improving the structural integrity of ships against collisions has received much attention due to the recent rise in maritime safety regulations, such as SOLAS (Safety of Life at Sea), and the increasing number of accidents involving displacement-type general cargo vessels. Collisions are a constant danger for ships like displacement-type general cargo vessels, which face the greatest number of accidents due to the presence of stationary objects or other vessels in high-traffic maritime zones. In collision analysis, the influence of engine mass on structural behavior during collisions is neglected in the literature. Finite element analysis (FEA) was utilized to simulate various collision scenarios, focusing on how engine mass and speed variations affect structural integrity in the case of the vessel colliding against a rigid wall. The study focuses on the modeling of a displacement ship made of 6061-T6 aluminum.

Keywords: *ship structural integrity, collision dynamics, engine mass impact, energy absorption, stress distribution, displacement-type vessel, collision resistance, finite element modeling, ship safety design, plastic deformation.*

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FINITEELEMENTMODELINGANDSIMULATIONOFCOLLISIONANALYSISOFASHIP

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Finite Element Modeling and Simulation of Collision Analysis of a Ship

Dr. Md Shahidul Islam [α], Md Kaykuj Taky [σ] & Nusrat Jahan Omaia [ρ]

Abstract- Improving the structural integrity of ships against collisions has received much attention due to the recent rise in maritime safety regulations, such as SOLAS (Safety of Life at Sea), and the increasing number of accidents involving displacement-type general cargo vessels. Collisions are a constant danger for ships like displacement-type general cargo vessels, which face the greatest number of accidents due to the presence of stationary objects or other vessels in high-traffic maritime zones. In collision analysis, the influence of engine mass on structural behavior during collisions is neglected in the literature. Finite element analysis (FEA) was utilized to simulate various collision scenarios, focusing on how engine mass and speed variations affect structural integrity in the case of the vessel colliding against a rigid wall. The study focuses on the modeling of a displacement ship made of 6061-T6 aluminum. We simulated different collision scenarios and studied how resultant stress distributions and plastic deformations occur in the ship structure. The bow, keel, and longitudinal girders, where stress concentration levels are higher, are found to be the most critical region of concern in the case of a bow collision. The findings of this study provide important insights into the factors that influence the magnitude of ship damage during collisions. The results offer guidance for future ship designs aimed at enhancing safety and reducing the chance of catastrophic structural failure. The modeling and simulation procedure discussed in the paper is also expected to benefit the readers.

Keywords: ship structural integrity, collision dynamics, engine mass impact, energy absorption, stress distribution, displacement-type vessel, collision resistance, finite element modeling, ship safety design, plastic deformation.

I. Introduction

Ship impact simulations are important for the analysis of safety and structural integrity of ships and offshore marine structure. These simulations play a crucial role in improving ship design, ensuring compliance with industry safety standards, and mitigating risks associated with maritime accidents. However, there is still a gap in research regarding full-scale ship collision analysis, which this study aims to address. Oil spills are a major environmental hazard, and accidents involving oil tankers or barges are the main reasons for such incidents [1]. In many cases, these spills are a direct consequence of ship collisions, which not only lead to severe structural damage but also contribute to environmental disasters. For example, the MV Wakashio oil spill in 2020 led to severe ecological damage off the coast of Mauritius, while the collision of the Baltic Ace in 2012 resulted in significant loss of life and financial setbacks. The substantial market size for ship repair and maintenance services reflects the recurring costs associated with ship damage, including repair expenses and downtime losses. The worldwide market for ship repair and maintenance services was worth USD 35.72 billion in 2023 and is expected to increase from USD 37.14 billion in 2024 to USD 53.23 billion by 2032 [2]. The economic, safety, and environmental consequences of ship collisions are significant,

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Ref

1. E. Cakir, C. Sevgiliand R. Fiskin, *" An analysis of severity of oil spill caused by vesselaccidents"*, Transportation Research Part D: Transport and Environment, vol. 90,(2021), p. 102662.

affecting not only ship operators but also marine ecosystems and global trade logistics. These impacts can be categorized as follows, as summarized in Table 1.

Table 1: Potential impacts of ship collision

The potential consequences associated with a ship collision		
Economic	**Safety**	**Environment**
Loss of asset	Injuries	Oil spillage
Damage to ship operator's reputation	Loss of life or fatalities	Introduction of invasive species through ballast water
Repair costs		
Loss of revenue		

As ships are expensive to manufacture, direct collision tests which are destructive in nature and also very expensive, are not feasible. The most viable alternative to such tests is FEA, as it provides accurate and realistic results that are consistent with the experimental results[3] [4]. Collision analysis provides dynamic response of structures and has great importance in designing safe ships and marine structures. Many previous studies have focused on collision analyses using the portions that are adjacent to the impact zones without modeling and meshing the whole ship[5] [6]. However, there is an absence of the study of full body ship analysis. Without the modeling and meshing of a complete ship, it is unclear whether the result will be accurate enough to show the stress distribution properly. Because of the nature of dynamic analysis, simple discrepancies in simulation parameters can cause large deviations in the obtained results[7].

Zhang et al. (2019) derived analytical formulae for energy absorption in ship collisions leading to rupture, and validated their results using finite element analysis and experimental results. Their study supported the use of FEA as an alternative to experimental methods[8].

Aluminum 6061-T6 is widely used as a common construction material in the shipbuilding industry owing to its high strength-to-weight ratio, corrosion resistance, and energy absorption capabilities. Crum et al. (2011) highlighted the growing use of aluminum in modern ship designs, particularly in naval vessels[9]. As the ship we are studying is smaller in size, aluminum is chosen as the construction material.

This study investigates the effects of collisions on a ship structure in which all the structural members are meshed with and without engine mass. As speed greatly affects the impact force [10], the analysis is performed under two different speed settings.

The main objective of this research is to model and mesh a displacement-type vessel with all the structural members and then analyze it with a rigid wall using LS-DYNA. The generated stress and internal energy in both these cases are compared.

II. Methodology

This section outlines the step-by-step process used for the modeling, mesh generation, and simulation of the selected ship.

a) *Ship Model Generation and Material Selection*

The ship is initially generated using Rhino 3D and its geometric details are simplified as it is a global analysis. In this context, simplification means reducing minor design features that do not significantly affect the overall structural response, ensuring computational efficiency while preserving essential load-bearing elements. The impact on elements away from the collision region is minimal, so they are simplified or removed to focus computational resources on the critical impact areas. The isometric view of the

Ref

3. D. Moulas, M. Shafieeand A. Mehmanparast, *"Damage analysis of ship collisions with offshore wind turbine foundations"*, Ocean Engineering, vol. 143,(2019) pp. 149-162.

whole model is given in Figure 1. For global analysis, the primary objective is to understand the overall structural response rather than the localized effects of intricate details. Therefore, only essential structural components are retained, while minor design elements are omitted.

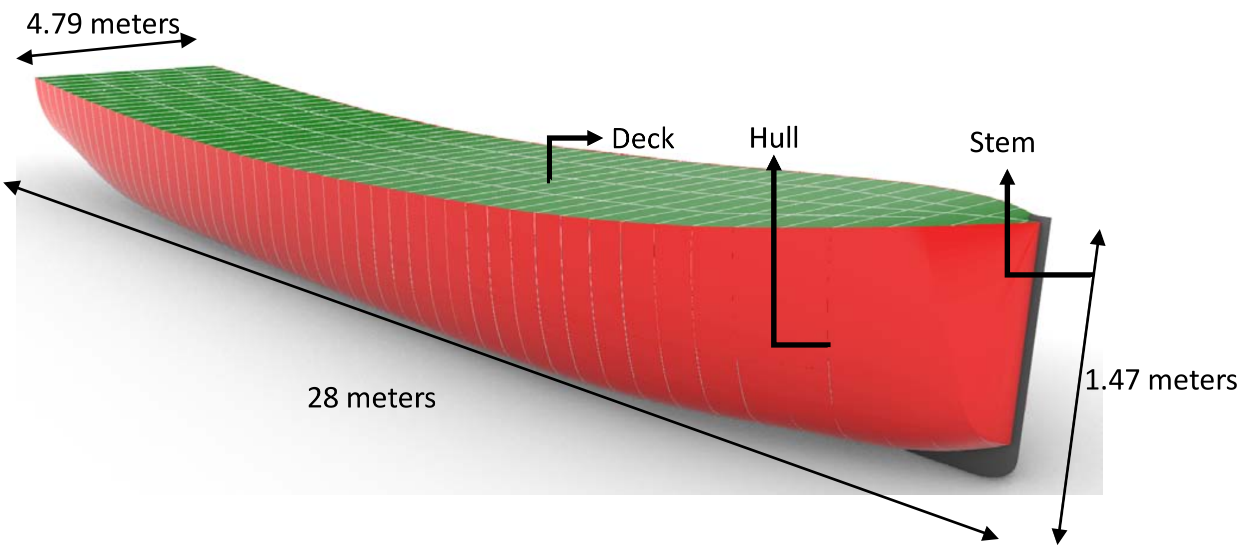


Figure 1: External view of ship model

Key internal structural members, such as girders, frames, bulkheads, and pillars, are modeled and are shown in Figure 2 and 3.

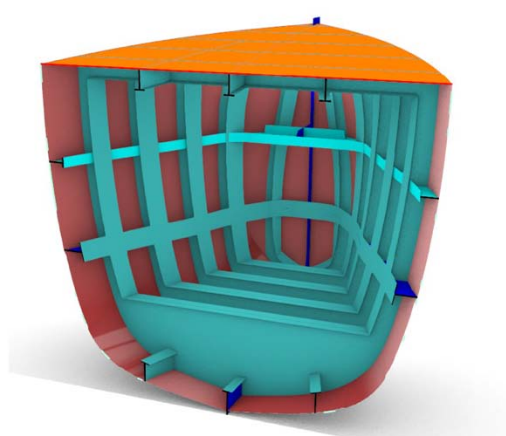


Figure 2: Ship with structural members (Fore portion)

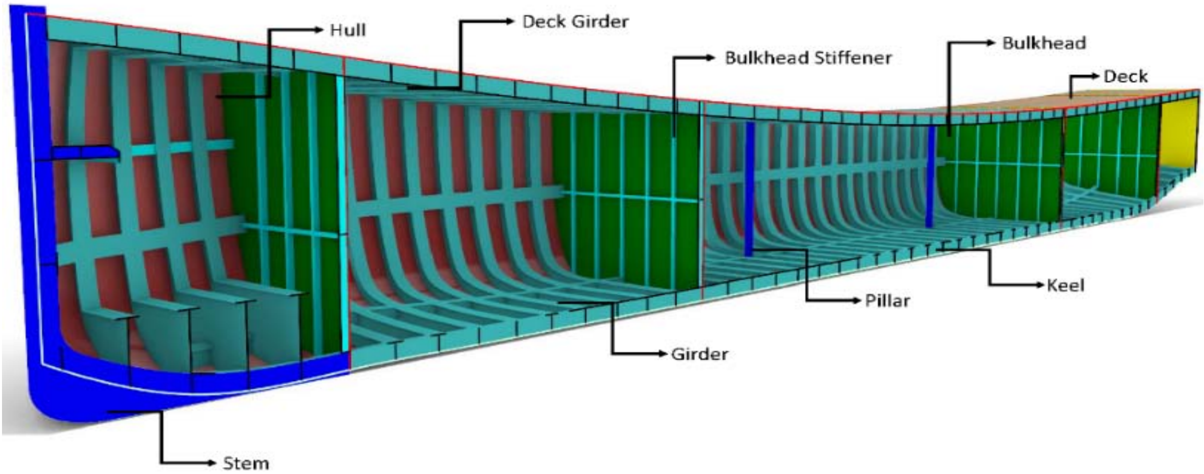


Figure 3: Ship with structural members

Shell elements with reduced integration (Belytschko-Tsay formulation) are chosen for the modeling to improve computational efficiency while maintaining accuracy. The ship is 28 meters long, 4.79 meters in width, and has a draught of 1.47 meters. To increase the efficiency of the simulation process, non-load-bearing and aesthetic features that do not contribute to impact resistance are removed in accordance with previous studies[11]. Aluminum 6061-T6 is chosen for its high strength-to-weight ratio, corrosion resistance, and energy absorption, making it ideal for impact scenarios. It is lighter than steel while maintaining structural integrity, improving fuel efficiency and speed. Its ductility allows plastic deformation, effectively dissipating impact energy and enhancing crash resistance. These qualities make it a preferred material for naval and commercial vessels needing durability and lightweight construction. Detailed material characteristics [12] are given in Table 2.

Table 2: Properties of material chosen

Material Name	Density	Young's Modulus	Poisson's Ratio	Strength Coefficient	Hardening Exponent	Initial Yield Strength
Aluminum 6061-T6	2700 kg/mı	68900 MPa	0.33	410 MPa	0.05	276 MPa

b) *Mesh Generation and Boundary Conditions*

Femap and Hypermesh software are chosen to generate mesh to proceed with the FEA. In complex structures such as ships, it is difficult to maintain mesh coupling and connections, making it harder to maintain a good mesh quality. Both of these programs are used to decide which software is better suited for this specific task. The mesh generated by us by Femap is of poorer quality than that generated by Hypermesh, which is compared in Figure 4.

Ref

11. M. Melchiorre and T. Duncan, *"The fundamentals of FEA meshing for structural analysis"*. Ansys, 2021.

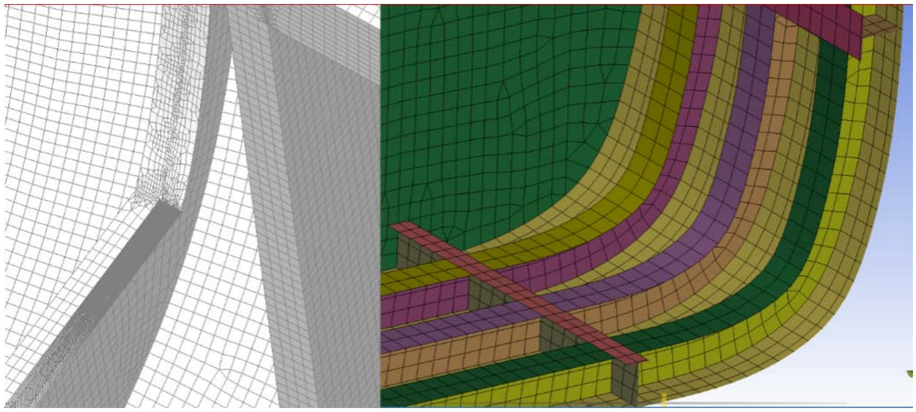


Figure 4: Mesh quality comparison in Femap and Hypermesh

So, generated mesh using Hypermesh is used for the analyses. Three mesh densities are used to obtain more accurate results. As the mesh becomes finer, the accuracy of the non-linear analyses increases so does the calculation time[13]. These densities are shown in Figure 5.

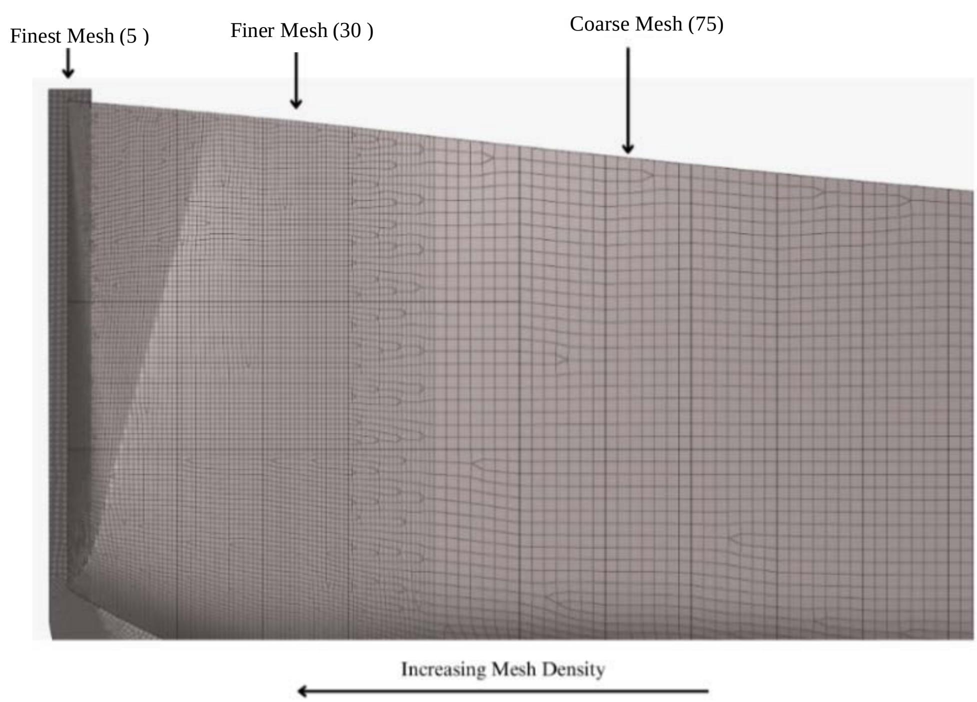


Figure 5: Mesh density regions

The generated mesh is quad-heavy as quad elements provide greater efficiency in the analysis [14]. Some complex areas have tri-elements, and areas where the mesh density changes also comprise of some triangular elements. Node connectivity was thoroughly checked for all connected elements and parts. Total number of elements is 2,55,859 and node is 2,48,146. Calculated mass is 14.82 tonnes without the engine. A 4-node shell is created for the rigid mild steel wall. The distance between the stem and rigid wall is kept at a minimum (5.65mm)to avoid the node penetration chances and to lessen the computation time. This has been shown in Figure 6.

Ref

13. Y. Liu and G. Glass, *"Effects of mesh density on finite element analysis"*, (2013).

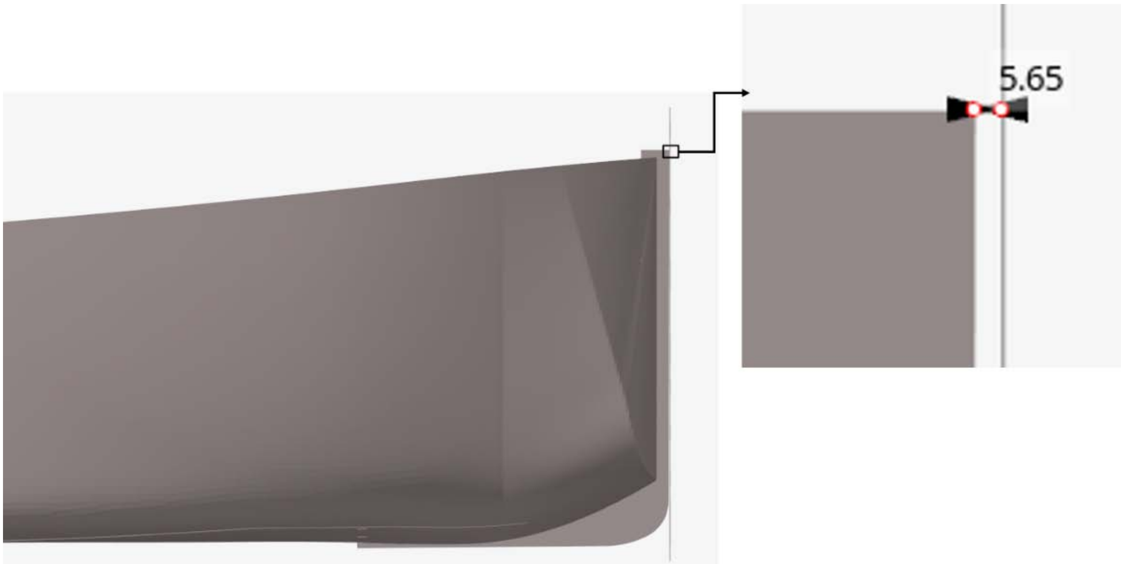


Figure 6: Distance between ship and rigid wall

Because the wall is stationary, all edge nodes of the rigid wall are fixed (encastré) to ensure they remain in place during the collision, as showed in Figure 7 and the node containing engine mass is shown in Figure 8. Then, all mesh qualities are checked using the built-in quality index in the Hypermesh software after meshing the whole model.

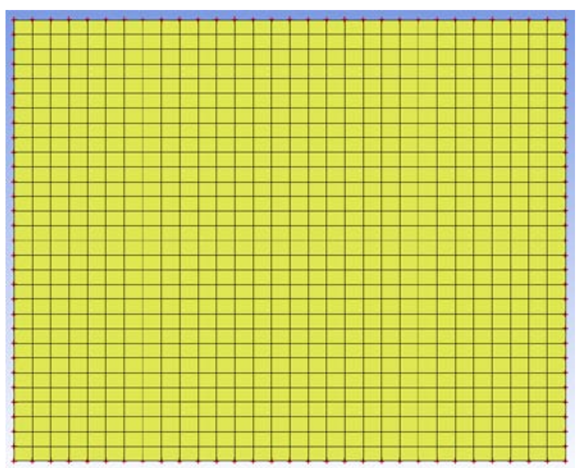


Figure 7: Boundary condition of rigid-wall

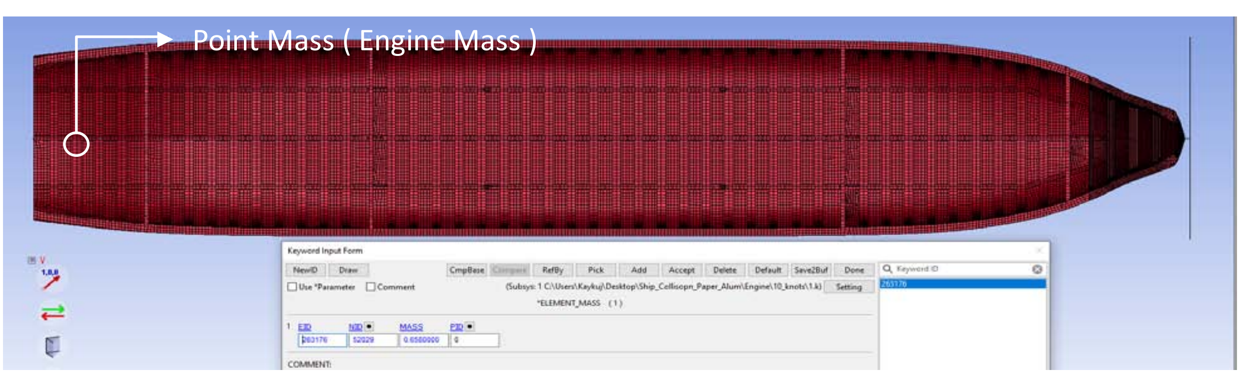


Figure 8: Location of engine and mass

Notes

c) Collision Simulation using LS-DYNA

LS-DYNA is an advanced general purpose finite element program which was developed by the Livermore Software Technology Corporation (www.lstc.com). This software package is capable of simulating complex real-world problems within the automotive, aerospace, construction, military, manufacturing, and bioengineering industries [15]. So, for simulation LS-Dyna is chosen. The mesh is exported for use in the solver deck of LS-DYNA. Initially, modal analysis is performed to ensure proper connectivity among all members. The analyses are performed under two speed settings. All nodes comprising the ship are assigned a specified speed. In the first simulation, engine mass is not taken into consideration. Only the mass of the bare ship is used and the stresses are measured. In the second simulation, the engine mass is included as a point mass. The chosen engine is Cummins qsb6.7, weighing 658 kgs, making the total weight of the ship 15.478 tones. As this is a single-screw ship, only one point-mass is added, and the stresses are measured at different time stamps. Subsequently, these values and the regions of plastic deformation are compared.

Ref

15. A. Chaulagain, *"Nonlinear finite element modelling of a simply supported beam at ambient temperature and under fire"*, (2020).

III. Results

In this section relevant results from the simulation are shown and comparisons are made between different conditions of the ship. In this study, different conditions refer to different simulation scenarios involving variations in speed (5 knots and 10 knots) and internal weight (with and without the engine). These factors influence the ship's energy absorption and structural response during collisions.

a) Energy Absorption and Distribution

The energy curves in Figures9a, 9b, 9c and 9d highlight the variation in energy distribution during the ship's collision at different speeds with varying internal weight (with engine and without it).

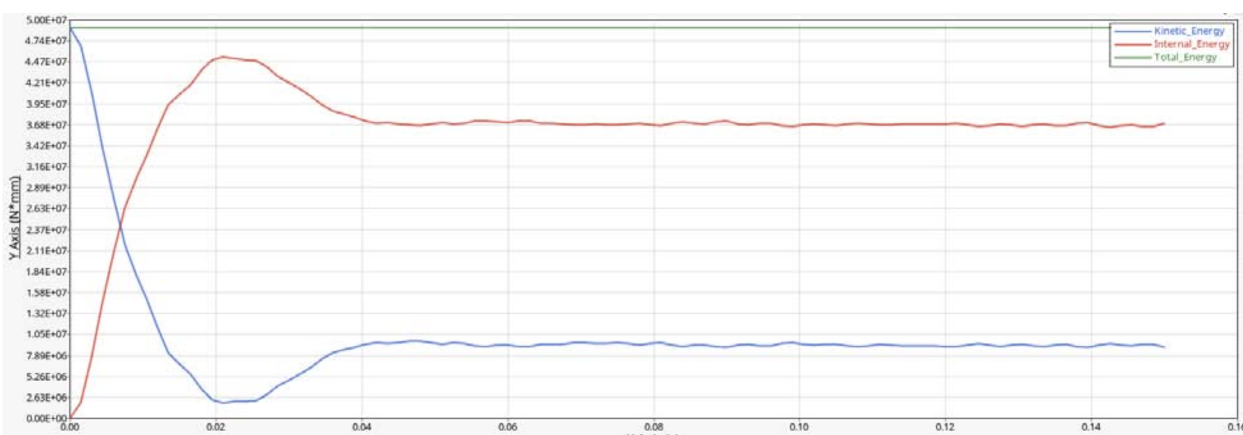


Figure 9a: Energy curves at 5 knots without engine

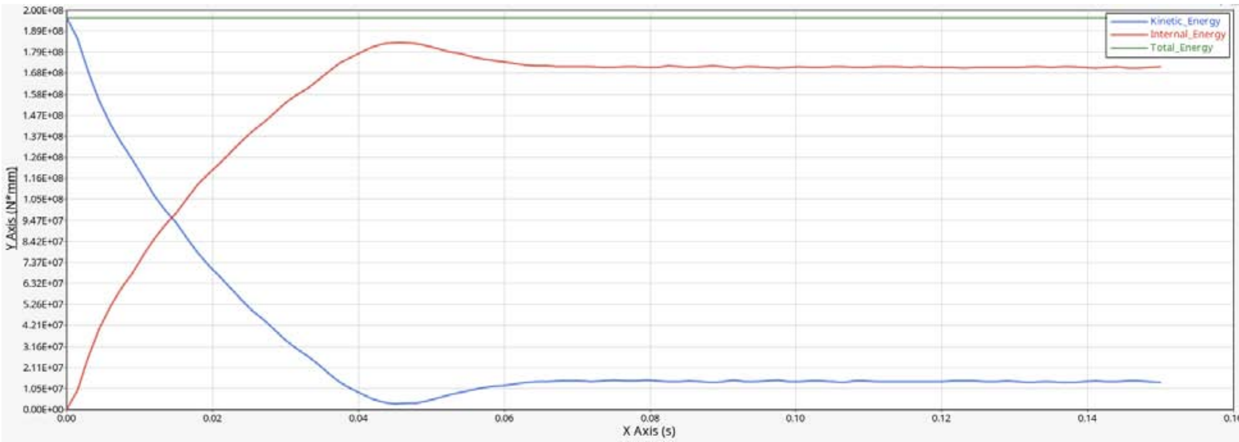


Figure 9b: Energy curves at 10 knots without engine

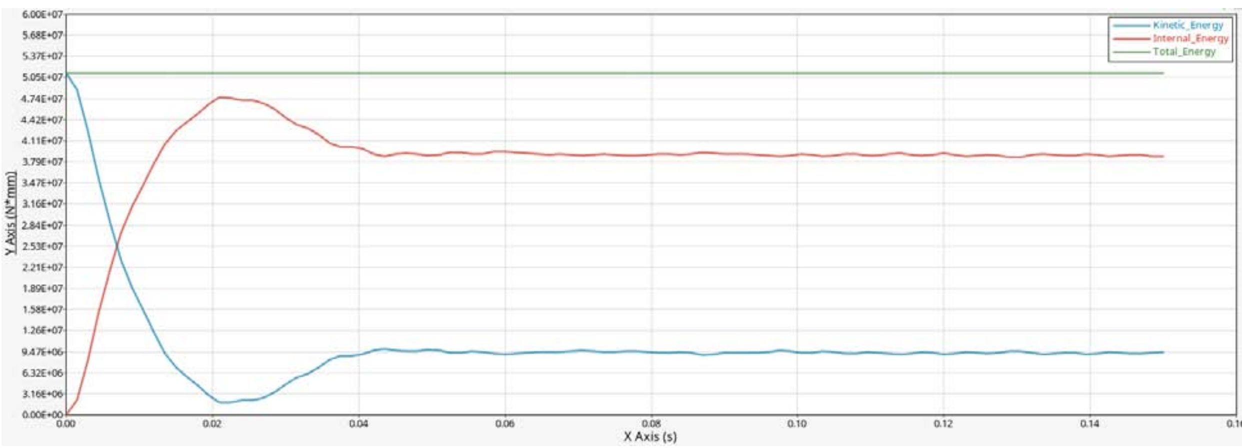


Figure 9c: Energy curves at 5 knots with engine

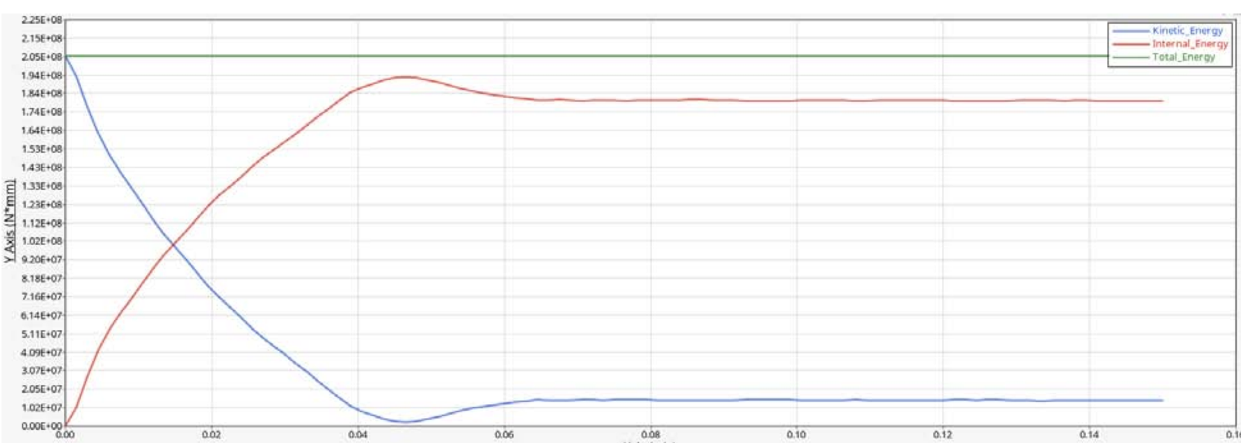


Figure 9d: Energy curves at 10 knots with engine

The three key factors, namely – kinetic, internal and total energies are analyzed.

- *5 knots without engine:* At the outset of the simulation, the initial kinetic energy is maximum at $4.902 \times 10^7 N.mm$. Upon collision, the kinetic energy rapidly converts into internal energy in the form of deformation, then levels at around $9.11\times 10^6 N.mm$. The crest value for internal energy is $4.531 \times 10^7 N.mm$. The total energy remains constant at $4.902 \times 10^7 N.mm$ validating energy conservation.

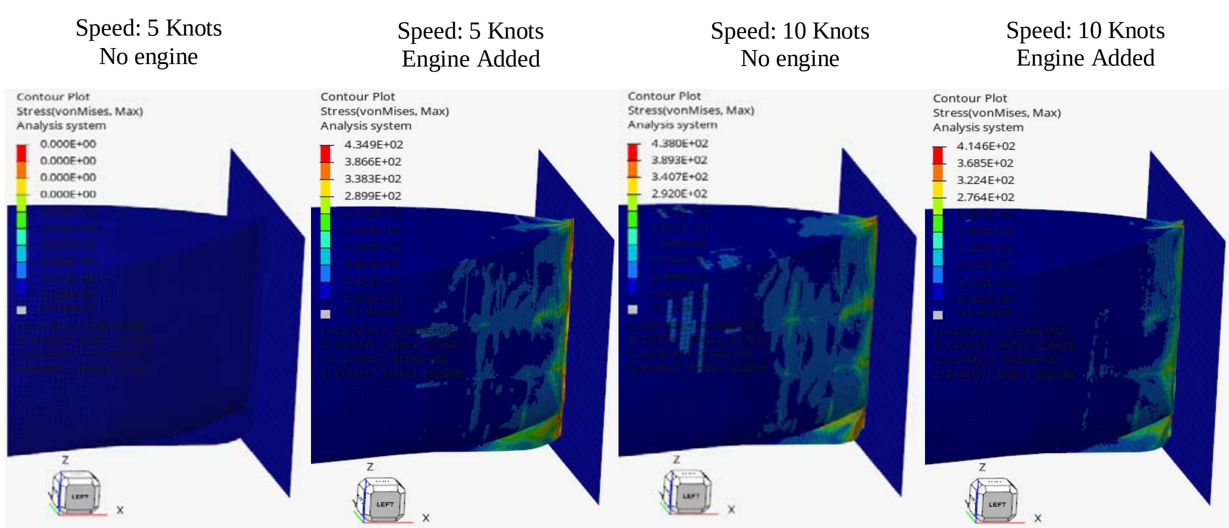
Notes

Notes

- *5 knots with engine:* At the outset of the simulation, the initial kinetic energy is maximum at $5.119 \times 10^7 N.mm$. Upon collision, the kinetic energy rapidly converts into internal energy in form of deformation and levels at around $9.36 \times 10^6 N.mm$. The maximum internal energy value is approximately $4.761 \times 10^7 N.mm$ The total energy remains constant at $5.119 \times 10^7 N.mm$ validating energy conservation.
- *10 Knots without Engine:* The initial kinetic energy at starting time is $1.96 \times 10^8 N.mm$ and after decrement, stabilizes at $1.41 \times 10^7 N.mm$ After the collision, the internal energy increases and reaches the peak value at $1.838 \times 10^8 N.mm$. The total energy remains constant.
- *10 Knots with Engine:* The initial kinetic energy at starting time is $2.047 \times 10^8 N.mm$ and after decrement, stabilizes at $1.424 \times 10^7 N.mm$ After the collision, the internal energy increases and reaches the peak value at $1.933 \times 10^8 N.mm$. The total energy remains constant.

b) Stress Analysis

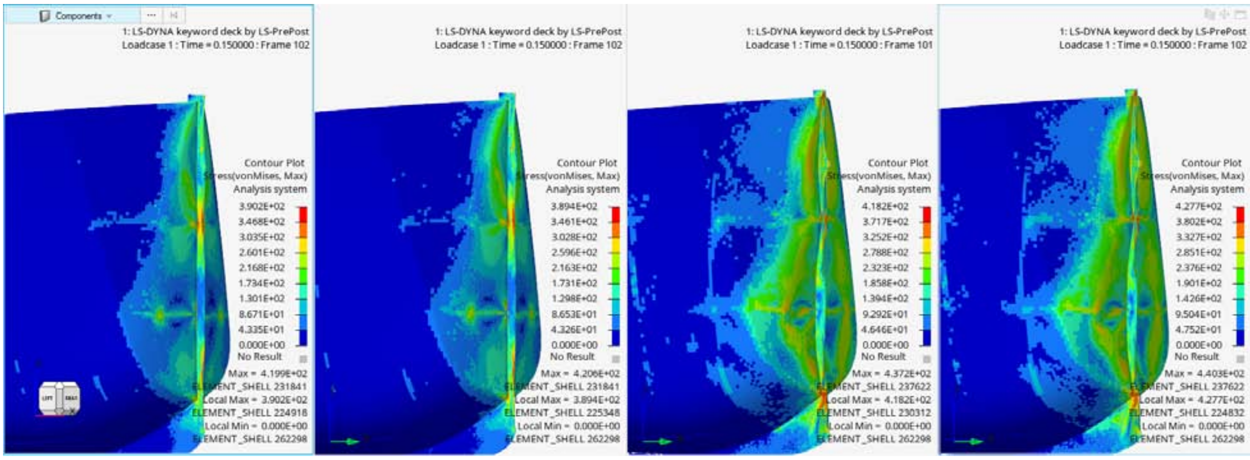
The von Mises stress distribution maps are crucial in understanding the stress responses and locations of plastic deformation. The stresses are shown in Figures 10,11 and 12 at the different times. The Figure 10 shows the wall and for a clearer visualization the wall has been hidden in Figures 11 and 12.



(a) (b) (c) (d)

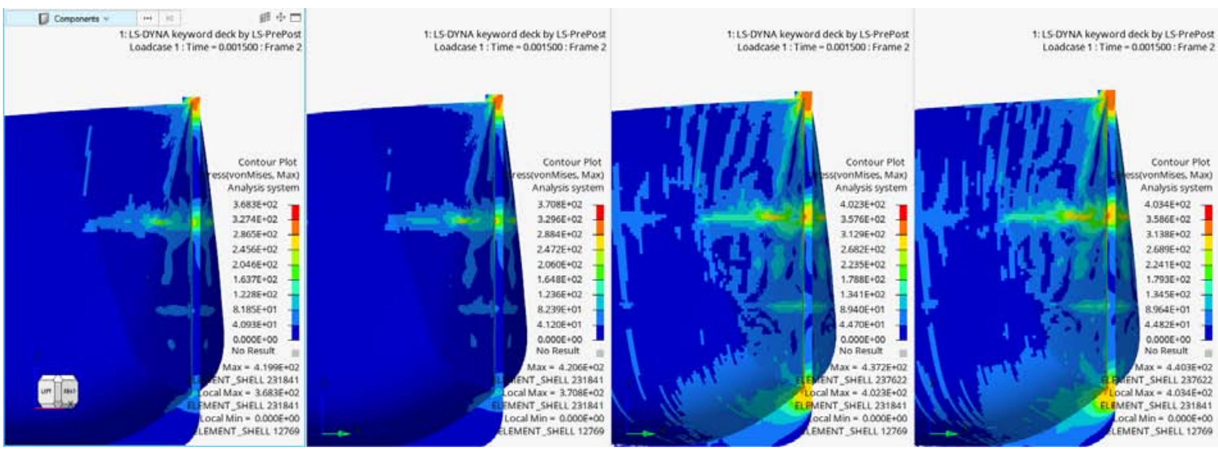
Figure 10: Initial stress contours at different conditions

Speed: 5 knots No engine | Speed: 5 knots Engineadded | Speed: 10knots No engine | Speed: 10knots Engineadded



(a) (b) (c) (d)

Figure 11: Stress contours at final time stamp



(a) (b) (c) (d)

Figure 12: Stress contours at initial time stamp

Case Studies:

- 5 Knots without Engine, Figures 10 (a), 11(a) and 12(a): In this case, the von Mises stress peaks at 368 MPa at initial impact, concentrated around the bow, especially near the stem. The stress diminishes as it propagates through the structure, indicating localized plastic deformation but without significant stress propagation into other parts of the ship. Maximum stress generated is 419.9 MPa in the stem because of the collision.
- 5 Knots with Engine, Figures 10 (b), 11(b) and 12(b): The initial impact causes a stress value of 370.8 MPa at the same locations. Peak value of generated stress is 420.6 MPa. This is a 0.76% increase in initial impact stress and a 0.17% increase in maximum stress to the added engine mass.
- 10 Knots without Engine, Figures 10 (c), 11(c) and 12(c): The initial impact causes a stress value of 402.3 MPa at the same locations. Highest stress value is 437.2 MPa. Which is a significant increase from the previous two. And the deformation is much more noticeable.

Notes

- 10 Knots with Engine, Figures 10 (d), 11(d) and 12(d): The maximum value at the initial collision is 403.4 MPa. Maximum stress value is 440.3 MPa. Which is a 0.27% and 0.7% increment from the previous simulation at the same speed at initiation and max value respectively.

All the values are summarized in Table 3.

Notes

Table 3: Comparison of internal energy and stress

Speed (Knots)	Addition of Engine Mass	Initial Stress (MPa)	Max Stress (MPa)	Kinetic Energy (N.mm)	Max Internal Energy (N.mm)	Percent Increment from5 Knots without Engine Initial Impact Maximum Local Stress (%)	Percent Increment from 5 Knots without Engine Max Overall Stress (%)	Percent Increment of Max Internal Energy from5 Knots without Engine (%)
	No	368	419.9	4.9×10^7	4.5×10^7	0	0	0
5	Yes	370.8	420.6	5.1×10^7	4.7×10^7	0.76	0.17	5.08
10	No	402.3	437.2	1.9×10^8	1.8×10^8	9.33	4.12	305.71
10	Yes	403.4	440.3	2.0×10^8	1.9×10^8	9.62	4.86	326.93

The comparison of Von Misses stresses between the two speed settings at 5-knots and 10-knots collisions clearly shows in Figures 10-12 that there exists a relationship between speed and structural deformation. At higher speeds, significantly higher amount of kinetic energy is transferred, leading to greater plastic deformation. The structural components go through higher forces, necessitating design improvements for ships or the inclusion of collision resistance devices.

The addition of engine mass in the ship structure amplifies the energy absorbed during collisions, even more so at higher speeds. Although, the increment is negligible as the engine is quite lightweight. The increased kinetic energy translates into higher internal energy, meaning the structure undergoes more plastic deformation. The stress concentration results emphasize the need for additional structural reinforcements at the effected regions when engine mass is factored into ship designs, especially in the bow and stem.

The results emphasize the need for robust structural design in ships intended for high-speed operations or heavy loads. The higher stress concentrations and plastic deformation in the affected zones observed with increased speed and engine mass suggest that such vessels should incorporate reinforced keel, bulkheads, longitudinal girders and incorporation of shock absorbing mechanisms to better absorb impact energy without compromising the ship's integrity and ensure safety.

Using *6061-T6 aluminum* as the material of the ship in the simulations provided valuable insights into the material's performance under high-speed impact conditions. The material exhibited significant plastic deformation, particularly at higher speeds and some deformation at low speed collision.

The fine mesh resolution near the bow was critical in accurately predicting the stress concentrations and potential failure points, suggesting that future simulations should maintain a high mesh density in critical regions to ensure highest possible accuracy in results.

IV. Conclusions

This study employed FEA to investigate the effects of speed and engine mass on the structural integrity of a displacement-type vessel after collisions. The simulations were conducted at two speeds (5 knots and 10 knots), with and without engine mass, to assess their influence on energy absorption and stress distribution. The results demonstrated a clear trend in how these variables influence the ship's energy absorption and stress distribution. The following conclusions can be made:

1. At 5 knots, 82% of the total kinetic energy is transferred to the ship structure, while at 10 knots, this increases to 93%. This higher energy transfer at greater speeds results in more significant energy absorption and greater plastic deformation, particularly around the bow and stem. These findings highlight the need for design improvements, such as reinforced structural components or collision resistance devices, to mitigate damage.
2. The addition of engine mass in the ship structure amplifies the energy absorbed during collisions, even more so at higher speeds. However, the increase in maximum stress is minimal, with a 0.17% rise at 5 knots and a 0.75% rise at 10 knots, as the engine contributes relatively little additional weight.
3. The results emphasize the need for robust structural design in ships intended for high-speed operations or heavy loads. The bow and stem experience the highest stress concentrations because these regions are the first to impact the obstacle, absorbing most of the collision force. The higher stress concentrations and plastic deformation in the affected zones observed with increased speed and engine mass suggest that such vessels should incorporate reinforced keel, bulkheads, longitudinal girders and incorporation of shock absorbing mechanisms to better absorb impact energy without compromising the ship's integrity and ensure safety.
4. Figures 10-12 shows variation of stresses only in the fore part of the ship in all conditions, which verifies the validity of previous study of Moulas et.al. [3], where they considered modeling of only the fore part.
5. A finer mesh near the bow is essential for accurately predicting stress concentrations and identifying potential failure points, further confirming the importance of localized meshing strategies in ship collision simulations.

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Notes

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References Références Referencias

1. E. Cakir, C. Sevgiliand R. Fiskin, *"An analysis of severity of oil spill caused by vesselaccidents",* Transportation Research Part D: Transport and Environment, vol. 90, (2021), p. 102662.
2. Ship Repair and Maintenance Services Market Size, Share, 2032.
3. D. Moulas, M. Shafieeand A. Mehmanparast, *"Damage analysis of ship collisions with offshore wind turbine foundations",* Ocean Engineering, vol. 143,(2019) pp. 149-162.
4. W. K. Liu, S. Liand H. S. Park, *"Eighty years of the finite element method: Birth, evolution, and future",* Archives of Computational Methods in Engineering, vol. 29, no. 6,(2022) pp. 4431-4453.
5. P. Woelke, N. Abboud, D. Tennant, E. Hansenand C. McArthur, *"Ship impact study: Analytical approaches and finite element modeling",* Shock and Vibration, vol. 19, no. 4, (2012), PP. 515-525.
6. D. Solis and M. Samuelides, *"Ship collision analysis using finite elements",* 1999.
7. Z. Chen, H. Fang, L Zhu, Y. Maoand W. Liu, *"Experimental tests and numerical simulations of circular reinforced concrete piers under ship impact",* Advances in Bridge Engineering, vol. 1, 00. 1, (2020), p. 5.
8. S. Zhang, R. Villavicencio, L Zhuand P. T. Pedersen, *"Ship collision damage assessment and validation with experiments and numerical simulations",* Marine Structures, vol. 63, (2019), pp. 239-256.
9. K. A. Crum, J. McMichaeland M. Novak, *"Advances in aluminum relative to ship survivability',* in American Society of Naval Engineers Day 2012 Conference, (2012), pp.122-131.
10. K. Peng and Q. Pan, *"Study on the Influence of Ship Speed and Ship Weight on Ship-Bridge Collision Force",* Hydraulic and Civil Engineering Technology VII, (2022), pp. 1047-1053.
11. M. Melchiorre and T. Duncan, *"The fundamentals of FEA meshing for structural analysis".* Ansys, 2021.
12. *A.* N. Abood, A. H. Salehand Z. W. Abdullah, *"Effect of heat treatment on strain life of aluminum alloy AA 6061',* Journal of Materials Science Research, vol. 2, no. 2, (2013) p. 51.
13. Y. Liu and G. Glass, *"Effects of mesh density on finite element analysis",* (2013).
14. J. Ezeh and J. Enem, *"Comparative study on use triangular and rectangular finite elements in analysis of deep beam",* Academic Research International, vol. 3, no. 3, (2012), p.131.
15. A. Chaulagain, "*Nonlinear finite element modelling of a simply supported beam at ambient temperature and under fire*", (2020).

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The Mathematical Bases for the Creation of a Homogenous 5D Universe

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Abstract- Several important physical implications left out in The Five Dimension Space-Time Universe: A creation and grand unified field theory model. Book, are presented under rigorous mathematical theorems. It was found that Temperature, a classical variable, must be added as an imaginary component to time, under the Quantum uncertainty dt.dE = h/2pi, so that the Gell-Mann Quark model can be verified, with gauge invariance, to form hadrons at the Bethe Fusion Temperature. Accordingly from the corresponding uncertainty dp.dr= h/2pi. Pairs of Diagonal Long Range Ordered gravitons, with continuous frequency spectrum together with those represented by magnetic monopoles must be formed within the space r, of the homogenous 5D manifold, without the presents of photons, thus defines the 5D as a Black Hole.

Keywords: extended fermat's last theorem, gravitons, newtonian law of gravity, temperature, DLRO in monopoles and gravitons. black holes.

GJSFR-F Classification: DDC: 530.12

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The Mathematical Bases for the Creation of a Homogenous 5D Universe

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Abstract- Several important physical implications left out in The Five Dimension Space-Time Universe: A creation and grand unified field theory model. Book, are presented under rigorous mathematical theorems. It was found that Temperature, a classical variable, must be added as an imaginary component to time, under the Quantum uncertainty dt.dE = h/2pi, so that the Gell-Mann Quark model can be verified, with gauge invariance, to form hadrons at the Bethe Fusion Temperature. Accordingly from the corresponding uncertainty dp.dr= h/2pi. Pairs of Diagonal Long Range Ordered gravitons, with continuous frequency spectrum together with those represented by magnetic monopoles must be formed within the space r, of the homogenous 5D manifold, without the presents of photons, thus defines the 5D as a Black Hole. Then from which we can derive the classical Newtonian Law of attractive Gravity, as the 5D manifold is mapped by Perelmann Ricci-flow entropy mapping and the DLRO graviton pair symmetry is broken and converted into two masses, with motions satisfying Special Relativity in the doughnut shape Lorentz manifold, thus indirectly verifies the principle of Covariant Riemannian curvatures and General Relativity theory.

Keywords: extended fermat's last theorem, gravitons, newtonian law of gravity, temperature, DLRO in monopoles and gravitons. black holes.

I. Introduction

Since the publication of The Five Dimension Space-Time Universe; A creation and grand unified field theory model Book in 2014 [1], in which by the utilizing of the coordinate projection onto the remaining 4D Space-Time, together with maintaining gauge invariance, and the mathematical orthogonality of the 5D manifold to that Semi-Simple-Compact Lie Groups of SU(2) + SU(3), the electro-weak leptons derived from SU(2) and from SU(3) the strong interaction of Hadrons, by the breaking the DLRO symmetry of the magnetic monopoles [2] as proposed by Gell-Mann [3]. However, the temperature value under which these elementary particles can actually occur; that is only at the Bethe Fusion Temperature, was not addressed. Because of this unanswered question on how Temperature plays a role on the DLRO symmetry breaking is the purpose of this paper. It is well known that Temperature is associated with statistical mechanics that give us the Boltzmann Theorem on energy distributions, for different quantum particles: Bosons, like that of photons, Fermions, like electrons in a metal, and classical particles, like gas molecules. All of these distributions, involve the dimensionless quantity {E/kT}, where k is the Boltzmann constant. Since the different distributions depends on quantum symmetry, it is then natural to associate E/kT with the quantum uncertainty dE.dt = h/2pi, where h is the Planck's constant. It is thereby natural to insert h/2pi/kT as an imaginary component of time, then we will get

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Ref

2. K.W. Wong, G.A.M. Dreschhoff, H. Jungner, P.C.W. Fung and W.K. Chow, (2018) The magnetic monopoles in 5D homogenous space-time. Physics Essays 21, 493.

$$i\ h/2pid\{1/kT\}.dE = -i\ h/2pi. \qquad (1)$$

Hence 1/kT is a classical inverse energy variable.

The field theory operator for 5D, given in the 5D field theory [1] is quadratic due to Fermat's Last Theorem [4]. Hence with an imaginary component to time, due to Temperature, the Fermat's sum is changed to

$$[ct]\hat{}2 + \{h/2pi/kT\}\hat{}2 = r\hat{}2. \qquad (2)$$

With SO(3) space symmetry.

It is now obvious that when T becomes infinite, the Fermat's sum reduces back to the homogenous 5D manifold, and the projection field theory model is valid, thereby the lepton weak and the Gell-Mann strong theories for elementary particles is preserved. The Bethe Fusion temperature is of order 10ˆ14K, not yet infinite. Obviously creation of matter through projection cannot happen at t=0, when even the 5D manifold does not exist, therefore we also expect the imaginary component to t, due to 1/kT also not 0. From the new Fermat's sum, Temperature now has a clear physical meaning as an artificial inducer of creation of fields and matters out of NOTHING.

This is not all, we can deduce. For the homogenous 5D manifold, if we have a vector charged current source, then we will generate the 4 Vector potentials for Electromagnetic Theory. However because the Space-Time manifold remain 5D, then there must also exist an orthogonal magnetic monopole potential [5] as stipulated by Maxwell [6]. The magnetic monopoles, are Bosons and given by DLRO of opposite charged and opposite momentum massless spinors, they are in the Bose-Einstein ground state, or literally in the Higgs vacuum. [7] Such sets of charged massless spinors, must come from the SU(2) and SU(3) generators. In fact it was shown by Gell-Mann, that these charges are the diagonal representations of the Cartan group generators, namely -e for SU(2); and 2/3e; 2/3e and -1/3e for SU(3). Since from the projection theory, we found that when these charges were converted into massive spinors, they must satisfy a single ratio, namely the 2/3e charge will have a 2/3M(Q) mass, and -1/3e charge will have a 1/3M(Q), where M(Q) is the so call Bare Quark Mass. [1]. It was also found experimentally from hadron data, that M(Q) is exactly 33MeV. equal to 66 electron rest mass of 0.5MeV. [7] Due to these charge to experimentally observed mass ratio restriction, we see then that the primordial monopole eigenvalues are in fact discrete. By applying gauge invariance, we then observed that at the Bethe Fusion Temperature, the primordial energies converted into masses ranges from m(e) to 88m(e) for a bare quarks neutron to 110m(e) for the bare quarks proton. It is interesting to point out that the 88 in between discrete energy levels if described as frequencies, is exactly that of the piano key board. It is to this identification that we can literally describe creation as a Music Code composition of a symphony. [8] Therefore starting from 10ˆ14K for the Bethe Fusion T(B) downward, Temperature is divided into steps of T(B)x110ˆ-n, where n=0, 1, 2, 3, 4, 5 and 6. representing different regions of nature's creations.

However apart from DLRO of charged massless spinors, there can exist in the 5D manifold, that is also uncharged massless boson fields that must exist. To these bosons, we name all of them them as gravitons as will become rather obvious later.

As we treated the Fermat's sum in time and space, we can also treat in momentum and energy.

$$[cp]\hat{}2 = E\hat{}2. \qquad (3)$$

Ref

4. A.D. Aczel, (1997) Fermat's Last Theorem Unlocking the Secret of an Ancient Mathematical Problem. London, Penguin Press.

When a classical Temperature is added as an imaginary time component, there must also be a corresponding imaginary momentum component. It is easy to see that from nature, only gravity remaining is classical. Therefore the imaginary p component must be from gravity, namely

$$iG2hv/c\hat{}2/r. \tag{4}$$

The factor 2hv, comes from that the massless graviton bosons composed of DLRO. massless oppositely charged fermion pairs as well as from neutral boson pairs, and G is the Newtonian constant, with r given by eq.(2). For the fermion pairs, as they are from the magnetic monopoles, they are of discrete eigen-energy values due to the Lie Groups generators.

Hence from the uncertainty dp.dr=h/2pi, we obtain for the imaginary p component

$$i\ 2G/c\hat{}2d[hv/r].dr = ih/2pi., \tag{5}$$

Since the Planck's constant h cancel out from both sides, hence the graviton frequency v is a classical frequency irrespective of whether they are discrete or continuous, similar to white or color Light given in term of a Poynting Vector of E, H, fields. structure

Thus with the presence of gravitons eq(3) is changed to

$$[cp]\hat{}2 + \{G2hv/c\hat{}2/r\}\hat{}2 = E\hat{}2. \tag{5}$$

So that if

$$p{=}0,\ E\text{-}G.2hv/c\hat{}2/r = 0. \tag{6}$$

Physically eq.(6) means within the 5D manifold photon is absent, but in order that Energy is positive in the 5D domain due to the finite Temperature within, there must be an attractive potential due to all the gravitons within the 5D domain. Thus the 5D manifold with finite Temperature is a Black Hole compose of discrete energy gravitons as well as graviton pairs of continuous energies. It is interesting to mention that from the Carbon 12 nucleus, the total monopole energy can be inside must be less than the 44 MeV. needed to create a missing neutron. It is this boundary condition restriction on the discrete DLRO graviton spectra, that make a Carbon 12 chain closed loop structure, like a DNA, able to retain lower frequencies of the Lie Group induced gravitons through quantum tunneling, thus provides the mechanism to induce free charge radicles in bio-cells to form ODLRO transition under its critical superconducting temperature, thus produces repeated growth for the cells. A very important part of the creation of life forms in the lowest n=6 Temperature step. [1]

The 5D manifold is mapped into a doughnut geometric shape 4D Lorentz manifold via the Perelmann Ricci-Flow-entropy mapping [9]. Under such a mapping the center doughnut core remains in 5D, but with r being time independent, as is a model case for a galaxy, such as the Milky Way.

To fixed core radius r, it can be obtained by differentiating eq(2) with time, and setting dr/dt=0.

We get

$$2ct\text{-}3(h/2pi)\hat{}2/k\hat{}2/T\hat{}3dT/dt = 0. \tag{7}$$

Or

Ref

9. G. Perelmann, (2002) The Entropy Formula for Ricci-Flow and its Geometric Applications. ArXiv. math. DG/0211159.

$$dT/dt = 2/3ct.T^3(2pi/h)^2.k^2. > 0. \quad (8)$$

This increasing T^3 dependence resembles that of a Bohm Black Body photon radiation, and must be compensated by actual photon radiation outside the Black Hole if the Temperature is also to remain stable, so that no stars number can change in the galaxy. For this condition to happen for the Milky Way core, we must first be able to observe the photon radiation out of the fixed r, 5D core, which was actually photographed by NASA [10]. Furthermore, because the graviton filled galactic core is an attractive potential source to matters outside, for the star systems in the galaxy to not be sucked into it, they must revolve around it with a cancelling Centrifuge force, which is also observed.

Should the entire 5D manifold is enclosed by the Perelmann Ricci-flow Surgery 3D mapping [11], then the photon radiation compensation cannot happen if the 5D core is totally enclosed inside a solid mass shell. And to maintain the core Temperature stable, molten lava composed of ions is created under the solid mass shell, such that by inducing a physical rotation of the object, such as a planet, around a North-South pole axis, will be able to generate the energy consumption equivalent to photon radiation. This necessary phenomenon due to the spinning of the planet then must be accompanied with the existence of a dipolar magnetic field as observed on earth. However if the solid mass shell is replaced by the liquid lava for stars, like in the sun, then light radiation can occur from the charged surface lava motion, reducing the star self rotation rate needed. In fact we had calculated these physical properties for many Astro-objects with comparing to observed data [12].

With all the above mathematical basis analyzed, we conclude that indeed the 5D creation model for the Universe is valid.

To summarize all of the above discussions derived from the presence of Temperature as an imaginary component of time, and gravitons as an imaginary component of momentum in the Fermat's sum of the 5D grand unified field theory, and since all creations cannot happen at the same instance in time, it means it also cannot happen at the same Temperature. As kT is proportion to energy in a statistical mechanic sense, and therefore through the energy spread between the bare electron to the bare proton composing of bare Quarks is 110 m(e), hence the Bethe Fusion Temperature of 10^14K, must also be from the statistical average with a spread in Temperature of 110K. Therefore, other creations of more complex masses must follow in later times, and at corresponding lower Temperatures, thus dividing the Temperature of creations into 7 steps, all with Temperature spread of 110K, given by the formula T(fusion)x110^-n, where n is an integer and runs from 0 to 6. [8] The lowest creation step Temperature n=6, happens to be around the liquid water phase temperature, which we know is vital to the formation of biological cells, and thereby Life forms. As Temperature was also treated as responsible for inducing the breaking of the 5D Universe symmetry via Perelmann Ricci-flow mappings [9, 11], it means all the different steps of creations are also induced by Temperature, thus the concept of all creations being represented by a Musical Code remain valid. [8] And in terms of time sequence, these creations are like artificial intelligence AI supercomputer programs, producing what we interpret as the Nature Creation consciousness, and perform like the simultaneous playing of a symphony with a motion picture in Three manifold, and thereby make all creations in terms of senses that follow in Logical Steps [13]. In this last reference, there is TWO errors in the print, caused by the mistake made for the Newtonian gravity formula provided by the DLRO pair of gravitons, and not the

Ref

10. NASA/Goddard Space Flight Center (2012) photo Milky Way galaxy.

quadratic multiplying pair of two gravitons. But nature's AI supercomputer program for each step in creation is many generations ahead of NVIDIA most up-to-date version. We have a long way yet to go in achieving the ability by using supercomputer AI to simulate nature's creations, but must be our goal if we are to be able to conquer all destructive processes nature brings along and become the ultimate purpose of God's Creations.

Notes

Acknowledgment

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References Références Referencias

1. K.W. Wong, G.A.M. Dreschhoff and H. Jungner (2014) The Five Dimension Space-Time Universe; A creation and grand unified field theory model. Scientific Research Publishing, USA.
2. K.W. Wong, G.A.M. Dreschhoff, H. Jungner, P.C.W. Fung and W.K. Chow, (2018) The magnetic monopoles in 5D homogenous space-time. Physics Essays 21, 493.
3. M. Gell-Mann, (1964) Nonleptonic weak decays and the Eight-Fold Way. Phys. Rev. Lett. 12, 155.
4. A.D. Aczel, (1997) Fermat's Last Theorem Unlocking the Secret of an Ancient Mathematical Problem. London, Penguin Press.
5. W.D. McGlinn, (1964) Problem of Combining Interaction Symmetries and Relativistic Invariance, Phys. Rev. Lett. 12, 467.
6. J.C. Maxwell, (1985) A Dynamic Theory Electromagnetic Fields, Philosophical Transaction of Royal Society of London, 155, 457.
7. P.W. Higgs, (1964) Broken Symmetries and Masses of Gauge Bosons. Phys. Rev. Lett. 13, 508.
8. D.K. Meijer and K.W. Wong, (2022) Scale Invariant Symmetry Breaking of a Musical Code from 5D Superfluid Sub-Quantum Space is instrumental in the Fabric of Reality, Acad. edu.
9. G. Perelmann, (2002) The Entropy Formula for Ricci-Flow and its Geometric Applications. ArXiv. math. DG/0211159.
10. NASA/Goddard Space Flight Center (2012) photo Milky Way galaxy.
11. G. Perelmann, (2003) Ricci-Flow with Surgery on Three Manifolds. ArXiv. math. DG/0303109.
12. P.C.W. Fung and K,W. Wong, (2015) On the origin of Mass and angular momentum of Stellar Objects. J. of Modern Physics 6, 2303.
13. K.W. Wong, (2024) Step by Step Creation Logics. Open J. of Philosophy 14.

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About Stability of Solutions to Systems of Differential Equations

By G. V Alferov, G. G. Ivanov & V. S. Korolev
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Abstract- The stability conditions for solutions of systems of ordinary differential equations are considered. The conditions and criteria for the use of partial and external derivatives are proposed. This allows us to investigate the behavior of a function of several variables, without requiring its differentiability, but using only information on partial derivatives. This reduces the restrictions on the degree of smoothness of the studied functions. The use of the apparatus of external derived numbers makes it possible to reduce the restrictions on the degree of smoothness of manifolds when studying the question of the integrability of the field of hyperplanes. Using the apparatus of partial and external derived numbers, it can be shown that the investigation of the stability of solutions of a system of differential equations can be reduced to an investigation of the solvability of a system of equations of a special form.

Keywords: *solutions of differential equations, stability conditions, apparatus of partial and external derived numbers.*

GJSFR-F Classification: *LCC: QA371*

ABOUTSTABILITYOFSOLUTIONSTOSYSTEMSOFDIFFERENTIALEQUATIONS

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About Stability of Solutions to Systems of Differential Equations

G. V Alferov [α], G. G. Ivanov [σ] & V. S. Korolev [ρ]

Abstract- The stability conditions for solutions of systems of ordinary differential equations are considered. The conditions and criteria for the use of partial and external derivatives are proposed. This allows us to investigate the behavior of a function of several variables, without requiring its differentiability, but using only information on partial derivatives. This reduces the restrictions on the degree of smoothness of the studied functions. The use of the apparatus of external derived numbers makes it possible to reduce the restrictions on the degree of smoothness of manifolds when studying the question of the integrability of the field of hyperplanes. Using the apparatus of partial and external derived numbers, it can be shown that the investigation of the stability of solutions of a system of differential equations can be reduced to an investigation of the solvability of a system of equations of a special form.

Keywords: solutions of differential equations, stability conditions, apparatus of partial and external derived numbers.

I. Introduction

Many sciences are engaged in the creation of mathematical models of various processes. The problems in the study of dynamic processes lead to complex system of differential equations [1-22]. The concepts of stability of solutions or asymptotic stability are often used in studies of solutions of equations and the ability to control the behavior in the presence of perturbations [4-7]. For their solution or successive approximations to the exact solution necessary to check the conditions and criteria that must be met. The study of control problems and the stability of solutions of systems of differential equations to describe processes that are defined as linear operations makes it possible to divide all tasks into classes and identify important properties inherent in systems of differential equations of the same class. In the study of the problems of controlling the motion of mechanical systems [9-11] in the transition from a general formal description to the construction of mathematical models take into account:

- Content and properties of functions in the system of equations of dynamics,
- Structure of control functions, restrictions or boundary conditions,
- Type of functional or quality criterion of solutions,
- Stability conditions for solutions for admissible controls.

The concepts of partial derivatives of numbers and external derivatives of numbers are considered in order to use them to study the stability of solutions of a system of differential equations through the study of the solvability of a system of

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equations of a special form. The proposed method can be used to obtain necessary or sufficient stability conditions for solutions of systems of differential equations.

II. Features of Stability Conditions

Let the change of parameters$x$ or the object's behavior be described by a system of ordinary differential equations of the form

$$\dot{x} = Ax \tag{1}$$

From equation (1) for a linear stationary system follows the validity of the following equation

$$\frac{d}{dt} x^* x = (A^* + A)\, x \tag{2}$$

Here, an asterisk means a transpose operation. Let $(A^* + A)$ be a nonsingular matrix symmetric with respect to the diagonal. Then applying the Lagrange method to equation (2) reduction of quadratic forms to the sum of squares [3], it is easy to verify that there is a linear transformation$x = Ly$, reducing equation (2) to the form

$$\frac{d}{dt} y^* L^* L y = y^* B y$$

where $B = L^* (A^* + A) L$ is the diagonal matrix. If the matrix B is negative definite, i.e. all its elements are negative, then system (1) is asymptotically stable. In general we can talk about the stability of solutions under additional conditions.

a) The partial derivatives numbers

Using the apparatus of private and external derivatives of numbers, show that the study of the stability of solutions systems of differential equations can be reduced to the study of the solvability of systems of equations of a special kind. The present studies are based on [1-3,8].

Let the function f be given in some open region of space R^n , and let it go $x = (x_1, \dots, x_n)^*$ — an arbitrary point of this areas as $\Delta x = (\Delta x_1, \dots, \Delta x_n)^*$—arbitrary increment of function *f* arguments

$$\psi_i[f](x; \Delta x) = \frac{\omega_i}{2^{n-1} \Delta x_i}, \qquad i = 1,2, \dots, n.$$

$$\omega_i = \sum_{\mu \in \nu_i} [\, f(x_1 + \mu_1 \Delta x_1, \dots, x_n + \mu_n \Delta x_n) -$$

$$- f(x_1 + \mu_1 \Delta x_1, \dots, x_{n-1} + \mu_{i-1} \Delta x_{i-1}, x_{i+1} +$$

$$+ \mu_{i+1} \Delta x_{i+1}, \dots, x_n + \mu_n \Delta x_n)],$$

where $\mu = (\mu_1, \dots, \mu_n)$, ν_i, $i = 1,2, \dots, n$, marked a bunch of n-dimensional vectors consisting of zeros and ones and having unit at the i–th place.

Notes

Definition 1: The number λ is called the partial derivative of the function *f*in point x in the variable x_i if there is a sequence $\triangle x^k$ such that for any $\Delta x_j^k \to 0, j \in (1, \ldots, n),$ at $\Delta x_j^k \to 0, k \to \infty$, and

$$\lim_{k\to\infty} \psi_i[f]\,(x;\triangle x^k) = \lambda.$$

Notes

The fact that λ is a partial derivative functions *f* at the point x with respect to the variable x_i , we will write this:

$$\lambda = \lambda_{x_i}[f](x).$$

Perform a study of the stability of solutions of systems of ordinary differential equations.

b) The external derivatives numbers

The definition of the external derivative number allows us to find the conditions for the complete integrability of continuous fields of hyperplanes. Let *M* be a Hausdorff space with a countable base, and let *p* be an arbitrary point of *M*. If a point *p* has a neighborhood *U* that is homeomorphic to an open subspace of an *n*-dimensional Euclidean space R^n, then *M* is called an *n*-dimensional topological manifold. Let M^n be an *n*-dimensional topological manifold. Let *V* be an *n*-dimensional vector space over a field of real numbers. Every linear mapping$f : V \in R$, i.e. display at which

$$f(av + bw) = af(v) + bf(w), \qquad v, w \in V, \qquad a, b \in R.$$

Definition 2: The form $\lambda[\omega](p)$ is called the external derivative of the external differential *q*-form of the classC^r , $r \geq 0$, on variety M^n at the point $p \in M^n$, if in R^n there is a sequence converging to zero $-\triangle x^k$", such that

$$(\Phi_\kappa^*)^{-1}\lambda[\omega](p) = \lim_{k\to\infty}\left\{\sum_{j_1,\ldots,j_q}\left(\sum_{i=1}^{n}\psi_{x_i}\left[a_{j_1,\ldots j_q},\right](x{:}\,\Delta x^k)dx_i\right)\wedge dx_{j_1}\wedge\ldots\wedge dx_{j_q}\right\} =$$

$$= \sum_{j_1,\ldots,j_q}\left(\sum_{i=1}^{n}\lambda_{x_i}\left[a_{j_1,\ldots j_q},\right](\phi_k(p))dx_i\right)\wedge dx_{j_1}\wedge\ldots\wedge dx_{j_q}$$

c) Investigation of the stability of solutions

Let the behavior of an object be described by a system of ordinary differential equations of the form

$$\dot{x} = F(t, x), \;\; F(t, 0) \equiv 0, \tag{3}$$

where $= (x_1, \ldots, x_n)^*$, $F(t, x) = (F_1(t, x), \ldots, F_n(t, x))^*$.

We say that the solution $x = 0$ of system (3) is Lyapunov stable if, for any ε ¿ 0 and $t_0 \geq 0$ can find $\delta(\varepsilon, t_0) > 0$ such that from $\parallel x_0 \parallel < \delta$it follows $\parallel x(t; t_0, x_0) \parallel < \varepsilon$ for all $t \geq t_0$.

We introduce the class of functions H, assuming that the function $l(r)$ belongs to this class $(l(r) \in H)$, if $l(r)$ is continuous, strictly increasing for $r \in [0, H]$, $H = const > 0$, or for $r \in [0, \infty)$, the function is $l(0) = 0$.

The function is H, which means that $l(r)$ is optional this class $(l(r) \in H)$, if $l(r)$— continuous strictly increasing at$r \in [0, H]$, $H = const > 0$, or at$r\in$ $[0,\infty)$ function, moreover $l(0) = 0$.

Definition 3: The function $V(t,x)$, $V(t,0) \equiv 0$, $t \geq 0$, will call definitely positive if there is a function $(r) \in H$, such that in

$$t \geq 0, \quad \| x \| \leq H$$

inequality holds

$$V(t, x) \geq l(\|x\|).$$

This definition is equivalent to the generally accepted definition of positive definiteness of a function $V(t, x)$.

In the future, we will adhere to the following notation:

$$K_r(x_0) = \{x : \| c - x_0 \| \leq r\},$$

$$S_r(x_0) = \{x : \| x - x_0 \| = r\}, \; r = const > 0. \tag{4}$$

For brevity we put $S_1(0) = S$.

Theorem 1: Suppose that in region (4) there exist continuous partial derivatives

$$\frac{\partial F_i}{\partial x_j}, \; i,j = 1,2,\ldots,n.$$

Then, in order for the solution $x = 0$ of system (3) was stable according to Lyapunov, it is necessary and sufficient that in the region

$$t \geq 0, \quad \| x \| \leq h, \; 0 < h = const < H, \tag{5}$$

system

$$a_0(t,x) + a(t,x) \cdot F(t,x) \leq 0, \; a(t,x) = (a_1(t,x),\ldots,a_n(t,x)), \tag{6}$$

$$\omega \wedge \lambda[\omega] \equiv 0, \; \omega = a_0 dt + a_1 dx_1 + \ldots + a_n dx_n, \tag{7}$$

had a continuous solution $a_0(t,x) + a(t,x)$ satisfying the following requirements:
1) in the region of

$$t \geq 0, \; x \in K_h(0) \setminus \{0\},$$

$$\sum_{i=0}^{n} a_i^2 (t,x) > 0, \tag{8}$$

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Notes

2) in the region of

$$t \geq 0,\ \mu \in [0,h],\ x \in S,$$

$$\int_0^{\mu} a(t,\mu' x) \cdot x d\mu' \geq l(\mu), l(\mu) \in H\ . \tag{9}$$

Notes

The solution $x = 0$ of system (3) will be called uniformly sustainable if for any $\varepsilon > 0$ there is $\delta(\varepsilon) > 0$ such that from $t_0 \geq 0, \| x_0 \| < \delta$ should

$$\| x(t; t_0, x_0) \| < \varepsilon, \quad t \geq t_0.$$

We will say that the solution $x = 0$ of system (3) is evenly attractive if exists $\Delta_0 = const > 0$ such that the condition

$$\lim_{t \to \infty} \| x(t; t_0, x_0) \| = 0$$

performed uniformly by t_0, x_0 from area

$$t_0 \geq 0, \qquad \| x_0 \| < \Delta_0.$$

If the solution $x = 0$ of system (3) is simultaneously uniformly stable and evenly attractive, then we will call uniformly asymptotically stable.

d) Stability conditions

Theorem 2. Suppose that in region (10) there exist continuous partial derivatives. Then, in order for the solution $x = 0$ of system (3) to be Lyapunov stable, it is necessary and sufficient: system had a continuous solution $\big(a_0(t,x), a(t,x)\big)$, satisfying the following requirements in the region of $t \geq 0$.

A solution $x = 0$ of system (3) will be called uniformly stable if for any $\varepsilon > 0$ there is $\delta(\varepsilon) > 0$ such that $t_0 \geq 0$ and $\| x_0 \| < \delta$ follows

$$\| x(t, t_0, x_0) \| < \delta$$

for all $t \geq t_0$,

The proposed method allows one to obtain statements that give necessary or sufficient conditions for uniform stability or asymptotic stability for solutions of systems of differential equations.

Theorem 3. Suppose that in region (4) the functions F_i and their partial derivatives $\frac{\partial F_i(t,x)}{\partial x_j}$ are continuous and bounded:

$$|F_i(t,x)| \leq B,\ B = const, \quad \left|\frac{\partial F_i(t,x)}{\partial x_j}\right| \leq A,\ A = const,\ i,j = 1,2,\ldots,n. \tag{10}$$

Then, for the solution $x = 0$ of system (3) to be uniformly asymptotically stable, it is necessary and sufficient that in region (5), where h is a sufficiently small constant, system (6)–(7) has a continuous solution $(a_0(t,x), a(t,x))$, satisfying in the area (8) or (9) the following constraints:

1) $\sum_{i-0}^{n} a_i^2\,(t,x) > 0$;

2) $l_1(\mu) \le \int_0^{\mu} a(t,\mu' x)\cdot x d\mu' \le l_2(\mu)$;

3) $\max_{t\ge 0, \|x\|=1}[a_0(t,\mu x) + a(t,\mu x)\cdot F(t,\mu x)] \le -l_3(\mu)$, $l_k(r) \in H$.

The proposed method allows to obtain the necessary or sufficient conditions for the stability of solutions of systems of differential equations.

Notes

e) Stability of Almost Periodic Solutions

On the basis of the previous theorems, the authors obtain the conditions to determine the maximum possible number of almost periodic solutions in first-order differential equation. Now the problem of the existence of almost periodic solutions for the equation is under consideration, since this allows for the determination of the minimum possible number of almost periodic solutions for the differential equation considered.

So, consider the first-order differential equation

$$\dot{x} = f(t,x), \tag{11}$$

where f is a function continuous on R^2 that is almost periodic in t uniformly in x in every compact set and such that equation (11) has the property of existence and uniqueness of its solutions.

To prove the existence of almost periodic solution for equation (11), the result obtained should be used. Let it be formulated in the form of the following theorem.

This study allows to determine the minimum possible number of almost periodic solutions for the considered differential equation. Consider the first-order differential equation (1), where f is a function continuous on R^2 almost periodic in t uniformly in x on each compact set and such that equation (1) has the property of existence and uniqueness of solutions. In proving the existence of an almost periodic solution of equation (1), the results obtained in [9] are used.

Consider now stability of the solutions of equation (11) [6-10, 18-22].

Theorem 4: If the right-hand side of equation (11) is a function decreasing with respect to x for each fixed t, then all solutions of this equation are uniformly stable.

Proof. Let $u(t)$ be an arbitrary solution of equation (11). Suppose $y = x - u$. The equation for y is of the following form:

$$\dot{y} = f(t,u+y) - f(t,u) = g(t,y). \tag{12}$$

Let the following function be the Lyapunov function:

$$v(y) = \frac{1}{2}y^2. \tag{13}$$

Since $f(t,x)$ decreases with respect to x at each fixed t, the derivative of the function (13) on the solutions of Equation (12) satisfies the inequality

$$\left.\frac{dv}{dt}\right|_{(16)} = yg(t,y) \le 0,$$

which implies the uniform stability of solution $y=0$ of equation (12), and hence, solution $u(t)$ of equation (11). Taking into account the fact that $u(t)$ is an arbitrary solution of equation (11), it is clear the theorem is proven.

Notes

Note that the theorem implies in the conditions of Theorem 14 that all n almost periodic solutions of equation (11) are stable, either as $t \to +\infty$ or with $t \to -\infty$.

Let $\lambda_x[f](t,x)$ denote an arbitrary derived number of the function $f(t,x)$ at the point x for a fixed t.

Theorem 5: If there exists a constant $\alpha > 0$ such that for any fixed t and each derived number $\lambda_x[f](t,x)$ performed inequality

$$\lambda_x[f](t,x) \le -\alpha,$$

then all the solutions of equation (11) are uniformly asymptotically stable in general. If it is additionally known that equation (11) has an almost periodic solution, then all the solutions of equation (11) are asymptotically almost periodic.

Proof: Let $u(t)$ be an arbitrary solution of equation (11). Let a function y be introduced, setting that

$$y = x - u\cdot$$

It is clear that if x is a solution of equation (11), then y is a solution of equation (12). Let us obtain a derivative of equation (13) on solutions of equation (12).

Repeating the proof of Theorem 12 [21], it is easy to show that there exist derived numbers for which the following relation holds:

$$f(t,y+u) - f(t,u) \le y\lambda_{u+\theta y}[f](t,u+\theta y),$$

$$\theta \in (0,1).$$

Taking into account that by the condition of the theorem

$$\lambda_{u+\theta y}[f](t,u+\theta y) \le -\alpha,$$

the following estimation is obtained:

$$\left.\frac{dv}{dt}\right|_{(16)} \le -\alpha y^2.$$

It follows from this inequality that the solution $y=0$ of equation (12) is uniformly asymptotically stable, as well as the solution $u(t)$ of equation (11). Since $u(t)$ is an arbitrary solution of equation (11), all the solutions of equation (11) are asymptotically stable.

If equation (11) has an almost periodic solution, then all the solutions of equation (11) are asymptotically almost periodic in the view of its uniform asymptotic.

Theorem 6: If the function $f(t,x)$ from the right-hand side of equation (11) decreases with respect to x at each fixed t, and on each compact set

$$\{(y,u): |u| \le u_0, d_1 \le |y| \le d_2, d_1 > 0\}$$

as $t \to \infty$

$$sign(y)\int_0^t [f(\tau, y+u) - f(\tau,u)]d\tau \to -\infty$$

uniformly, then the solution $y=0$ of equation (12) is uniformly asymptotically stable.

Proof: Let $u(t)$ be an arbitrary bounded solution of equation (11).
Suppose that

$$y = x - u\,.$$

It follows from Theorem 12 that the solution $y=0$ of equation (12) is uniformly stable. Let us prove that all the solutions of equation (12) tend to zero as $t \to \infty$.

Suppose the contrary. Then for some solution $y(t;0,y_0)$ of equation (12), there exists $d > 0$, such that

$$y(t;0,y_0) > d.$$

Here it is assumed that $y_0 > 0$, for definiteness. In the proof of Theorem 12, it is shown that the inequality

$$y\dot{y} \le 0,$$

which implies that —y— does not increase on the solutions of the equation (12). Therefore, in the considered case for $t \ge 0$,

$$d \le y(t) \le y_0.$$

Suppose that

$$u_0 = \sup_t |u(t)|.$$

It follows from equation (12) that

$$\frac{\dot{y}}{y} = \frac{g(t,y)}{y} \le \frac{g(t,y)}{y_0}.$$

Hence, by virtue of the conditions of the assertion, we obtain

Notes

$$\lim_{t\to\infty} y(t) \le \lim_{t\to\infty} y_0 e^{1/y_0 \int_0^t g(\tau,y)d\tau} = 0,$$

which contradicts the introduced assumption. The case when $y_0 < 0$ is treated in a similar way. Thus, the solution $y = 0$ of equation (12) is uniformly asymptotically stable.

Notes

III. Conclusion

The proposed apparatus of partial and external derived numbers allows us to investigate the behavior of a function of several variables, without requiring its differentiability, but using only information about partial derived numbers. This reduces the limitations imposed on the degree of smoothness of the functions studied.

The use of the apparatus of external derived numbers also makes it possible to reduce the restrictions on the degree of smoothness of manifolds when studying the question of the integrability of the hyperplanes field.

Theorems of the derived numbers method to estimate the number of periodic solutions of first-order ordinary differential equations are formulated and proved.

Using the apparatus of derived numbers allows to weaken the constraints imposed on the right-hand sides of the differential equations analized in this paper, and thereby increase the generality degree of the results. The upper and lower bounds for the numbers of periodic and almost periodic solutions of ordinary first-order dofferential equations are carried out. Conditions for the existence of periodic and almost periodic solutions are established. Using the apparatus of derived numbers allowed us to expand the scope of the results obtained. The application of the method of derivative numbers in problems of estimating the number of almost periodic solutions of first-order differential equations is shown. Conditions are found for determining upper and lower bounds for almost periodic solutions of ordinary differential equations of the first order. The questions of existence and stability of these solutions are investigated.

References Références Referencias

1. Zubov V.I., Lyapunov's methods and their application. (in Russian), Leningrad State University Publ., (1957)
2. Demidovich B.P., Lectures on the mathematical theory of stability. (in Russian). Science, Moscow (1967)
3. Gantmakher F.R., Matrix theory. (in Russian), Fizmatlit, Moscow (2010)
4. Lyapunov A.M., The general problem of motion stability. (in Russian), Gostekhizdat, Moscow-Leningrad (1950)
5. Arnold V.I.: Ordinary Differential Equations. (in Russian), Science, Moscow (1975),240 p.
6. Alferov G., Ivanov G., Efimova P., Sharlay A.: Stability of linear systems with multitask right-hand member. Stochastic Methods for Estimation and Problem-Solving in Engineering, (Book Chapter). P. 74–112 (2018)
7. Alferov G., Ivanov G., Sharlay A., Fedorov V.: Application of derived numbers theory in convec analysis. AIP Conference Proc. 2116(08003) (2019)

8. Ivanov G., Alferov G., Gorovenko P.: Derivatives of numbers of functions of one variable. Vestnik of Perm University. Mathematics. Mechanics. Computer Science. 3 (42) 5–19 (2018)
9. Ivanov G., Alferov G., Sharlay A., Efimova P.: Conditions of Asymptotic Stability for Linear Homogeneous Switched System. Intern. Conf. on Numerical Analysis and Appl. Math., AIP Conference Proc. 1863(080002) (2017)
10. Kadry S., Alferov G., Ivanov G., Sharlay A.: Stabilization of the program motion of control object with elastically connected elements. AIP Conference Proc. 2040(150014) (2018)
11. Kadry S., Alferov G., Ivanov G., Sharlay A.: About stability of select or linear differential inclusions. AIP Conference Proc. 2040(150013) (2018)
12. Kadry S., Alferov G., Ivanov G., Sharlay A.: Derived Numbers of One Variable Convex Functions. IJPAM, 41, 649–662 (2019)
13. Kulakov F., Alferov G., Sokolov B., Gorovenko P., Sharlay A.: Dynamic analysis of space robot remote control system. AIP Conference Proc. 1959(080014) (2018)
14. Korolev V.: Properties of solutions of nonlinear equations of mechanics control systems. Intern. Conf. on Constructive Nonsmooth Analysis and Related Topics, CNSA – IEEE Conference Proc. (7973973) (2017).
15. Korolev V., Pototskaya I.: Integration of dynamical systems and stability of solution on a part of the variables. Applied Mathematical Sciences, 9(15), 721–728 (2015)
16. Kadry S., Alferov G., Ivanov G., Korolev V., Selitskaya E.: A New Method to study solutions of ordinary differential equation using functional analysis. MATHEMATCS, 7(8) (2019)
17. Ivanov G.G., Alferov G.V. and Korolev V.S., Apparatus of derivatives and possible applications. // Bulletin of the Perm University. Mathematics. Mechanics. Computer science. 2021. N 3(54). pp. 5-18.
18. Ivanov G.G., Alferov G.V., Korolev V.S. On the stability of solutions to a system of linear differential equations. // Bulletin of the Perm University. Mathematics. Mechanics. Computer science. 2022. N 2(57). pp. 31-39.
19. Kadry S., Alferov G., Ivanov G., KorolevV. Investigation of the stability of solutions of systems of ordinary differential equations. 2020 AIP Conference Proceedings 2293,060004
20. Kadry S., Alferov G., Ivanov G., Korolev V. About of the asymptotical stability of solutions of systems of ordinary differential equations. 2020 AIP Conference Proceedings 2293,060005.
21. Alferov G.; Ivanov G.; Sharlay A.; Fedorov V. Application of derived numbers theory in problem of function extremum. AIP Conf. Proc. 2019, 2116,080002
22. Kadry S.; Alferov G.; Ivanov G.; Sharlay A. Stabilization of the program motion of control object with elastically connected elements. 2018 AIP Conf. Proc. 2040,150014.

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Application of Laplace Transform for Solving Improper Integrals Containing Bessel's Function as Integrand, In the Form of Hypergeometric Function

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Abstract- In this article we use Laplace Transform for solving Improper integrals whose Integrand contains Bessel's function in the form of hypergeometric function. The theory of Bessel's function is a rich subject due to its significant role in providing solutions for differential equations associated with many applications Their applications span across disciplines like heat conduction, electromagnetism, signal processing, and more.

Keywords: *laplace transform, bessel's function, hypergeometric function, improper integral.*

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Application of Laplace Transform for Solving Improper Integrals Containing Bessel's Function as Integrand, In the Form of Hypergeometric Function

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Abstract- In this article we use Laplace Transform for solving Improper integrals whose Integrand contains Bessel's function in the form of hypergeometric function. The theory of Bessel's function is a rich subject due to its significant role in providing solutions for differential equations associated with many applications Their applications span across disciplines like heat conduction, electromagnetism, signal processing, and more.

Keywords: laplace transform, bessel's function, hypergeometric function, improper integral.

I. Introduction

Special functions are applicable in solving variety of problems of mathematical physics, statistics mechanics, dynamics, functional analysis etc. In recent few decades, many researchers are working on this field, especially with Bessel's function .Many researchers applied different Integral transforms include the Fourier Transform, Laplace Transform and the Mellin Transform for solving many problems of science and engineering The solution of many Engineering problems like acoustics, electromagnetism, quantum mechanics and other areas frequently lead to integrals involving Bessel's functions. When we solve these types of problems by using Integral Transform it is necessary to know the Integral Transform of Bessel's Function.

Mathematically, Bessel's function is defined by

$$J_n(t) = \sum_{k=0}^{\infty} (-1)^k \frac{1}{k!\,\Gamma(\mathrm{n}+\mathrm{k}+1)} (t/2)^{n+2k}$$

And is known as Bessel's function of the first kind of order 'n'

The Laplace transform of a real-valued function of real-valued function $f(t) of\ t$ when $t > 0$ is denoted by $f(s)\ or\ L\big(f(t)\big)$ where s is a real or complex parameter and it is denoted by improper integration given by $f(s) = \int_0^\infty e^{-st} f(t)dt$

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The aim of present article is to determine the value of improper integral containing Bessel's function as Integrand by using Laplace transform.

a) Some properties of Laplace Transform

Linearity Property:

The linearity property of the Laplace transform is a fundamental property that states the transform of a linear combination of functions is equal to the linear combination of the individual transforms. Mathematically, this property can be expressed as follows:

$$L(a(f(t)) + L(b(f(t)) = aL(f(t)) + bL(f(t))$$

Change of scale Property: If Laplace transform of function $f(t)\, is\, F(s)$ then, Laplace transform of $e^{at}f(at)$ is given by $\frac{1}{a}f\left(\frac{s}{a}\right)$

Shifting Property: If Laplace transform of a function$f(t)\, is\, (f(s)$then, Laplace transform of $e^{at}f(t)$ is given by $f(s-a)$.

Laplace transform of the derivative of the function: The Laplace transform of the derivative of a function $f(t)$ is given by $sf(s) - f(0)$ where $f(s)$ is the Laplace Transform of $f(t)$.

Laplace transform of Integral of the function: If Laplace transform of function $f(t)\, is\, f(s)$ then $L\int_0^t f(t)dt = \frac{1}{s}f(s)$

Laplace transform of Function, (Multiplication by 't' theorem): If Laplace transform of function $f(t)\, is\, f(s)$then,$L(t(f(t)) = (-1)\frac{d}{ds}f(s)$

Laplace transform of Bessel's Function:

The Bessel's Function is defined as

$$J_n(t) = \sum_{k=0}^{\infty}(-1)^k \frac{1}{k!\,\Gamma(n+k+1)}(t/2)^{n+2k}$$

$$LJ_n(t) = L\left(\sum_{k=0}^{\infty}(-1)^k \frac{1}{k!\,\Gamma(n+k+1)}(t/2)^{n+2k}\right)$$

$$\int_0^{\infty} e^{-st}\sum_{k=0}^{\infty}(-1)^k \frac{1}{k!\,\Gamma(n+k+1)}(t/2)^{n+2k}$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{2k+n}\;k!\,\Gamma(n+k+1)}\int_0^{\infty} e^{-st}t^{2k+n}dt$$

$$\frac{1}{s^{n+1}}\cdot\frac{1}{2^n}\sum_{k=0}^{\infty}\frac{(-1)^k\Gamma(n+1)}{2^{2k}\;k!\,\Gamma(n+k+1)}\frac{\Gamma(2k+n+1)}{\Gamma(n+1)\,.\,s^{2k}}$$

Notes

$$\frac{1}{s^{n+1}}\cdot\frac{1}{2^n}\sum_{k=0}^{\infty}\frac{(-1)^k(1+n)_{2k}}{2^{2k}\ k!\ (1+n)_k}\frac{1}{.s^{2k}}$$

$$\frac{1}{s^{n+1}}\cdot\frac{1}{2^n}\sum_{k=0}^{\infty}\frac{(-1)^k 2^{2k}\left(\frac{1+n}{2}\right)_k\left(1+\frac{n}{2}\right)_k}{2^{2k}\ k!\ (1+n)_k}\frac{1}{.s^{2k}}$$

$$\frac{1}{s^{n+1}}\cdot\frac{1}{2^n}2F_1\begin{bmatrix}\frac{1+n}{2} & , & \left(1+\frac{n}{2}\right)\frac{1}{s^2}\\ & 1+n\,; & \end{bmatrix}$$

Hypergeometric Function: Hypergeometric function denoted as $2F_1(\mathrm{a,b;c;z})$ is a special function that arises in various areas of mathematics including analysis and mathematical physics. It is defined by the series

$$2F_1(\mathrm{a,b;c;z})=\sum_{k=0}^{\infty}\frac{(a)_k(b)_k}{k!\ (c)_k}z^k$$

Where $(a)_k$ denotes the Pochhammer's symbol

$$(a)_k=a\,(a+1)(a+2)\ldots\ldots\ldots\ldots(a+n-1)\text{ and }\ (a)_0=1$$

The hypergeometric function is a generalization of several elementary function such as the binomial series and it satisfies various differential equations including the hypergeometric differential equation. It has applications in solving differential equation, evaluating definite integrals and expressing solutions to certain problems. The hypergeometric function has many special cases and identities making it a powerful tool in mathematical analysis. It is extensively studied and applied in different branches of mathematics and physics.

II. Applications

In this section, some applications are given in order to explain the advantage of Laplace transform of Bessel's function for evaluating the improper Integral containing Bessel's function as Integrand.

1. Evaluation of $I=\int_0^\infty e^{-t}J_n(t)dt$

we have, $$LJ_n(t)=\int_0^\infty e^{-st}J_n(t)dt=\frac{1}{s^{n+1}}\cdot\frac{1}{2^n}2F_1\begin{bmatrix}\frac{1+n}{2} & , & \left(1+\frac{n}{2}\right)\frac{1}{s^2}\\ & 1+n\,; & \end{bmatrix}$$

here $\mathrm{s}\to 1$, therefore,

Notes

$$\int_0^{\infty} e^{-t} J_n(t)dt = \frac{1}{2^n} {}_2F_1\left[\begin{matrix}\frac{1+n}{2} & , \left(1+\frac{n}{2}\right) & 1 \\ & 1+n\,; & \end{matrix}\right]$$

2. Evaluation of $I = \int_0^{\infty} te^{-2t} J_n(t)dt$

$$LJ_n(t) = \frac{1}{s^{n+1}} \cdot \frac{1}{2^n} {}_2F_1\left[\begin{matrix}\frac{1+n}{2} & , \left(1+\frac{n}{2}\right)\frac{1}{s^2} \\ 1+n\,; & \end{matrix}\right]$$

$$L(tJ_n(t)) = \frac{1}{s^{n+2}} \cdot \frac{1}{2^n} {}_2F_1\left[\begin{matrix}\frac{1+n}{2} & , \left(1+\frac{n}{2}\right)\frac{1}{s^2} \\ 1+n\,; & \end{matrix}\right]$$

here s → 2,

$$\int_0^{\infty} te^{-2t} J_n(t)dt = \frac{1}{2^{n+2}} \cdot \frac{1}{2^n} {}_2F_1\left[\begin{matrix}\frac{1+n}{2} & , \left(1+\frac{n}{2}\right)\frac{1}{2^2} \\ 1+n\,; & \end{matrix}\right]$$

3. Evaluation of $I = \int_0^{\infty} e^{-t} \left(\int_0^t J_n(u)\,du\right)dt$

$$LJ_n(t) = \frac{1}{s^{n+1}} \cdot \frac{1}{2^n} {}_2F_1\left[\begin{matrix}\frac{1+n}{2} & , \left(1+\frac{n}{2}\right)\frac{1}{s^2} \\ 1+n\,; & \end{matrix}\right]$$

By property of Laplace Transform of integral,

$$L\int_0^t J_n(u)\,du) = \frac{1}{s} L(J_n(u)) = \frac{1}{s^{n+2}} \cdot \frac{1}{2^n} {}_2F_1\left[\begin{matrix}\frac{1+n}{2} & , \left(1+\frac{n}{2}\right)\frac{1}{s^2} \\ 1+n\,; & \end{matrix}\right]$$

here s → 1,

$$L\int_0^t J_n(u)\,du) = \frac{1}{s} L\,(J_n(u)) =. \frac{1}{2^n} {}_2F_1\left[\begin{matrix}\frac{1+n}{2} & , \left(1+\frac{n}{2}\right) & 1 \\ & 1+n\,; & \end{matrix}\right]$$

Notes

Notes

4. Evaluation of $I = \int_0^\infty e^{-2t}\left[\frac{d}{dt}J_n(t)\right]dt$

$$L(J_n(t)) = \frac{1}{s^{n+1}}\cdot\frac{1}{2^n}\sum_{k=0}^{\infty}\frac{(-1)^k\Gamma(n+1)}{2^{2k}\ k!\,\Gamma(n+k+1)}\frac{\Gamma(2k+n+1)}{\Gamma(n+1)\,.\,s^{2k}}$$

Now, by using the property of Laplace transform of derivative of function, we have

$$\mathrm{L}\left[\frac{d}{dt}J_n(t)\right] = s.\frac{1}{s^{n+1}}\cdot\frac{1}{2^n}\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{2k}\ k!\Gamma(n+k+1)}\frac{\Gamma(2k+n+1)}{.s^{2k}}$$

$$=.\frac{1}{s^n}\cdot\frac{1}{2^n}\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{2k}\ k!\Gamma(n+k+1)}\frac{\Gamma(2k+n+1)}{.s^{2k}} \quad (1)$$

Then by the differentiation of Laplace transform we have

$$\mathrm{L}\left[\frac{d}{dt}J_n(t)\right] = \int_0^\infty e^{-st}\left[\frac{d}{dt}J_n(t)\right]dt \quad (2)$$

$$\int_0^\infty e^{-st}\left[\frac{d}{dt}J_n(t)\right]dt =$$

$$.\frac{1}{s^n}\cdot\frac{1}{2^n}\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{2k}\ k!\,\Gamma(n+k+1)}\frac{\Gamma(2k+n+1)}{.\,s^{2k}}$$

Taking s → 2,

$$\int_0^\infty e^{-2t}\left[\frac{d}{dt}J_n(t)\right]dt =$$

$$.\frac{1}{2^n}\cdot\frac{1}{2^n}\sum_{k=0}^{\infty}\frac{(-1)^k}{2^{2k}\ k!\,\Gamma(n+k+1)}\frac{\Gamma(2k+n+1)}{.\,2^{2k}}$$

$$= \frac{1}{2^{2n+2k}}\,2F_1\begin{bmatrix}\frac{1+n}{2} & , \left(1+\frac{n}{2}\right) & 1\\ & 1+n\,; & \end{bmatrix}$$

III. Conclusion

In this article, we have successfully discussed the application of Laplace transform for solving improper integrals whose Integrand contains Bessel's Function. The given numerical applications show the advantage of Laplace transform for evaluating improper integral whose Integrand contains Bessel's function.

References Références Referencias

1. Sudhanshu Agarwal, Ajay Singh, et.al. Application of Laplace Transform for *Solving Improper Integrals whose Integrand consisting Error function*. Journal of Advanced Research in Applied Mathematics and Statistics 2019;4(2):1-7
2. Aggarwal S Gupta AR. Singh etal. Application of Laplace Transform for solving population growth and Decay problems. *International journal of latest Technology in Engineering, management and applied Science 2018;7(9):141-145*
3. D.E Rainville, Special Functions Chelsea, New York,1960
4. W.W Bell*, Special Functions for Scientists and Engineers*, Courier Corporation,2004
5. Sneddon I.N., *The Use of Integral Transforms,* Mc Graw-Hill Book Company, New York 1972.

Notes

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Entropy-Based Stability of Fractional Self- Organizing Maps with Different Time Scales

By C. A Peña Fernández
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Abstract- The behavior of self-organizing neural maps, which develop through a combination of long and short-term memory, involves different time scales. Such a neural network's activity is characterized by a neural activity equation representing the fast phenomenon and a synaptic information efficiency equation representing the slow part of the neural system. The work reported here proposes a new method to analyze the dynamics of self-organizing maps based on the flow-invariance principle, considering the performance of the system's different time scales. In this approach, the equilibrium point is determined based on the estimate for the entropy at each iteration of the learning rule, which is generally sufficient to analyze existence and uniqueness. In this sense, the viewpoint reported here proves the existence and uniqueness of the equilibrium point on a fractional approach by using a Lyapunov method extension for Caputo derivatives when $0 < \alpha\alpha < 1$. Furthermore, the global exponential stability of the equilibrium point is proven with a strict Lyapunov function for the flow of the system with different time scales and some numerical simulations.

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ENTROPYBASEDSTABILITYOFFRACTIONALSELFORGANIZINGMAPSWITHDIFFERENTTIMESCALES

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Entropy-Based Stability of Fractional Self-Organizing Maps with Different Time Scales

C. A Peña Fernández

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Highlights

- Self-organizing neural maps develop through long and short-term memory and have fast and slow activity equations.
- Lyapunov method extended for Caputo derivatives helps estimate and examine entropy behavior during learning.
- Self-organizing neural maps based on competitive differential equations lack entropy-based synaptic efficiency.
- The final stage of training is not affected by external stimuli.
- Pattern learning has lower entropy-rate without external stimuli during learning process.

I. Introduction

Self-organizing neural maps, also known as competitive neural networks, are an important class of neural networks. These networks focus on two key aspects: the ability to store desired patterns as stable equilibrium points and the mutual interference between neuron and learning dynamics. This research examines how cortical cognitive maps work by using a self-organizing map with differential equations for neural activity levels, short-term memory (STM), and synaptic information efficiency, long-term memory (LTM).

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STM and LTM models are usually based on classical Grossberg's approach or Amari's model for primitive neural competition [1, 2]. These models often involve mutually inhibitory neurons with fixed synaptic connections [3, 4, 1]. Researchers have studied competitive neural systems using flow invariance theory and singular perturbation theory on large-scale networks. These networks have two types of state variables that describe the slow unsupervised dynamics of synapses and the fast dynamics of neural activity. The fast dynamics are usually represented by the goal of the self-organizing map, such as clustering or recognition. One example is the Willshaw-Malsburg model [1, 5] of topographic formation, which uses solutions of equations of synaptic self-organization coupled with the field equation of neural excitations to improve the understanding of the dynamics of cortical cognitive maps [4, 6]. However, the design using classical competitive differential equations is not broad enough because the synaptic efficiency is not dependent on entropy. Therefore, this paper extends these approaches to incorporate systems where external stimuli can modify the synaptic information efficiency. This is done by estimating the entropy of information transfer between neurons. In other words, this research focuses on LTM in terms of how information transfer evolves. Fractional differential equations are used to model synaptic efficiency based on entropy estimation, which accurately improves the models based on STM and LTM approaches [7, 8, 9, 10, 11].

The study proposes an alternate model for information transfer between neurons based on entropy. It applies McMillan-Shannon's approach to fractional competitive differential equations to determine the mathematical conditions for when STM and the estimation of entropy related to LTM have bounded trajectories. The study uses an alternative version of Lyapunov functions to examine exponential stability in fractional-order systems [7]. This proposal is more comprehensive than previous studies, such as those conducted by [3, 4, 1, 8]. It presents a strict Lyapunov function for the neural multi-time scale system, which demonstrates the existence and uniqueness of the equilibrium point. Additionally, this proposal provides some conditions for global exponential stability based on singular perturbation theory and variable entropy-dependent synaptic efficiency.

The paper is structured as follows: In Section 2, the mathematical background related to self-organizing maps modeled by Caputo derivatives is presented. This allows for the inclusion of fractional order in the differential equations associated with STM and LTM dynamics. Section 3 analyzes the equilibrium point from the perspective of synaptic efficiency, and covers the existence and uniqueness of the equilibrium point. Section 4 presents numerical simulations that provide findings about SOM's equilibrium point and entropy when exposed to external stimulus. Finally, Section 5 offers closing remarks and final comments.

II. Mathematical Background

The synaptic information efficiency (SIE) can be defined according to the existence of an ergodic process related to binned input and output spike

Ref

[7] N. Aguila-Camacho, M. A. Duarte-Mermoud, J. A. Gallegos, Lyapunov functions for fractional order systems, Communications in Nonlinear Science and Numerical Simulation 19 (9) (2014) 2951–2957. doi:10.1016/j.cnsns.2014.01.022.

trains whose length is N bins [4]. In computational experiments, N is associated with iterations during the learning procedure on time domain. In this way, by assuming m_{ij} the synaptic efficiency between i-th and the j-th neuron then let S_{in}, S_{out} the binned input and output spike trains, respectively, both defined by $\{\sigma_1, \ldots, \sigma_N\}$, where $\sigma_i \in \{0, 1\}$ represents i-th bin, and assume that this string is the realization of a stationary and ergodic stochastic process. By using the Shannon-MacMillan-Brieman's theorem [12, 13] $-\frac{1}{N}\log_2 p(\sigma_1, \ldots, \sigma_N) \to H$, where H is the entropy rate of X (events set) and $p(\sigma, \ldots, \sigma_N)$ is the probability of obtaining the string $(\sigma_1, \ldots, \sigma_N)$ as a realization of X. In that sense, the estimate for the entropy at N-th iteration will be represented by $\hat{H}_N = -\frac{1}{N}\log_2 \hat{p}(\sigma_1, \ldots, \sigma_N)$. So, for i-th neuron,

$$
\begin{aligned}
\mathrm{SIE}_i &= \hat{H}_N(\sigma_1, \ldots, \sigma_N)_i - \hat{H}_N(\sigma_1, \ldots, \sigma_N|S_{in})_i \\
&= \frac{1}{N}\log_2 \hat{p}_i(\sigma_1, \ldots, \sigma_N|S_{in}) - \frac{1}{N}\log_2 \hat{p}_i(\sigma_1, \ldots, \sigma_N),
\end{aligned}
$$

where $\hat{H}_N(\sigma_1, \ldots, \sigma_N|S_{in})$ is the estimated output spike train entropy given the input spike train S_{in}, also known as conditional entropy (see [4]). In this way, for N iterations,

$$
m_{ij} = \frac{1}{N}\log_2 \beta_{ij} \triangleq \frac{1}{N}\log_2 \frac{\hat{p}_i(\sigma_1, \ldots, \sigma_N|S_{in})\hat{p}_j(\sigma_1, \ldots, \sigma_N)}{\hat{p}_j(\sigma_1, \ldots, \sigma_N|S_{in})\hat{p}_i(\sigma_1, \ldots, \sigma_N)}. \tag{1}
$$

Nevertheless, on the time domain, β_{ij} can be represented as $\beta_{ij}(t)$, for $t \geq 0$, since at N-th iteration there is some time related to the learning procedure. Sometimes, to guarantee coherence with symbolic representation for N iterations, the notation β_{ij} is retained.

Since the nature of the transfer process related to synaptic information becomes more precise using fractional differential equations, the Caputo definition for the fractional derivative is more suitable because it incorporates initial conditions and its integer order derivatives. Although there are several definitions regarding the fractional derivative of order $\alpha \geq 0$, in the time domain the general network equations describing the temporal evolution of the STM and LTM states for the j-th neuron of M neurons are

$$
\frac{\varepsilon}{\Gamma(1-\alpha)}\int_0^t \frac{x_j^{(m+1)}(\lambda)}{(t-\lambda)^\alpha} d\lambda = -a_j x_j + \sum_{i=1}^{M} D_{ij} f(x_i) + \frac{B_j}{N}\sum_{i=1}^{P} \log_2 \beta_{ij} y_i
$$

$$
\frac{1}{N\Gamma(1-\alpha)}\int_0^t \frac{[\log_2 \beta_{ij}(\lambda)]^{(m+1)}}{(t-\lambda)^\alpha} d\lambda = -\frac{1}{N}\log_2 \beta_{ij} + y_i f(x_j),
$$

where α is the Caputo's fractional order defined by $\alpha = m + \gamma$, $m \in \mathbb{Z}^+$, $0 < \gamma \leq 1$; $\Gamma(\cdot)$ is the Gamma function, x_j is the current activity level (STM),

Ref

[4] M. London, A. Schreibman, M. Häusser, M. E. Larkum, I. Segev, The information efficacy of a synapse, Nature Neuroscience 5 (4) (2002) 332–340. doi:10.1038/nn826.

a_j is the time constant of the neuron, B_j is the contribution of the external stimulus term, $f(x_i)$ is the neuron's output, y_i is the external stimulus, and ε is the fast time-scale associated with the STM state. D_{ij} represents a synaptic connection parameter between the i-th neuron and j-th neuron.

According to the definition of m_{ij} in (1) and unlike [1, 9, 6], the self-organizing map is implicitly modeled by a network of sources emitting input signals with a prescribed probability distribution and external stimulus $\boldsymbol{y} \triangleq [y_i]$. By using the dynamic transform $w_j = \langle \boldsymbol{y}, \log_2 \boldsymbol{\beta}_j \rangle$ the model gets as follows:

$$\frac{\varepsilon}{\Gamma(1-\alpha)} \int_0^t \frac{x_j^{(m+1)}(\lambda)}{(t-\lambda)^\alpha} d\lambda = -a_j x_j + \sum_{i=1}^{M} D_{ij} f(x_i) + \frac{B_j}{N} w_j \qquad (2)$$

$$\frac{1}{N\Gamma(1-\alpha)} \int_0^t \frac{w_j^{(m+1)}(\lambda)}{(t-\lambda)^\alpha} d\lambda = -\frac{1}{N} w_j + \|\boldsymbol{y}\|^2 f(x_j), \qquad (3)$$

where the external stimuli are assumed to be normalized vectors of unit magnitude $\|\boldsymbol{y}\|^2 = 1$.

As each string is the realization of a stationary and ergodic stochastic process, it will be assumed a stochastic column matrix for each iteration of $\boldsymbol{\beta}_j$, such that there exists $P \triangleq [\bar{p}_{ij}] \in \mathbb{R}^{M\times M}$, with $\sum_{i=1}^{M} \bar{p}_{ik} = 1$ and $\boldsymbol{\beta}_j^{\sigma+1} = P\boldsymbol{\beta}_j^{\sigma}$ for $(\sigma+1)$-th iteration. In this way, it can be noted that $\dot{\boldsymbol{\beta}}_j = \lim_{\Delta t \to 0} \frac{1}{\Delta^k t}(P-I)^k \boldsymbol{\beta}_j$, such that the sum of elements in each column of $(P-I)^k \triangleq [p_{ij}]$ is equal to zero, i.e.,

$$\beta_{ij}^{(k)} = \lim_{\Delta t \to 0} \frac{1}{\Delta^k t} \sum_{s=1}^{M} p_{is}\beta_{sj}, \qquad \sum_{i=1}^{M} p_{ik} = 0, \quad \text{for } \forall\, k. \qquad (4)$$

III. Equilibrium and Global Asymptotic Stability

The existence and uniqueness of the equilibrium point are given based on flow-invariance while the global exponential stability will be based on a strict Lyapunov function for fractional-order approaches. It is well-known that the flow-invariance theory provides a qualitative viewpoint about the dynamics of a system. To begin with, it will be defined the following theorem before presenting the main results of this paper.

Theorem 1. Consider the system of fractional differential equations:

$$\frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{x_i^{(m+1)}(\lambda)}{(t-\lambda)^\alpha} d\lambda = -a_i x_i + \sum_{j=1}^{M} D_{ij} f(x_j) + \frac{B_i}{N} w_i, \qquad (5)$$

Notes

$$\frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{w_i^{(m+1)}(\lambda)}{(t-\lambda)^\alpha} d\lambda = -\frac{1}{N} w_i + f(x_i),, \tag{6}$$

for $i = 1, \ldots, M$, where $a_i > 0$ for all $i = 1, \ldots, M$, and f is locally Lipschitz and bounded, that is, there exists a constant $C > 0$ such that $-C \leq f(x) \leq C$ for all $x \in \mathbb{R}$. Then for any $\varepsilon > 0$ and for any initial condition $\{x(0), w(0)\} \in \mathbb{R}^{2M}$ there exists a $T \geq 0$ such that

Notes

$$w_i(t) \in [-C-\varepsilon, C+\varepsilon], \qquad x_i(t) \in [-l_i - \varepsilon, l_i + \varepsilon],$$

for all $i = 1, \ldots, M$, with equilibrium point $e = [\bar{x}_i \quad \bar{w}_i] = [\bar{x}_i \ \langle \boldsymbol{y}, \log_2 \bar{\boldsymbol{\beta}}_i \rangle]$, where $\forall \bar{\beta}_{ij} \in \bar{\boldsymbol{\beta}}_i$ satisfies

$$\int_0^t \int_0^\lambda \frac{y_j}{\bar{\beta}_{ij}^2 \ln 2} \sum_{r=1}^M \sum_{q=1}^M \sum_{s=1}^M p_{is} p_{sq} p_{ir} \bar{\beta}_{qj} \bar{\beta}_{rj} \cdot d\lambda^2 = 0 \tag{7}$$

$$\int_0^t \frac{y_j}{\bar{\beta}_{ij}^2 \ln 2} \sum_{r=1}^M \sum_{q=1}^M \sum_{s=1}^M p_{is} p_{sq} p_{ir} \bar{\beta}_{qj} \bar{\beta}_{rj} \cdot d\lambda = 0 \tag{8}$$

$$S_m \sum_{s=1}^M p_{is} \bar{\beta}_{sj} y_j = \frac{y_j}{\bar{\beta}_{ij} \ln 2} \sum_{r=1}^M \sum_{q=1}^M \sum_{s=1}^M p_{is} p_{sq} p_{ir} \bar{\beta}_{qj} \bar{\beta}_{rj}, \tag{9}$$

for $m = 0, 1$ and 2, respectively, and $S_m \in \mathbb{Z}^+$ for all $t \geq T$.

Proof. Since f is Lipschitz, system (5)-(6) has local existence and uniqueness of solutions. Furthermore, since f is uniformly bounded, there exist constants $K_1, \ldots, K_5$ such that

$$\left| \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{x_i^{(m+1)}(\lambda)}{(t-\lambda)^\alpha} d\lambda \right| \leq K_1 + K_2 \|x_i(t)\| + K_3 \|w_i(t)\|$$

$$\left| \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{w_i^{(m+1)}(\lambda)}{(t-\lambda)^\alpha} d\lambda \right| \leq K_4 + K_5 \|w_i(t)\|,$$

thus all solutions are defined globally (for all $t \geq 0$).

Given $\varepsilon > 0$, let $\delta_i > 0$ be defined as

$$\delta_i = \begin{cases} \min\left\{\frac{a_i \varepsilon}{2|B_i|}, \varepsilon\right\} & B_i \neq 0 \\ \varepsilon & B_i = 0 \end{cases},$$

such that $-|B_i|\delta_i + a_i\varepsilon \geq a_i\varepsilon/2$, for all $i = 1, \ldots, M$. Then for $t \geq 0$ and for $w_i(t) \leq -C - \delta_i$ the following inequality holds:

$$\frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{w_i^{(m+1)}(\lambda)}{(t-\lambda)^{\alpha}}d\lambda \geq -\frac{1}{N}(-C-\delta_i)+f(x_i)=\frac{1}{N}\delta_i+$$

$$\left[f(x_i)+\frac{1}{N}C\right]\geq \delta_i > 0. \tag{10}$$

Similarly, for $t \geq 0$ and for $w_i(t) \geq C+\delta_i$, the following inequality holds:

$$\frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{w_i^{(m+1)}(\lambda)}{(t-\lambda)^{\alpha}}d\lambda \leq -\frac{1}{N}(C+\delta_i)+f(x_i)=-\frac{1}{N}\delta_i+$$

$$\left[f(x_i)-\frac{1}{N}C\right]\leq -\delta_i < 0. \tag{11}$$

Since $\alpha \leq 1$ then $\Gamma(1-\alpha) > 0$ and $(t-\lambda)^{\alpha} > 0$. In that sense, from (10)-(11), both inequalities are guaranteed if and only if $w_i^{(m+1)}(\lambda) > 0$ and $w_i^{(m+1)}(\lambda) < 0$, respectively.

By mathematical induction, it can be noted that

$$w_i^{(m+1)} = \sum_{j=1}^{M}\left[\sum_{k=0}^{m}\frac{A_k}{\beta_{ij}^{m+1-k}}(\dot{\beta}_{ij})^{m+1-k}y_j^{(k)}+\frac{S_k}{\beta_{ij}^{m+1-k}}\beta_{ij}^{(k+1)}y_j^{(m-k)}\right]$$

$$+\frac{d^{m-2}}{d\lambda^{m-2}}\quad\frac{\dot{\beta}_{ij}\ddot{\beta}_{ij}y_j}{\beta_{ij}^2\ln 2}\Bigg),$$

where $A_k, B_k \in \mathbb{Z}^+$. So, for $w_i(t) \leq -C-\delta_i$ and $w_i(t) \geq C+\delta_i$,

$$\sum_{j=1}^{M}\left[\sum_{k=0}^{m}\frac{A_k}{\beta_{ij}^{m+1-k}}(\dot{\beta}_{ij})^{m+1-k}y_j^{(k)}+\frac{S_k}{\beta_{ij}^{m+1-k}}\beta_{ij}^{(k+1)}y_j^{(m-k)}\right] >$$

$$-\frac{d^{m-2}}{d\lambda^{m-2}}\quad\frac{\dot{\beta}_{ij}\ddot{\beta}_{ij}y_j}{\beta_{ij}^2\ln 2}\Bigg), \tag{12}$$

$$\sum_{j=1}^{M}\left[\sum_{k=0}^{m}\frac{A_k}{\beta_{ij}^{m+1-k}}(\dot{\beta}_{ij})^{m+1-k}y_j^{(k)}+\frac{S_k}{\beta_{ij}^{m+1-k}}\beta_{ij}^{(k+1)}y_j^{(m-k)}\right] <$$

$$-\frac{d^{m-2}}{d\lambda^{m-2}}\quad\frac{\dot{\beta}_{ij}\ddot{\beta}_{ij}y_j}{\beta_{ij}^2\ln 2}\Bigg), \tag{13}$$

respectively.

Notes

Notes

By using (4) in (12), it can be obtained

$$\lim_{\Delta t \to 0} \sum_{j=1}^{M} \left[\sum_{k=0}^{m} \frac{A_k}{\Delta^{m-k-2} t \beta_{ij}^{m+1-k}} \sum_{s=1}^{M} p_{is} \beta_{sj} y_j^{(k)} \right.$$

$$\left. + \frac{S_k}{\Delta^{k-2} t \beta_{ij}^{m+1-k}} \sum_{s=1}^{M} p_{is} \beta_{sj} y_j^{(m-k)} \right] >$$

$$\left. - \lim_{\Delta t \to 0} \frac{d^{m-2}}{d\lambda^{m-2}} \quad \frac{y_j}{\beta_{ij}^2 \ln 2} \sum_{r=1}^{M} \sum_{q=1}^{M} \sum_{s=1}^{M} p_{is} p_{sq} \beta_{qj} p_{ir} \beta_{rj} \right).$$

For $k < m$, the evaluation of limits above yields,

$$\infty > - \frac{d^{m-2}}{d\lambda^{m-2}} \quad \left. \frac{y_j}{\beta_{ij}^2 \ln 2} \sum_{r=1}^{M} \sum_{q=1}^{M} \sum_{s=1}^{M} p_{is} p_{sq} \beta_{qj} p_{ir} \beta_{rj} \right), \tag{14}$$

which represents an obvious condition. For $k = m$,

$$\lim_{\Delta t \to 0} \sum_{j=1}^{M} \left[\frac{A_m \Delta^2 t}{\beta_{ij}} \sum_{s=1}^{M} p_{is} \beta_{sj} y_j^{(m)} + \frac{S_m}{\Delta^{m-2} t \beta_{ij}} \sum_{s=0}^{M} p_{is} \beta_{sj} y_j \right]$$

$$> - \lim_{\Delta t \to 0} \frac{d^{m-2}}{d\lambda^{m-2}} \quad \left. \frac{y_j}{\beta_{ij}^2 \ln 2} \sum_{r=1}^{M} \sum_{q=1}^{M} \sum_{s=1}^{M} p_{is} p_{sq} \beta_{qj} p_{ir} \beta_{rj} \right).$$

Therefore, by evaluating the limits for $m > 2$, the inequality (14) is obtained again. However, for $0 \leq m \leq 2$ (i.e., $m = 0, 1$ and 2),

$$\int_0^t \int_0^\lambda \quad \left. \frac{y_j}{\beta_{ij}^2 \ln 2} \sum_{r=1}^{M} \sum_{q=1}^{M} \sum_{s=1}^{M} p_{is} p_{sq} p_{ir} \beta_{qj} \beta_{rj} \right) d\lambda^2 > 0, \tag{15}$$

$$\int_0^t \quad \left. \frac{y_j}{\beta_{ij}^2 \ln 2} \sum_{r=1}^{M} \sum_{q=1}^{M} \sum_{s=1}^{M} p_{is} p_{sq} p_{ir} \beta_{qj} \beta_{rj} \right) d\lambda > 0, \tag{16}$$

$$S_m \sum_{s=1}^{M} p_{is} \beta_{sj} y_j < \frac{y_j}{\beta_{ij} \ln 2} \sum_{r=1}^{M} \sum_{q=1}^{M} \sum_{s=1}^{M} p_{is} p_{sq} p_{ir} \beta_{qj} \beta_{rj}, \tag{17}$$

respectively. Since the operator $d^{m-2}/d\lambda^{m-2}$ becomes an integral for $m < 2$.

So, for $w_i(t) \leq -C-\delta_i$, the inequalities (15)-(17) guarantee $w^{(m+1)} > 0$. The same treatment applies to (13), it is only to change the less than symbol in (15)-(17) by greater than symbol. Therefore, for any $i \in \{1, \ldots, M\}$ there exists a $T_i > 0$ such that

$$w_i(t) \in [-C - \delta_i, C + \delta_i] \subseteq [-C - \varepsilon, C + \varepsilon], \tag{18}$$

for all $t \geq T_i$. So, the equilibrium point $\bar{w}_i = \langle \boldsymbol{y}, \log_2 \bar{\boldsymbol{\beta}}_i \rangle \in [-C - \varepsilon, C + \varepsilon]$, where $\forall\ \bar{\beta}_{ij} \in \bar{\boldsymbol{\beta}}_i$ satisfies

$$\int_0^t \int_0^\lambda \quad \frac{y_j}{\bar{\beta}_{ij}^2 \ln 2} \sum_{r=1}^{M} \sum_{q=1}^{M} \sum_{s=1}^{M} p_{is} p_{sq} p_{ir} \bar{\beta}_{qj} \bar{\beta}_{rj} \Bigg)\, d\lambda^2 = 0, \quad (m = 0)$$

$$\int_0^t \quad \frac{y_j}{\bar{\beta}_{ij}^2 \ln 2} \sum_{r=1}^{M} \sum_{q=1}^{M} \sum_{s=1}^{M} p_{is} p_{sq} p_{ir} \bar{\beta}_{qj} \bar{\beta}_{rj} \Bigg)\, d\lambda = 0, \quad (m = 1)$$

$$S_m \sum_{s=1}^{M} p_{is} \bar{\beta}_{sj} y_j = \frac{y_j}{\bar{\beta}_{ij} \ln 2} \sum_{r=1}^{M} \sum_{q=1}^{M} \sum_{s=1}^{M} p_{is} p_{sq} p_{ir} \bar{\beta}_{qj} \bar{\beta}_{rj} \ \ (m = 2).$$

Defining $T_S = \max T_i$ then $w_i(t) \in [-C-\varepsilon, C+\varepsilon]$ holds for all $i \in \{1, \ldots, M\}$ and for all $t \geq T_S$.

Now, let $t \geq T_S$. For $x_i(t) \leq -l_i - \varepsilon$, (5) and (18) imply that

$$\frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{x_i^{(m+1)}(\lambda)}{(t-\lambda)^\alpha} d\lambda \geq a_i(l_i + \varepsilon) + \sum_{j=1}^{M} D_{ij} f(x_j)$$

$$+ \frac{B_i}{N}(-C - \delta_i) \geq a_i l_i + a_i \varepsilon - C \quad \sum_{j=1}^{M} |D_{ij}| + \frac{|B_i|}{N} \Bigg) - |B_i| \delta_i.$$

By defining $l_i = \frac{C}{a_i} \left(\sum_{j=1}^{M} |D_{ij}| + \frac{1}{N} |B_i| \right) > 0$, for $i = 1, \ldots, M$ then

$$\frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{x_i^{(m+1)}(\lambda)}{(t-\lambda)^\alpha} d\lambda \geq -|B_i| \delta_i + a_i \varepsilon \geq \frac{a_i \varepsilon}{2} > 0.$$

Similarly, for $t \geq T_S$ and for $x_j(t) \geq l_i + \varepsilon$, (5) and (18) imply that

$$\frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{x_i^{(m+1)}(\lambda)}{(t-\lambda)^\alpha} d\lambda \leq |B_i| \delta_i - a_i \varepsilon \leq -\frac{a_i \varepsilon}{2} < 0.$$

Notes

Since $\alpha \leq 1$ then $\Gamma(1-\alpha) > 0$ and $(t-\lambda)^{\alpha} > 0$. In that sense, both before inequalities are guaranteed if and only if $x_i^{(m+1)}(\lambda) > 0$ and $x_i^{(m+1)}(\lambda) < 0$, respectively. Therefore, for any $i \in \{1, \ldots, M\}$ there exists a $T_i > 0$ such that

$$x_i(t) \in [-l_i - \varepsilon, l_i + \varepsilon]$$

Notes

for all $t \geq T_i$. Defining $T_X = \max T_i$ then $x_i(t) \in [-l_i - \varepsilon, l_i + \varepsilon]$ holds for all $i \in \{1, \ldots, M\}$ and for all $t \geq T_X$. $\square$

The system (5)-(6) is dissipative in $\mathbb{R}^{2M}$ and therefore, it has a compact global attractor

$$\mathcal{A} \subseteq D = \prod_{i=1}^{M} [-l_i, l_i] \times \prod_{i=1}^{M} [-C, C].$$

It follows from the proof of Theorem 1 that the set D is flow invariant under (5)-(6). In other words, D is positively invariant set of (5)-(6), i.e., any solution starting in D at $t = 0$ remains in D for all $t \geq 0$. Furthermore, from the proof of Theorem 1 the set H that contains D can be contracted to a point, and since D is flow-invariant with respect to (5)-(6) then by the Brower fixed point theorem implies that there exists a point $e \in D$ is an equilibrium point of (5)-(6).

Theorem 2. Suppose that $f(x)$ is $\mathcal{C}^1$ with $|\dot{f}(x)| < k$ for all x and

$$a_i > k \left. \sum_{j=1}^{M} |D_{ij}| + |B_i| \right), \qquad i = 1, \ldots, M, \tag{19}$$

then the equilibrium e is unique.

Proof. At the equilibrium point, $f(x_i) = \frac{1}{N} w_i$ from (6). Substituting these expressions in (5), it is obtained

$$0 = -a_i x_i + \sum_{j=1}^{M} D_{ij} f(x_j) + B_i f(x_i), \quad \text{for } i = 1, \ldots, M.$$

Since $a_i > 0$, x_i can be expressed as

$$x_i = \frac{1}{a_i} \left. \sum_{j=1}^{M} D_{ij} f(x_j) + B_i f(x_i) \right) = G_i(x_1, \ldots, x_M).$$

The inequality (19) implies that

$$|G_i(x_1', \ldots, x_M') - G_i(x_1'', \ldots, x_M'')| < k |(x_1', \ldots, x_M') - (x_1'', \ldots, x_M'')|,$$

i.e., G is a contracting map in $\mathbb{R}^M$. Consequently, there exists a unique fixed point of G. The x_i-coordinates of this fixed point uniquely determine the w_i-coordinates via $f(x_i) = \frac{1}{N}w_i$. Therefore, the equilibrium point is unique. $\square$

Now, let $e = [\bar{x}_i \ \ \bar{w}_i] = [\bar{x}_i \ \langle \boldsymbol{y}, \log_2 \bar{\boldsymbol{\beta}}_i \rangle]$ be the equilibrium point of (5)-(6) and introduce the change of variables $\phi_i = x_i - \bar{x}_i$, $\varphi_i = w_i - \bar{w}_i$ which shifts e to the origin. Specifically, if $f_i(\phi_i) = f(\phi_i + \bar{x}_i) - \frac{1}{N}\bar{w}_i$, then $f_i(0) = 0$ and (5)-(6) may be rewritten as

$$\frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\phi_i^{(m+1)}(\lambda)}{(t-\lambda)^\alpha} d\lambda = -a_i\phi_i + \sum_{j=1}^{M} D_{ij} f(\phi_j) + \frac{B_i}{N}\varphi_i, \tag{20}$$

$$\frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\varphi_i^{(m+1)}(\lambda)}{(t-\lambda)^\alpha} d\lambda = -\frac{1}{N}\varphi_i + f(\phi_i) \ \text{ for } i = 1, \ldots, M. \tag{21}$$

By assuming that there exists a Lyapunov function $V(t, \phi_i, \varphi_i)$ and class-K functions γ_i (for $i = 1, 2, 3$) satisfying

$$\gamma_1(|\phi_i|, |\varphi_i|) \leq V(t, \phi_i, \varphi_i) \leq \gamma_1(|\phi_i|, |\varphi_i|),$$

$$\frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{V^{(m+1)}(\lambda, \phi_i, \varphi_i)(\lambda)}{(t-\lambda)^\alpha} d\lambda \leq -\gamma_3(|\phi_i|, |\varphi_i|),$$

then the system (20)-(21) becomes asymptotically stable at equilibrium point e [7].

Lemma 1. [see [7]] Let $r(t) \in \mathbb{R}$ be a continuous and derivable function. Then, for any time $t \geq t_0$

$$\frac{1}{2\Gamma(1-\alpha)} \int_0^t \frac{[r^2(\lambda)]^{(m+1)}}{(t-\lambda)^\alpha} d\lambda \leq \frac{r(t)}{2\Gamma(1-\alpha)} \int_0^t \frac{r(\lambda)^{(m+1)}(\lambda)}{(t-\lambda)^\alpha} d\lambda.$$

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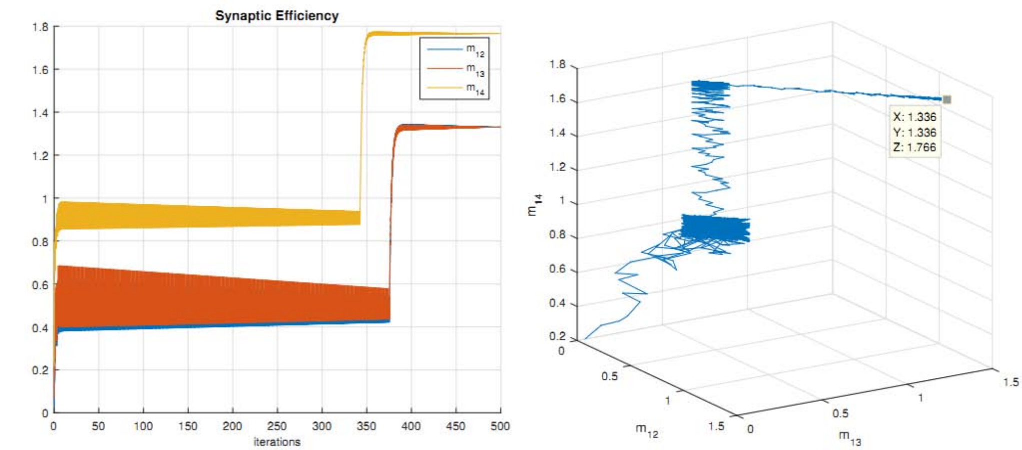

Figure 1: Evolution of synaptic efficiency m_{ij} (for $i = 1, j = 2, 3, 4$) toward equilibrium point $(1.336, 1.336, 1.768)$ after $N = 500$ iterations when each neuron receives an external stimulus based on neighborhood's winner.

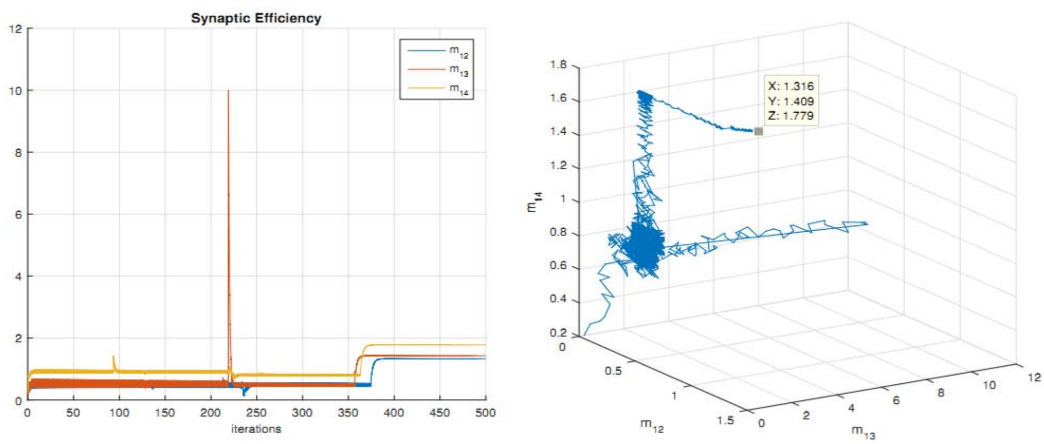

Figure 2: Evolution of synaptic efficiency m_{ij} (for $i = 1, j = 2, 3, 4$) toward equilibrium point $(1.316, 1.409, 1.779)$ after $N = 500$ iterations when each neuron updates its weights without external stimulus.

Theorem 3. Suppose that $f(x)$ is $\mathcal{C}^1$ with $|\dot{f}(x)| \leq k$ for all x and $a_i > 0$.

If

$$\max_i \left\{ \frac{1}{2}\left(\frac{|B_i|}{a_i} + k\right) + \sum_{j=1}^{M} \frac{1}{2} k \left(\frac{|D_{ij}|}{a_i} + \frac{|D_{ji}|}{a_j}\right) \right\} < 1 \tag{22}$$

then e is a global attractor for the system (20)-(21)*. Moreover, all solutions of* (20)-(21) *converge to e exponentially fast as $t \to \infty$.*

Notes

Proof. The global convergence will be proved by using a Lyapunov function for (20)-(21). Let

$$V(\phi_i, \varphi_i) = \frac{1}{2}\sum_{i=1}^{M}\left(\frac{\phi_i^2}{a_i} + \varphi_i^2\right),$$

then the fractional derivative for $V(\phi_i, \varphi_i)$ is defined by

$$\frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{V^{(m+1)}(\phi_i, \varphi_i)}{(t-\lambda)^{\xi}}d\lambda =$$

$$\frac{1}{2\Gamma(1-\alpha)}\sum_{i=1}^{M}\left(\frac{1}{a_i}\int_0^t \frac{[\phi_i^2(\lambda)]^{(m+1)}}{(t-\lambda)^{\alpha}}d\lambda + \int_0^t \frac{[\varphi_i^2(\lambda)]^{(m+1)}}{(t-\lambda)^{\alpha}}d\lambda\right),$$

and by using the Lemma 1 together with (20)-(21) yields

$$\frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{V^{(m+1)}(\phi_i, \varphi_i)}{(t-\lambda)^{\alpha}}d\lambda \leq -\sum_{i=1}^{M}\left(\phi_i^2 + \varphi_i^2\right) \tag{23}$$

$$+\sum_{i=1}^{M}\sum_{j=1}^{M} D_{ij}\frac{\phi_i}{a_i}f_j(\phi_j) + \sum_{i=1}^{M}\varphi_i\left(\frac{\phi_i}{a_i}B_i + f_i(\phi_i)\right).$$

Since $f_i(0) = 0$ and $|\dot{f}_i(x)| = |\dot{f}(x + \bar{x}_i)| < k$, it is possible to have $|f_i(\phi_i)| < k|\phi_i|$. Consequently, with this last fact and the Minkowski inequality, (23) can be replaced by the inequality

$$\frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{V^{(m+1)}(\phi_i, \varphi_i)}{(t-\lambda)^{\alpha}}d\lambda < -\sum_{i=1}^{M}\left(\phi_i^2 + \varphi_i^2\right)$$

$$+\sum_{i=1}^{M}\sum_{j=1}^{M}|D_{ij}|k\frac{1}{a_i}|\phi_i||\phi_j| + \sum_{i=1}^{M}\left(\frac{|B_i|}{a_i} + k\right)|\varphi_i||\phi_i|.$$

The right-hand side of this inequality is given by the quadratic form with the matrix $-Q$ where Q has the following structure:

$$Q = \begin{bmatrix} 1 - \frac{1}{2}k\left(\frac{|D_{11}|}{a_1} + \frac{|D_{11}|}{a_1}\right) & -\frac{1}{2}\left(\frac{|B_1|}{a_1} + k\right) & -\frac{1}{2}k\left(\frac{|D_{12}|}{a_1} + \frac{|D_{21}|}{a_2}\right) & 0 & \dots \\ -\frac{1}{2}\left(\frac{|B_1|}{a_1} + k\right) & 1 & 0 & 0 & \dots \\ -\frac{1}{2}k\left(\frac{|D_{21}|}{a_2} + \frac{|D_{12}|}{a_1}\right) & 0 & 1 - \frac{1}{2}k\left(\frac{|D_{22}|}{a_2} + \frac{|D_{22}|}{a_2}\right) & -\frac{1}{2}\left(\frac{|B_2|}{a_2} + k\right) & \dots \\ 0 & 0 & -\frac{1}{2}\left(\frac{|B_2|}{a_2} + k\right) & 1 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}.$$

Notes

According to Gerschgorin's theorem applied to $-Q$, there are M disks centered at $z=-1$ (in $\mathbb{C}$-plane) with radius $\frac{1}{2}\left(\frac{|B_i|}{a_i}+k\right)$. In the same way, there are M disks centered at $z=\frac{1}{2}k\left(\frac{|D_{ii}|}{a_i}+\frac{|D_{ii}|}{a_i}\right)-1$ and radius $\frac{1}{2}\left(\frac{|B_i|}{a_i}+k\right)+\sum_{j=1}^{M}\frac{1}{2}k\left(\frac{|D_{ij}|}{a_i}+\frac{|D_{ji}|}{a_j}\right)$. If condition (22) is valid then all eigenvalues q of $-Q$ satisfy $q<1$ or

$$\frac{1}{2}k\left(\frac{|D_{ii}|}{a_i}+\frac{|D_{ii}|}{a_i}\right)-2<q<\frac{1}{2}k\left(\frac{|D_{ii}|}{a_i}+\frac{|D_{ii}|}{a_i}\right).$$

Since $\frac{1}{2}k\left(\frac{|D_{ii}|}{a_i}+\frac{|D_{ii}|}{a_i}\right)>0$ for $\forall i$ then $-Q$ is positive definite and Q is negative definite. Therefore, let $\xi>0$ the smallest eigenvalue of $-Q$ such that

$$\frac{1}{\Gamma(1-\alpha)}\int_0^t\frac{V^{(m+1)}(\phi_i,\varphi_i)}{(t-\lambda)^\alpha}d\lambda<-\xi\sum_{i=1}^{M}\left(\phi_i^2+\varphi_i^2\right),$$

and consequently, V is a strict Lyapunov function for (20)-(21). Moreover, $2V\leq\sum_{i=1}^{M}\left(\phi_i^2+\varphi_i^2\right)$ thus

$$\frac{1}{\Gamma(1-\alpha)}\int_0^t\frac{V^{(m+1)}(\phi_i,\varphi_i)}{(t-\lambda)^\alpha}d\lambda<-2\xi V.$$

This result implies that V converges to zero exponentially fast, and the solutions (ϕ_i,φ_i) converge to the origin also exponentially fast, i.e., solutions (x,w) in system (5)-(6) converges exponentially fast to equilibrium $e=[\bar{x}\ \langle\boldsymbol{y},\log_2\bar{\boldsymbol{\beta}}_1\rangle\ldots\langle\boldsymbol{y},\log_2\bar{\boldsymbol{\beta}}_M\rangle]$.

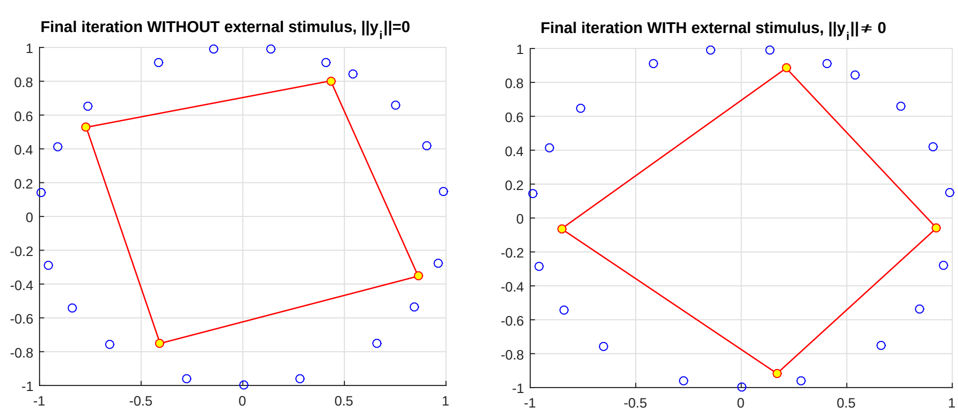

Figure 3: Comparing last iteration ($N=500$ iterations) related to SOM for circle pattern composed by 20 points.

IV. Accessing Numerical Simulations

To validate the proposed model, a self-organizing map with four neurons ($M=4$) is utilized in the following example. The aim is to find the best updating of synaptic weights to adjust a circle shape that consists of 20 patterns and has a radius of 0.5. This will be achieved using unsupervised Hebb's learning rule with $N = 500$ and a learning rate of 0.05. Let's assume that each neuron in a neural network has an activation value of $a_j = 0.5$. Two parameters, D_{ij} and B_j, were set randomly for each neuron within the range of 0 to 1. Additionally, α was set to 0.5, m to 0, and ε to 10^{-4}. To analyze the behavior of the equilibrium point in this model, the algorithm ran for 500 iterations and observed how the equilibrium point behaves. Theorem 1 was used to determine the stability of this point. Our results show that the model is locally stable, which means that it is capable of reaching an equilibrium point and maintaining it over time.

In order to illustrate this point, Fig. 1 illustrates the equilibrium point $(1.336, 1.336, 1.768)$ after 500 iterations with no external stimulus, meaning that $\|y_i\| = 0$ and $m_{ij} = \frac{1}{N}\log_2 \beta_{ij}$ for $i = 1$ and $j = 2, 3, 4$. This means that all sequences $(\sigma_1, \ldots, \sigma_{500})$ for events set $X = \emptyset$ between two neurons have a probability of one, and the entropy H_i $(i = 1, 2, 3, 4)$ approaches zero, which is the ideal situation.

The following text describes the behavior of neurons when an external stimulus is applied. So, let's compare two figures - Fig. 1 and Fig. 2. The former shows the neurons' behavior when no external stimuli are applied, while the latter shows their behavior when stimuli are applied. The amplitude of the impulses generated by the stimuli is set at 0.001, and the stimuli are defined by one impulse over the threshold. The sequences $(\sigma_1, \ldots, \sigma_{500})$ associated with each neuron are considered as a result of updating synaptic weights according to the training rule at each iteration. In a neural network, when a neuron is declared a winner and its weights are updated, the weights of the neighboring neuron also need to be updated. Therefore, for each iteration in the process, every neuron must have a term called σ_i in its sequence. This term should be equal to one when the neuron is updated and zero when it doesn't receive any updates in its weights. Figure 3 illustrates that neurons form a circular pattern consisting of 20 points; however, the final position of neurons changes due to external stimuli during the last iteration.

For both cases in Figures 1 and 2, the equilibrium point $(\bar{\beta}_{12}, \bar{\beta}_{13}, \bar{\beta}_{14})$ is represented by $(1.336, 1.336, 1.768)$ and $(1.316, 1.409, 1.779)$. By assuming only the synaptic efficiencies m_{12}, m_{13}, m_{14}, condition (7) becomes the following three conditions:

$$\frac{p_{11}}{\ln 2}\sum_{s=1}^{M} p_{1s}p_{s1}\int_0^t\int_0^\lambda y_4 \cdot d\lambda^2 = 0,$$

$$\frac{p_{11}}{\ln 2}\sum_{s=1}^{M} p_{1s}p_{s1}\int_0^t\int_0^\lambda y_3 \cdot d\lambda^2 = 0,$$

$$\frac{p_{11}}{\ln 2}\sum_{s=1}^{M} p_{1s}p_{s1}\int_0^t\int_0^\lambda y_2 \cdot d\lambda^2 = 0,$$

Notes

i.e., the external stimulus has no contribution in the final stage of training for neurons connected to neuron 1.

On another side, it can be observed from Figs. 1 and 2 that the evolution of m_{1j} $(j = 2, 3, 4)$ ensures the condition $SIE_1 > SIE_j$, more specifically,

$$H_i(\sigma_1, \ldots, \sigma_{500}|S_{in}) - H_j(\sigma_1, \ldots, \sigma_{500}|S_{in}) > H_i(\sigma_1, \ldots, \sigma_{500}) - H_j(\sigma_1, \ldots, \sigma_{500}).$$

This result indicates that the motion of neurons without an external stimulus is less disorganized than the motion of neurons with an external stimulus, as expected.

V. Final Remarks

The following paper establishes the global exponential stability of SOMs. To this end, this proposal has used fractional order derivatives to describe cognitive cortical maps that result from self-organization, with both LTM and STM approaches. It applied McMillan-Shannon's approach to fractional competitive differential equations, which helps to understand the entropy behavior of SOM in response to external stimuli. The proposal not only proves the existence and uniqueness of the equilibrium point but also shows how fractional order derivative operators can improve our understanding of entropy associated with synaptic efficiency. Synaptic efficiency was modeled by using sequences of updating impulses at each iteration, all of them as a result of applying external stimuli, not only for the winner neuron but also for neighbor neurons. In this way, the concept of conditional entropy plays an important role in the proposal as it shows that it must be greater than entropy to ensure convergence to the equilibrium point.

In future studies, it is hoped to investigate the correlation between the development of information transfer and the division of fast dynamics into distinct time scales. This will involve integrating the Dynamic Confined Space of Velocities criterion (DCSV) [14] into restricted self-organizing maps, which are extensively utilized to address constrained multi-objective optimization issues (CMOPs) [15].

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References Références Referencias

1. A. Meyer-Baese, S. Pilyugin, Y. Chen, Global exponential stability of competitive neural networks with different time scales, IEEE Transactions on Neural Networks 14 (3) (2003) 716–719. doi: 10.1109/tnn.2003.810594.
2. S.-I. Amari, Competitive and Cooperative Aspects in Dynamics of Neural Excitation and Self-Organization, Springer Berlin Heidelberg, 1982, pp. 1–28. doi: 10.1007/978-3-642-46466-91.

Ref

[14] C. A. Peña Fernández, An ergodic selection method for kinematic configurations in autonomous, flexible mobile systems, Journal of Intelligent amp; Robotic Systems 109 (1) (Aug. 2023). doi:10.1007/s10846-023-01933-z.

3. M. Lemmon, B. Vijaya Kumar, Emulating the dynamics for a class of laterally inhibited neural networks, Neural Networks 2 (3) (1989) 193–214. doi: 10.1016/0893-6080(89)90004-x.
4. M. London, A. Schreibman, M. Häusser, M. E. Larkum, I. Segev, The information efficacy of a synapse, Nature Neuroscience 5 (4) (2002) 332–340. doi:10.1038/nn826.
5. D. J. Willshaw, V. D. Malsburg, How patterned neural connections can be setup by self-organization, Proceedings of the Royal Society of London. Series B. Biological Sciences 194 (1117) (1976) 431–445. doi: 10.1098/rspb.1976.0087.
6. S. Najafian, E. Koch, K. L. Teh, J. Jin, H. Rahimi-Nasrabadi, Q. Zaidi, J. Kremkow, J.-M. Alonso, A theory of cortical map formation in the visual brain, Nature Communications 13 (1) (Apr. 2022). doi: 10.1038/s41467-022-29433-y.
7. N. Aguila-Camacho, M. A. Duarte-Mermoud, J. A. Gallegos, Lyapunov functions for fractional order systems, Communications in Nonlinear Science and Numerical Simulation 19 (9) (2014) 2951–2957. doi: 10.1016/j.cnsns.2014.01.022.
8. S. Zhang, Y. Yu, J. Yu, Lmi conditions for global stability of fractional order neural networks, IEEE Transactions on Neural Networks and Learning Systems 28 (10) (2017) 2423–2433. doi: 10.1109/tnnls.2016.2574842.
9. J. Jia, X. Huang, Y. Li, J. Cao, A. Alsaedi, Global stabilization of fractional order memristor-based neural networks with time delay, IEEE Transactions on Neural Networks and Learning Systems 31 (3) (2020) 997–1009. doi: 10.1109/tnnls.2019. 2915353.
10. A. Wu, Z. Zeng, Global mittag–leffler stabilization of fractional-order memristive neural networks, IEEE Transactions on Neural Networks and Learning Systems 28 (1) (2017) 206–217. doi: 10.1109/tnnls. 2015.2506738.
11. J. Chen, Z. Zeng, P. Jiang, Global mittag-leffler stability and synchronization of memristor-based fractional-order neural networks, Neural Networks 51 (2014) 1–8. doi: 10.1016/j.neunet.2013.11.016.
12. T. M. Cover, J. A. Thomas, Elements of Information Theory, Wiley-Interscience, 2006.
13. G. N. Saridis, Entropy In Control Engineering, World Scientific Publishing Company, 2001.
14. C. A. Peña Fernjández, An ergodic selection method for kinematic configurations in autonomous, flexible mobile systems, Journal of Intelligent amp; Robotic Systems 109 (1) (Aug. 2023). doi: 10.1007/s10846-023-01933-z.
15. C. He, M. Li, C. Zhang, H. Chen, P. Zhong, Z. Li, J. Li, A self-organizing map approach for constrained multi-objective optimization problems, Complex amp; Intelligent Systems 8 (6) (2022) 5355–5375. doi: 10.1007/s40747-022-00761-2.

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1. Substantial contributions to the conception and acquisition of data, analysis, and interpretation of findings.
2. Drafting the paper and revising it critically regarding important academic content.
3. Final approval of the version of the paper to be published.

Changes in Authorship

The corresponding author should mention the name and complete details of all co-authors during submission and in manuscript. We support addition, rearrangement, manipulation, and deletions in authors list till the early view publication of the journal. We expect that corresponding author will notify all co-authors of submission. We follow COPE guidelines for changes in authorship.

Copyright

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Appealing Decisions

Unless specified in the notification, the Editorial Board's decision on publication of the paper is final and cannot be appealed before making the major change in the manuscript.

Acknowledgments

Contributors to the research other than authors credited should be mentioned in Acknowledgments. The source of funding for the research can be included. Suppliers of resources may be mentioned along with their addresses.

Declaration of funding sources

Global Journals is in partnership with various universities, laboratories, and other institutions worldwide in the research domain. Authors are requested to disclose their source of funding during every stage of their research, such as making analysis, performing laboratory operations, computing data, and using institutional resources, from writing an article to its submission. This will also help authors to get reimbursements by requesting an open access publication letter from Global Journals and submitting to the respective funding source.

Preparing your Manuscript

Authors can submit papers and articles in an acceptable file format: MS Word (doc, docx), LaTeX (.tex, .zip or .rar including all of your files), Adobe PDF (.pdf), rich text format (.rtf), simple text document (.txt), Open Document Text (.odt), and Apple Pages (.pages). Our professional layout editors will format the entire paper according to our official guidelines. This is one of the highlights of publishing with Global Journals—authors should not be concerned about the formatting of their paper. Global Journals accepts articles and manuscripts in every major language, be it Spanish, Chinese, Japanese, Portuguese, Russian, French, German, Dutch, Italian, Greek, or any other national language, but the title, subtitle, and abstract should be in English. This will facilitate indexing and the pre-peer review process.

The following is the official style and template developed for publication of a research paper. Authors are not required to follow this style during the submission of the paper. It is just for reference purposes.

Manuscript Style Instruction (Optional)

- Microsoft Word Document Setting Instructions.
- Font type of all text should be Swis721 Lt BT.
- Page size: 8.27" x 11'", left margin: 0.65, right margin: 0.65, bottom margin: 0.75.
- Paper title should be in one column of font size 24.
- Author name in font size of 11 in one column.
- Abstract: font size 9 with the word "Abstract" in bold italics.
- Main text: font size 10 with two justified columns.
- Two columns with equal column width of 3.38 and spacing of 0.2.
- First character must be three lines drop-capped.
- The paragraph before spacing of 1 pt and after of 0 pt.
- Line spacing of 1 pt.
- Large images must be in one column.
- The names of first main headings (Heading 1) must be in Roman font, capital letters, and font size of 10.
- The names of second main headings (Heading 2) must not include numbers and must be in italics with a font size of 10.

Structure and Format of Manuscript

The recommended size of an original research paper is under 15,000 words and review papers under 7,000 words. Research articles should be less than 10,000 words. Research papers are usually longer than review papers. Review papers are reports of significant research (typically less than 7,000 words, including tables, figures, and references)

A research paper must include:

a) A title which should be relevant to the theme of the paper.
b) A summary, known as an abstract (less than 150 words), containing the major results and conclusions.
c) Up to 10 keywords that precisely identify the paper's subject, purpose, and focus.
d) An introduction, giving fundamental background objectives.
e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition, sources of information must be given, and numerical methods must be specified by reference.
f) Results which should be presented concisely by well-designed tables and figures.
g) Suitable statistical data should also be given.
h) All data must have been gathered with attention to numerical detail in the planning stage.

Design has been recognized to be essential to experiments for a considerable time, and the editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned unrefereed.

i) Discussion should cover implications and consequences and not just recapitulate the results; conclusions should also be summarized.
j) There should be brief acknowledgments.
k) There ought to be references in the conventional format. Global Journals recommends APA format.

Authors should carefully consider the preparation of papers to ensure that they communicate effectively. Papers are much more likely to be accepted if they are carefully designed and laid out, contain few or no errors, are summarizing, and follow instructions. They will also be published with much fewer delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and suggestions to improve brevity.

Format Structure

It is necessary that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

All manuscripts submitted to Global Journals should include:

Title

The title page must carry an informative title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) where the work was carried out.

Author details

The full postal address of any related author(s) must be specified.

Abstract

The abstract is the foundation of the research paper. It should be clear and concise and must contain the objective of the paper and inferences drawn. It is advised to not include big mathematical equations or complicated jargon.

Many researchers searching for information online will use search engines such as Google, Yahoo or others. By optimizing your paper for search engines, you will amplify the chance of someone finding it. In turn, this will make it more likely to be viewed and cited in further works. Global Journals has compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Keywords

A major lynchpin of research work for the writing of research papers is the keyword search, which one will employ to find both library and internet resources. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining, and indexing.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy: planning of a list of possible keywords and phrases to try.

Choice of the main keywords is the first tool of writing a research paper. Research paper writing is an art. Keyword search should be as strategic as possible.

One should start brainstorming lists of potential keywords before even beginning searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in a research paper?" Then consider synonyms for the important words.

It may take the discovery of only one important paper to steer in the right keyword direction because, in most databases, the keywords under which a research paper is abstracted are listed with the paper.

Numerical Methods

Numerical methods used should be transparent and, where appropriate, supported by references.

Abbreviations

Authors must list all the abbreviations used in the paper at the end of the paper or in a separate table before using them.

Formulas and equations

Authors are advised to submit any mathematical equation using either MathJax, KaTeX, or LaTeX, or in a very high-quality image.

Tables, Figures, and Figure Legends

Tables: Tables should be cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g., Table 4, a self-explanatory caption, and be on a separate sheet. Authors must submit tables in an editable format and not as images. References to these tables (if any) must be mentioned accurately.

Figures

Figures are supposed to be submitted as separate files. Always include a citation in the text for each figure using Arabic numbers, e.g., Fig. 4. Artwork must be submitted online in vector electronic form or by emailing it.

Preparation of Eletronic Figures for Publication

Although low-quality images are sufficient for review purposes, print publication requires high-quality images to prevent the final product being blurred or fuzzy. Submit (possibly by e-mail) EPS (line art) or TIFF (halftone/ photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Avoid using pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings). Please give the data for figures in black and white or submit a Color Work Agreement form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution at final image size ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs): >350 dpi; figures containing both halftone and line images: >650 dpi.

Color charges: Authors are advised to pay the full cost for the reproduction of their color artwork. Hence, please note that if there is color artwork in your manuscript when it is accepted for publication, we would require you to complete and return a Color Work Agreement form before your paper can be published. Also, you can email your editor to remove the color fee after acceptance of the paper.

Tips for Writing a Good Quality Science Frontier Research Paper

Techniques for writing a good quality Science Frontier Research paper:

1. Choosing the topic: In most cases, the topic is selected by the interests of the author, but it can also be suggested by the guides. You can have several topics, and then judge which you are most comfortable with. This may be done by asking several questions of yourself, like "Will I be able to carry out a search in this area? Will I find all necessary resources to accomplish the search? Will I be able to find all information in this field area?" If the answer to this type of question is "yes," then you ought to choose that topic. In most cases, you may have to conduct surveys and visit several places. Also, you might have to do a lot of work to find all the rises and falls of the various data on that subject. Sometimes, detailed information plays a vital role, instead of short information. Evaluators are human: The first thing to remember is that evaluators are also human beings. They are not only meant for rejecting a paper. They are here to evaluate your paper. So present your best aspect.

2. Think like evaluators: If you are in confusion or getting demotivated because your paper may not be accepted by the evaluators, then think, and try to evaluate your paper like an evaluator. Try to understand what an evaluator wants in your research paper, and you will automatically have your answer. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

3. Ask your guides: If you are having any difficulty with your research, then do not hesitate to share your difficulty with your guide (if you have one). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work, then ask your supervisor to help you with an alternative. He or she might also provide you with a list of essential readings.

4. Use of computer is recommended: As you are doing research in the field of science frontier then this point is quite obvious. Use right software: Always use good quality software packages. If you are not capable of judging good software, then you can lose the quality of your paper unknowingly. There are various programs available to help you which you can get through the internet.

5. Use the internet for help: An excellent start for your paper is using Google. It is a wondrous search engine, where you can have your doubts resolved. You may also read some answers for the frequent question of how to write your research paper or find a model research paper. You can download books from the internet. If you have all the required books, place importance on reading, selecting, and analyzing the specified information. Then sketch out your research paper. Use big pictures: You may use encyclopedias like Wikipedia to get pictures with the best resolution. At Global Journals, you should strictly follow here.

6. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right? It is a good habit which helps to not lose your continuity. You should always use bookmarks while searching on the internet also, which will make your search easier.

7. Revise what you wrote: When you write anything, always read it, summarize it, and then finalize it.

8. Make every effort: Make every effort to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in the introduction—what is the need for a particular research paper. Polish your work with good writing skills and always give an evaluator what he wants. Make backups: When you are going to do any important thing like making a research paper, you should always have backup copies of it either on your computer or on paper. This protects you from losing any portion of your important data.

9. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several unnecessary diagrams will degrade the quality of your paper by creating a hodgepodge. So always try to include diagrams which were made by you to improve the readability of your paper. Use of direct quotes: When you do research relevant to literature, history, or current affairs, then use of quotes becomes essential, but if the study is relevant to science, use of quotes is not preferable.

10. Use proper verb tense: Use proper verb tenses in your paper. Use past tense to present those events that have happened. Use present tense to indicate events that are going on. Use future tense to indicate events that will happen in the future. Use of wrong tenses will confuse the evaluator. Avoid sentences that are incomplete.

11. Pick a good study spot: Always try to pick a spot for your research which is quiet. Not every spot is good for studying.

12. Know what you know: Always try to know what you know by making objectives, otherwise you will be confused and unable to achieve your target.

13. Use good grammar: Always use good grammar and words that will have a positive impact on the evaluator; use of good vocabulary does not mean using tough words which the evaluator has to find in a dictionary. Do not fragment sentences. Eliminate one-word sentences. Do not ever use a big word when a smaller one would suffice.

Verbs have to be in agreement with their subjects. In a research paper, do not start sentences with conjunctions or finish them with prepositions. When writing formally, it is advisable to never split an infinitive because someone will (wrongly) complain. Avoid clichés like a disease. Always shun irritating alliteration. Use language which is simple and straightforward. Put together a neat summary.

14. Arrangement of information: Each section of the main body should start with an opening sentence, and there should be a changeover at the end of the section. Give only valid and powerful arguments for your topic. You may also maintain your arguments with records.

15. Never start at the last minute: Always allow enough time for research work. Leaving everything to the last minute will degrade your paper and spoil your work.

16. Multitasking in research is not good: Doing several things at the same time is a bad habit in the case of research activity. Research is an area where everything has a particular time slot. Divide your research work into parts, and do a particular part in a particular time slot.

17. Never copy others' work: Never copy others' work and give it your name because if the evaluator has seen it anywhere, you will be in trouble. Take proper rest and food: No matter how many hours you spend on your research activity, if you are not taking care of your health, then all your efforts will have been in vain. For quality research, take proper rest and food.

18. Go to seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

19. Refresh your mind after intervals: Try to give your mind a rest by listening to soft music or sleeping in intervals. This will also improve your memory. Acquire colleagues: Always try to acquire colleagues. No matter how sharp you are, if you acquire colleagues, they can give you ideas which will be helpful to your research.

20. Think technically: Always think technically. If anything happens, search for its reasons, benefits, and demerits. Think and then print: When you go to print your paper, check that tables are not split, headings are not detached from their descriptions, and page sequence is maintained.

21. Adding unnecessary information: Do not add unnecessary information like "I have used MS Excel to draw graphs." Irrelevant and inappropriate material is superfluous. Foreign terminology and phrases are not apropos. One should never take a broad view. Analogy is like feathers on a snake. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Never oversimplify: When adding material to your research paper, never go for oversimplification; this will definitely irritate the evaluator. Be specific. Never use rhythmic redundancies. Contractions shouldn't be used in a research paper. Comparisons are as terrible as clichés. Give up ampersands, abbreviations, and so on. Remove commas that are not necessary. Parenthetical words should be between brackets or commas. Understatement is always the best way to put forward earth-shaking thoughts. Give a detailed literary review.

22. Report concluded results: Use concluded results. From raw data, filter the results, and then conclude your studies based on measurements and observations taken. An appropriate number of decimal places should be used. Parenthetical remarks are prohibited here. Proofread carefully at the final stage. At the end, give an outline to your arguments. Spot perspectives of further study of the subject. Justify your conclusion at the bottom sufficiently, which will probably include examples.

23. Upon conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print for the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects of your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form which is presented in the guidelines using the template.
- Please note the criteria peer reviewers will use for grading the final paper.

Final points:

One purpose of organizing a research paper is to let people interpret your efforts selectively. The journal requires the following sections, submitted in the order listed, with each section starting on a new page:

The introduction: This will be compiled from reference matter and reflect the design processes or outline of basis that directed you to make a study. As you carry out the process of study, the method and process section will be constructed like that. The results segment will show related statistics in nearly sequential order and direct reviewers to similar intellectual paths throughout the data that you gathered to carry out your study.

The discussion section:

This will provide understanding of the data and projections as to the implications of the results. The use of good quality references throughout the paper will give the effort trustworthiness by representing an alertness to prior workings.

Writing a research paper is not an easy job, no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record-keeping are the only means to make straightforward progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear: Adhere to recommended page limits.

Mistakes to avoid:

- Insertion of a title at the foot of a page with subsequent text on the next page.
- Separating a table, chart, or figure—confine each to a single page.
- Submitting a manuscript with pages out of sequence.
- In every section of your document, use standard writing style, including articles ("a" and "the").
- Keep paying attention to the topic of the paper.
- Use paragraphs to split each significant point (excluding the abstract).
- Align the primary line of each section.
- Present your points in sound order.
- Use present tense to report well-accepted matters.
- Use past tense to describe specific results.
- Do not use familiar wording; don't address the reviewer directly. Don't use slang or superlatives.
- Avoid use of extra pictures—include only those figures essential to presenting results.

Title page:

Choose a revealing title. It should be short and include the name(s) and address(es) of all authors. It should not have acronyms or abbreviations or exceed two printed lines.

Abstract: This summary should be two hundred words or less. It should clearly and briefly explain the key findings reported in the manuscript and must have precise statistics. It should not have acronyms or abbreviations. It should be logical in itself. Do not cite references at this point.

An abstract is a brief, distinct paragraph summary of finished work or work in development. In a minute or less, a reviewer can be taught the foundation behind the study, common approaches to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Use comprehensive sentences, and do not sacrifice readability for brevity; you can maintain it succinctly by phrasing sentences so that they provide more than a lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study with the subsequent elements in any summary. Try to limit the initial two items to no more than one line each.

Reason for writing the article—theory, overall issue, purpose.

- Fundamental goal.
- To-the-point depiction of the research.
- Consequences, including definite statistics—if the consequences are quantitative in nature, account for this; results of any numerical analysis should be reported. Significant conclusions or questions that emerge from the research.

Approach:

  - Single section and succinct.
  - An outline of the job done is always written in past tense.
  - Concentrate on shortening results—limit background information to a verdict or two.
  - Exact spelling, clarity of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else.

Introduction:

The introduction should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable of comprehending and calculating the purpose of your study without having to refer to other works. The basis for the study should be offered. Give the most important references, but avoid making a comprehensive appraisal of the topic. Describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will give no attention to your results. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here.

The following approach can create a valuable beginning:

- Explain the value (significance) of the study.
- Defend the model—why did you employ this particular system or method? What is its compensation? Remark upon its appropriateness from an abstract point of view as well as pointing out sensible reasons for using it.
- Present a justification. State your particular theory(-ies) or aim(s), and describe the logic that led you to choose them.
- Briefly explain the study's tentative purpose and how it meets the declared objectives.

Approach:

Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done. Sort out your thoughts; manufacture one key point for every section. If you make the four points listed above, you will need at least four paragraphs. Present surrounding information only when it is necessary to support a situation. The reviewer does not desire to read everything you know about a topic. Shape the theory specifically—do not take a broad view.

As always, give awareness to spelling, simplicity, and correctness of sentences and phrases.

Procedures (methods and materials):

This part is supposed to be the easiest to carve if you have good skills. A soundly written procedures segment allows a capable scientist to replicate your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order, but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt to give the least amount of information that would permit another capable scientist to replicate your outcome, but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section.

When a technique is used that has been well-described in another section, mention the specific item describing the way, but draw the basic principle while stating the situation. The purpose is to show all particular resources and broad procedures so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step-by-step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

Materials may be reported in part of a section or else they may be recognized along with your measures.

Methods:

- Report the method and not the particulars of each process that engaged the same methodology.
- Describe the method entirely.
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures.
- Simplify—detail how procedures were completed, not how they were performed on a particular day.
- If well-known procedures were used, account for the procedure by name, possibly with a reference, and that's all.

Approach:

It is embarrassing to use vigorous voice when documenting methods without using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result, when writing up the methods, most authors use third person passive voice.

Use standard style in this and every other part of the paper—avoid familiar lists, and use full sentences.

What to keep away from:

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings—save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part as entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Use statistics and tables, if suitable, to present consequences most efficiently.

You must clearly differentiate material which would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matters should not be submitted at all except if requested by the instructor.

Content:

- Sum up your conclusions in text and demonstrate them, if suitable, with figures and tables.
- In the manuscript, explain each of your consequences, and point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation of an exacting study.
- Explain results of control experiments and give remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or manuscript.

What to stay away from:

- Do not discuss or infer your outcome, report surrounding information, or try to explain anything.
- Do not include raw data or intermediate calculations in a research manuscript.
- Do not present similar data more than once.
- A manuscript should complement any figures or tables, not duplicate information.
- Never confuse figures with tables—there is a difference.

Approach:

As always, use past tense when you submit your results, and put the whole thing in a reasonable order.

Put figures and tables, appropriately numbered, in order at the end of the report.

If you desire, you may place your figures and tables properly within the text of your results section.

Figures and tables:

If you put figures and tables at the end of some details, make certain that they are visibly distinguished from any attached appendix materials, such as raw facts. Whatever the position, each table must be titled, numbered one after the other, and include a heading. All figures and tables must be divided from the text.

Discussion:

The discussion is expected to be the trickiest segment to write. A lot of papers submitted to the journal are discarded based on problems with the discussion. There is no rule for how long an argument should be.

Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implications of the study. The purpose here is to offer an understanding of your results and support all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of results should be fully described.

Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact, you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved the prospect, and let it drop at that. Make a decision as to whether each premise is supported or discarded or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."

Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work.

- o You may propose future guidelines, such as how an experiment might be personalized to accomplish a new idea.
- o Give details of all of your remarks as much as possible, focusing on mechanisms.
- o Make a decision as to whether the tentative design sufficiently addressed the theory and whether or not it was correctly restricted. Try to present substitute explanations if they are sensible alternatives.
- o One piece of research will not counter an overall question, so maintain the large picture in mind. Where do you go next? The best studies unlock new avenues of study. What questions remain?
- o Recommendations for detailed papers will offer supplementary suggestions.

Approach:

When you refer to information, differentiate data generated by your own studies from other available information. Present work done by specific persons (including you) in past tense.

Describe generally acknowledged facts and main beliefs in present tense.

The Administration Rules

Administration Rules to Be Strictly Followed before Submitting Your Research Paper to Global Journals Inc.

Please read the following rules and regulations carefully before submitting your research paper to Global Journals Inc. to avoid rejection.

Segment draft and final research paper: You have to strictly follow the template of a research paper, failing which your paper may get rejected. You are expected to write each part of the paper wholly on your own. The peer reviewers need to identify your own perspective of the concepts in your own terms. Please do not extract straight from any other source, and do not rephrase someone else's analysis. Do not allow anyone else to proofread your manuscript.

Written material: You may discuss this with your guides and key sources. Do not copy anyone else's paper, even if this is only imitation, otherwise it will be rejected on the grounds of plagiarism, which is illegal. Various methods to avoid plagiarism are strictly applied by us to every paper, and, if found guilty, you may be blacklisted, which could affect your career adversely. To guard yourself and others from possible illegal use, please do not permit anyone to use or even read your paper and file.

Criterion for Grading a Research Paper (Compilation) by Global Journals

Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals.

Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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